Chapter 8: Gravitation, Electric Charges, Fields and Energy

8-1 Introduction

In this chapter, we will be investigating electric charges, fields, and energy. This chapter marks a distinct departure from the kinds of things we have been discussing in the previous chapters, because the material becomes much more theoretical and abstract. While things like forces, motion, and even torques are relatively intuitive to visualize, things like fields and potentials are difficult, at times, to wrap your mind around. It is very important that you fully understand each concept, and spend some time and intellectual effort to truly grasp the ideas before moving on to the next one. Each idea will build on the one before it, and it is very common for students to miss one idea and then get lost along the way. By the time we reach voltage, very often we find that students don't really know what it is, but are just struggling to solve the numeric problems without understanding them. A bit of effort, a successful grasp of what each idea mean, and some imagination are often all it takes to really understand each idea and enjoy the topics and solve the problems.

Although we are focused on electric charges, fields, and energy, we will begin this section with a review of gravitation. Some physics text books have gravitation immediately preceding electricity, while others lump it in with forces and mechanics. We did cover a good deal of gravitation in mechanics, but we will do so again here simply because it is good way to bridge into the more abstract concepts in the chapter and apply the same ideas to electricity.

8.2 - Gravitation Review

The Four Forces

Up to this point, we have been discussing very concrete concepts, such as force, rotation and motion. It is now time to turn our attention to more general and more abstract notions. During the next few chapters, we will be focusing on the fundamental forces, the four forces that are the cause of all the effects in the universe. Instead of things like ropes and pushes, we will try to understand things like gravity and electromagnetism, which are the two most obvious of the four fundamental forces. As we progress, the concepts will become more general and harder to visualize, thus I implore the student to pay particular attention and be sure to understand each topic before going on to the next one. We are moving out of everyday life and into the most basic laws that govern the universe. We are beginning to look at the whole picture at once, instead of focusing on particular situations.

It was mentioned earlier that there are only four fundamental forces in the universe. It is believed that everything that occurs can be explained by the existence of the four forces. Thus far, at least, nothing has discovered that these four forces could not explain. This notion leads us to the conclusion that no other forces exist. It is important to keep in mind, however, that if something is discovered that is unexplainable through these forces, we would have to rethink our theories. Keep looking.

The four forces are: gravity, electromagnetism and the strong and weak nuclear forces. We will discuss gravity and electromagnetism in depth in the next few sections, but it remains to make at least a comment about the other two before continuing.

Everyone knows that the nucleus contains protons, or positive charges. In earlier science classes your were probably taught that like charges repel. How then, can the protons stay contained in the nucleus when they are repelling each other? It was this very question that led scientists to propose the existence of the strong nuclear force. This is an attractive force that only acts at very close range (dropping off very quickly to zero at distances further than the size of a nucleus). Thus, while the protons are being repelled by the electric force, they are also being attracted to each other by the (very) strong nuclear force. Much is still not understood about this force, and what is known is a very complicated mess of facts. For example: the nuclear force only affects certain types of particles (protons and neutrons) while other particles are immune to it's pull (electrons), protons and neutrons seem to be affected the same by this force, and other properties (such as atomic spin) seem to also play a role. Because of its complexity, we will not treat the strong nuclear force in detail. What you should remember for now is that the strong nuclear force is an attractive force that affects protons and neutrons only in close proximity.

The weak nuclear force is another force about which little is known. It turns out that there is one phenomena which the other forces cannot explain. This phenomena is called beta-decay (β-decay) and is a type of nuclear reaction where electrons or positrons are emitted from the nucleus of one element as it changes to a different element. The weak nuclear force was postulated to explain this event.

For the last few decades, physicists have been attempting to unify all of the four forces into one. They believe that there is only one force in the universe and what we are seeing is different manifestations of that one force. At the moment, they think that at very high temperatures, the four forces would "merge" back to one fundamental force and they believe this was the case when the big bang occurred. It seems that the weak force and the electromagnetic force do merge at high temperatures (it is then called the electro-weak force) and it also appears that the strong nuclear force joins them at even higher temperatures. Gravity still remains a mystery.

Newton's Universal Law of Gravitation

It is with this mystery that we begin. Gravity is the weakest of all the four forces (although after these sections, an astute student might ask how we can appropriately compare different forces) and it seems to be the strangest. It is unlike the other forces because it always attracts and appears to be linked into the fabric of space and time (!). However, it is the one force that most people are familiar with since it's workings are obvious to us.

Gravitation is the force of attraction between any two objects that have mass. Thus any two objects in the universe, no matter how big or small, will attract each other. Right now you are attracted to this paper, however, the force of attraction is drowned out by the other forces around you that are much stronger. If it was only you and this paper in the entire universe (a lonely proposition), you would see the paper float towards you. Gravity also has an infinite range, although it weakens with distance. Thus even if you and the paper were on opposite sides of that theoretical, lonely, universe, you would end up finding each other.

Gravity was first explained satisfactorily by Isaac Newton, when he (supposedly) saw the apple fall. It was then that he realized that the apple falling and the moon orbiting could be caused by the same effect, one of attraction between two masses. He later came up with a mathematical relation stating:

> "The force of attraction between two objects is directly proportional to the product of the two masses and inversely proportional to the distance between their two centers squared."

mathematically:

$$\underline{F} = -Gm_1m_2/\underline{r}^2$$

Where m_1 and m_2 are the two masses, r is the distance between the two centers of mass and G is a constant of proportionality called the Universal Gravitational Constant. Today we know G as having the value of:

$$G = 6.67 \times 10^{-11} \ \mathrm{N.m^2/kg^2}.$$

There is a negative sign in the above equation because gravity is attractive and because it is a vector, the equation needs a negative to yield the proper direction. Let us do one simple problem involving Newton's Universal Law of Gravitation (NULG).

Example 8.2.1

What is the force of attraction between the earth and the moon? Where is this force acting?

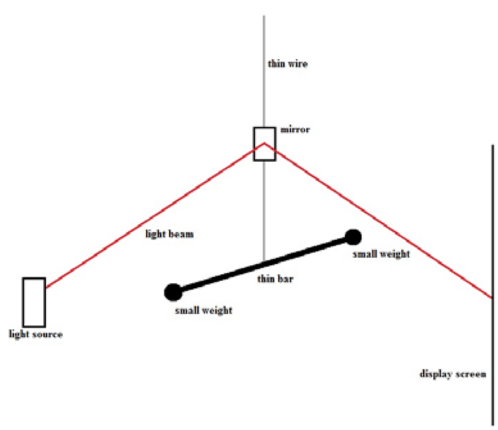
Once again, I would like to stress the importance of Newton's formulation of a theory of gravity, in terms of what it meant to the

Aristotelian theory of motion. As was mentioned in a previous chapter, by developing one law for both the heavens and the earth, Newton dispelled the myth of there being separate, corrupted and non-corrupted universes. He united heaven and earth together for the first time by saying they are subject to the same laws. In doing so, it can be said that he was making a first tentative step towards a universe that is governed by reason, not superstition. If the heavens are a part of the earth, and no longer a spiritual realm, they are not to be feared. Notice that this means that religions whose deities preciously had their gods seated in the heavens, had to move them to another location. Nowadays, we have a different dichotomy, a separation of the physical and the spiritual world, where the spiritual world is no longer a part of "the universe" and thus no longer subject to the rules or investigations of science. Newton's Universal Law of Gravitation was just that, the first Universal law of the scientific age.

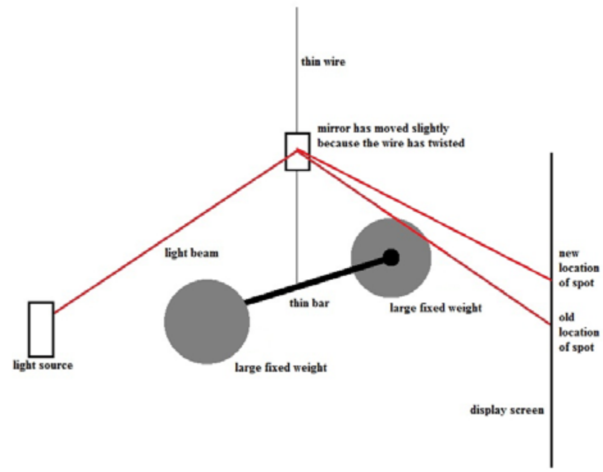
The Cavendish Experiment

Before we proceed with our discussion of gravity, we should take a minute to discuss the measurement of the constant G. When Newton first formulated the Universal Law of Gravitation, he was only able to make a rough guess at the value of the Gravitational Constant. If we consider the equation $F=-Gm_1m_2/r^2$ for a moment, we can see that to measure G we would need to know all the other variables. We can easily measure mass and distance between two object in a lab, but what about force? Consider two object that can be easily brought into a lab, say masses of about 100 kg (roughly 220 lbs) and consider a reasonable distance of say about 1 m (remember that the distance is between their centers of mass). This gives a force of about 10^{-7} N (or on the order of 10^{-8} lbs or about a hundred millionth of a pound). Certainly we can see that this is not an easy measurement to make in a lab. After all, how would you measure a force this small? We can get better by increasing the masses or decreasing the distance, but it still remains on the order of millionths of a pound.

The first true and accurate measurement of G came in 1798 by a man named Henry Cavendish. He accomplished this by setting two small masses on the end of a long, thin rod and suspending them on a wire from the center of the rod. On the wire he attached a mirror and bounced a beam of light off the mirror to a distant screen.



Once the setup was established, he placed two larger masses which were fixed in place next to the small masses. This caused the rod to rotate because of the gravitational attraction between the masses and it caused the wire to twist until its internal forces balanced the gravitational forces. When the wire twisted, so did the mirror, sending the light beam to a different spot on the screen.



By knowing how much the beam moved, he could determine how much the wire twisted and by knowing the properties of the wire he could determine the torque caused by the gravity acting at each end. From this torque he could determine the force of gravity between the masses. The success of this experiment cannot be overstated. What Cavendish managed to do was to find a way to accurately measure forces as low as a millionth of a pound. Whenever I discuss this experiment, it impresses upon me the ingenuity and creativeness necessary for a good scientist. To me, thinking up, designing and carrying out this experiment was nothing short of brilliant. Later, this same design was used to measure other constants.

The Gravitational Field

The next step in understanding gravity is to understand the concept of a gravitational field. The idea of a field is very abstract and rarely understood by beginning physics students (and even some advanced ones). However, it is an essential concept in all of physics. Often I see students struggling with fields and just blundering their way through problems without any true understanding of what is going on. If you intend to be a serious physics student, I strongly suggest that you try your best to understand this concept and make sure each point makes sense before you move on (if you are not interested in being a serious physics student, I suggest you just blunder through, feebly understand what is going on and try to get a C-).

A field is a measure of the force an object can create. Often it is described as the area around the object where its force can act, although this is a little confusing. A field is not a space, it is an effect that could occur. Since we are dealing with gravity, we will use the gravitational field as an example. The student should remember that gravity is just an example and the concept of a field can be applied to other forces. What a field is is interwoven with the reason why we need the concept of a field to begin with. Imagine that we wanted to compare the gravity caused by two different objects, say the Earth and Jupiter. Using the force of gravity, we simply cannot compare them. Force is something that exists between two objects. One object cannot have a force. It is nonsensical to talk about the force of gravity of the earth. We can talk about the force of gravity between the earth and the sun or between the earth and a cat, but not "of the earth". Therefore, it is handy to have some concept related to the gravity of ONE object. This is a field.

Think back to our example of the Earth and Jupiter. We could compare their gravities if we put the same object on each planet and measured the force of the planet on the object. In fact, if we had one standard object, we could compare the gravity of any two objects in the universe. Although this is one way to accomplish this, it is not the easiest or most practical. Instead, we need a concept that divorces the second object from the first altogether. The field is found by finding the force between object 1 and the standard object and then dividing out the influence of the standard object. The influence of the standard object is given by the aspect of the object that is affected by the field.

Although what I have said above is fairly general, it may not make sense at first. Using gravity as our example, we would find the force in the following way: place a standard test object on the planet, measure the force between the test object and the planet and then divide that force by the mass of the test object. This would give us the field of the original object. Mathematically,

$$\text{Gravitational Field of Object 1} = \frac{\text{Force Between Objects 1 and 2}}{\text{Mass of Object 2}}$$

or,

$$g_1 = F_{12}/m_2.$$

Notice that we have used the small letter "g" for the gravitational field. This is no accident. It turns out that the gravitational field and the acceleration due to gravity are one and the same. However, this only works for gravity (the electric field is not the acceleration due to the electric force, for example) and this only works because gravitational mass and inertial mass appear to be the same (recall the discussion of the two types of masses earlier - the student should be able to figure out the reasoning).

There is one other nice thing about the field, and that is that it can be calculated theoretically without having to actually place an object anywhere. Consider NULG:

$$F_{12} = -Gm_1m_2/r^2$$

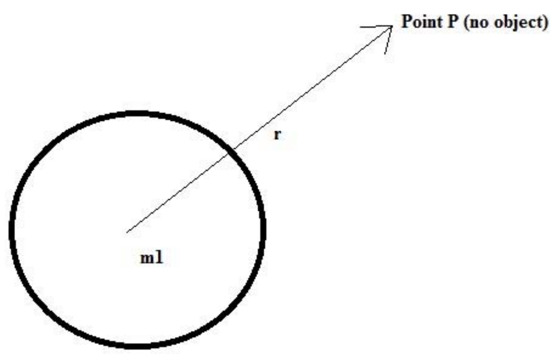
and consider

$$g_1 = F_{12}/m_2.$$

combining we get:

$$g_1 = -Gm_1/r^2.$$

This is the field of object 1. Notice, however, that the variable r has lost its meaning. It used to be the distance between the centers of mass of the two objects. It now means the distance to the point in question. In other words, the equation above give the field of object one at a distance r from its center.



Before we continue and do problems regarding this concept, there are four points I wish to discuss.

The first is simply a reminder about what a field is. A field is a measure of the gravity (or other force) the object would create if something was placed in its area of effect. It is the gravity of one object. It is debatable whether or not fields exist. After all, it is only a measure of what the object could do if something else were there.

Secondly, remember the defining equation of a field. It is one of the simplest and most often forgotten equations in basic physics. The field is the force divided by the mass.

$$g = F/m.$$

Thus, the force is the field times the mass.

$$F = mg.$$

The fact that the force is the field times the aspect of the object in the field that reacts to the field is true for all fields. For example, in electricity, $F = qE$. Where q is the charge and E is the electric field the charge is placed in.

The third point is rather theoretical. It has to do with the definition. We have referred to the object placed in the field to measure it as the standard object. There is another requirement we did not mention. This object must be very small and is called the test mass (or test object). The reason for this is that large objects will actually change the gravitational field they are placed in. Therefore, we imagine the test object to be infinitesimally small so that it has no effect on the original field.

Finally, I would like to take a minute to discuss the concept of a field equation. We have been talking about the field at a particular location, but it would be handier to discuss the field around an entire object. We do this with a field equation; one simple equation that mathematically describes the field all around the object. Then, if we wanted to know the field value at a location, we could just plug in the numbers. Obviously, the only variables in such an equation should be the location of the point in question. All the other quantities should be known. For example,

$$g = Gm/r^2$$

is the field equation for a planet. G and m must be known, then we can plug in a value for r to find the field at point r. In physics, often we are in search of a field equation. Once that is found, the rest is assumed to be easy (it often isn't). The field equation for a planet is simple (actually, the above equation only applies to a uniform, spherical planet, with noting else in the near vicinity), but imagine the field equation for the solar system. At each point there would be nine planets plus the sun and numerous comets and asteroids all pulling at the test object. Moving the test object around would greatly change the force in a complicated (non linear) way. Besides that, the planets are moving at different rates. Such an equation would me monstrous indeed (and essentially out of reach to imagine it as one formula). We will discuss field equations in more detail when we discuss the electric force, since there exist many different combinations of charged objects that can create complicated electric fields.

With the proper substitution of variables, this equation above should give us the value of g at the surface of the earth (9.8 m/sec^2). However, we should point out a few obvious problems with that value. First, we would need to use an average value for r, since different locations are different heights above sea level. In fact, since the earth is not actually spherical, but rather bulged around the equator, further adjustments would be needed to give an accurate response. Secondly, the equation above works only for a sphere of uniform density. Gravity at a specific location can vary due to the density of rock beneath the spot where you are standing. Thirdly, since the earth is a non-inertial reference frame, fictitious forces are needed to properly achieve an answer. After saying all that, the average value of g at the surface of the earth is 9.8 m/sec^2, and while it does vary slightly, for most common problems, the variation is negligible.

Let us take some time to practice using these simple equations.

Example 8.2.2

Given that the gravitational field strength at sea level is 9.803 m/sec^2 and at Java it is 9.782 m/sec^2, what is the distance from the center of the earth to (a.) sea level and (b.) Java?

Example 8.2.3

Find the gravitational field strength on the surface of Jupiter. What would be the force of gravity on a 70 kg object on the surface of the planet? How long would it take that object to fall 10 m?

The last thing to mention about gravitational fields is that they are vector quantities. The actual equation for a gravitational field is:

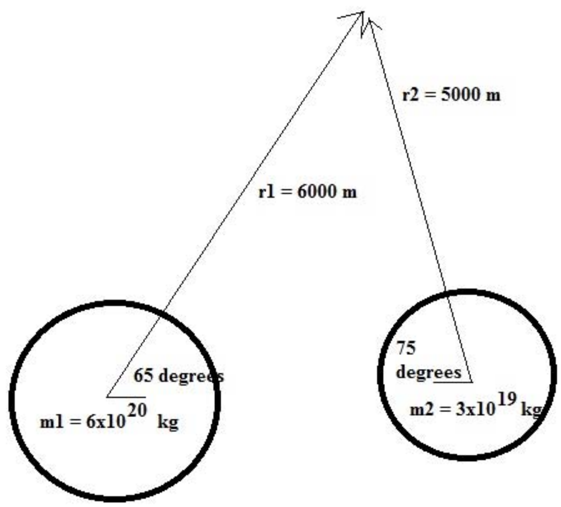
$\underline{g} = \underline{F}/m$.

Which shows you that the gravitational field, $\underline{g}$ is found by dividing another vector ($\underline{F}$) by a scalar (m). This means that the gravitational field is in the same direction as the force (which means pointing towards the center of mass of the object causing the field). It is interesting to note that one single equation can give the field strength at any location, but the direction must be figured out logically. There is a method involving vectors that will give the direction, but for now we will gloss over it.

Since fields are vectors, they add as vectors. Consider the example below.

Example 8.2.4

Find the field strength and direction for the gravitational field at point P caused by the two masses shown.



Gravitational Potential Energy

There is one final topic related to gravity that we should discuss: gravitational potential energy. In the past we have discussed gravitational potential energy and defined it as $\Delta U=mgh$. However, you may recall that we stated that this equation was only valid if g remained constant. We would like to have some way of determining the potential energy in a situation that does not have a constant g. Using calculus, we can derive the following result:

$$U = -Gm_1m_2/r$$

Where this equation gives the gravitational potential energy of an object (m_2) in the gravitational field of another object (m_1) at some point r. Notice how this equation is not a change of potential energy, but rather the potential energy itself. When we first discussed potential energy we said it was only possible to discuss it in terms of changes and that comment is still true. The above equation is actually the change in potential energy from the point in question out to infinity. This technique is common in physics, when we have a concept that involves changes, we often standardize it using an infinite distance as one of the reference points. The above equation should actually be written as:

$$\Delta U_{r\infty} = -Gm_1m_2/r$$

Which says that the change in potential energy between the point r (a distance r from the center of the object causing the field) out to infinity (where the gravity can be taken to be zero) is equal to negative G times the masses divided by the distance r.

> Example 8.2.5
>
> Find the potential energy of the space shuttle (mass = 50000 kg) when it is in orbit 300 miles above the surface of the earth.

Physically, this potential energy is the energy stored in the bond between the two masses. By finding the energy between r and infinity, you have actually calculated the amount of energy necessary to remove the object from its orbit and send it out to infinity. If we wanted to send a space probe from the surface of the earth out to deep space, this is the energy we would have to supply it (not counting the extra energy needed to overcome air resistance). Thus the energy you have just calculated is the energy needed for the space shuttle to leave orbit and reach deep outer space. This same concept is also used in electricity, it is the amount of energy stored in the bonds between nuclei and electron, or as it is called, the ionization energy of an element.

> Example 8.2.6
>
> What is the “ionization energy” of the moon? In other words, how much energy would it take to remove the moon completely from the earth’s orbit?

Another interesting aspect of this concept is that it can be used to give you the escape velocity of a planet. The escape velocity is the velocity at which you would have to throw something up in the air and have it escape the gravitational pull of the planet. If we simply imagine throwing something up in the air, we would expect it to eventually fall back down. The faster we throw something, the longer it would take to fall back down and the higher it would go. But as it rises, the gravity of the earth would get weaker. Imagine that you could throw something fast enough so that it would rise to the point where the earth's gravity went to zero. It would never come back down. Since gravity is infinite in range, that height would be at infinity. The speed you would have to throw an object for it to reach infinity is called the escape velocity. What goes up at that speed need not come down! In other words, the gravity of the planet would get weaker faster than the object would slow down, causing to never stop and having it end up at infinity. Think of it this way: finding the energy of an object at the surface of the earth would tell you how much energy is needed for it to leave the surface and escape the planets pull of gravity. That energy could be supplied in the form of kinetic, thus there is an associated velocity.

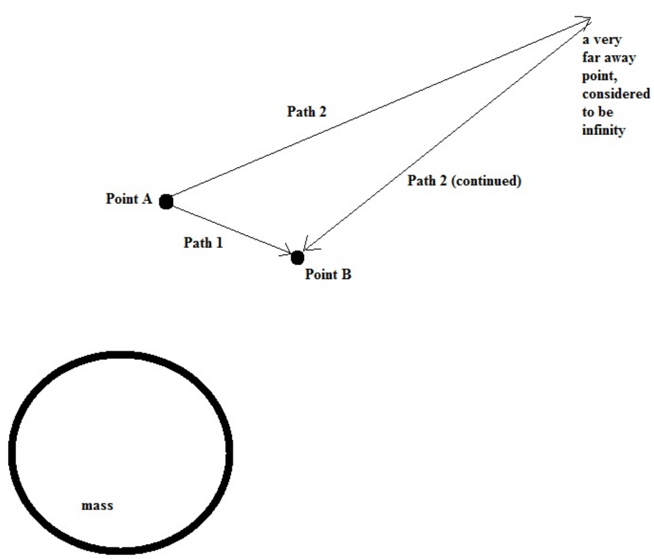
> Example 8.2.7
>
> Determine the escape velocity of the earth.

One last comment about escape velocity should be made before we move on. This is the velocity an unpowered object would need to break free of the earth. Rockets are not launched at escape velocity, since they are powered along their trip.

The concept of potential energy between a point and infinity can be extended to find the change in potential energy between any two points in a field, since energies are additive. For example:

$$\Delta U_{AB} = \Delta U_{A\infty} + \Delta U_{\infty B} = \Delta U_{A\infty} - \Delta U_{B\infty}$$

schematically:



In this diagram, we are saying that if want to calculate the energy as an object moves from point A to point B, we could use path 2 because we have a simple equation for that, while we do not have any equations for the energy along path 1. The two paths are equivalent.

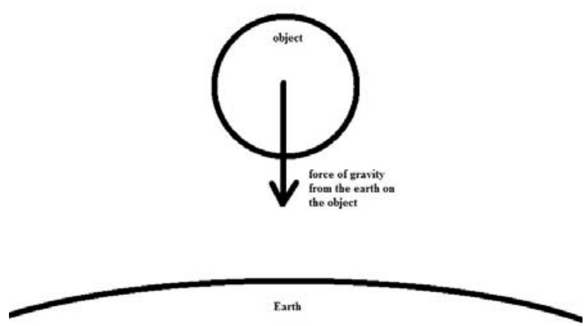
Example 8.2.8

How much energy is needed for the shuttle to change from a 300 mile orbit to a 400 mile orbit? How much energy was needed to put the shuttle in its 300 mile orbit in the first place?

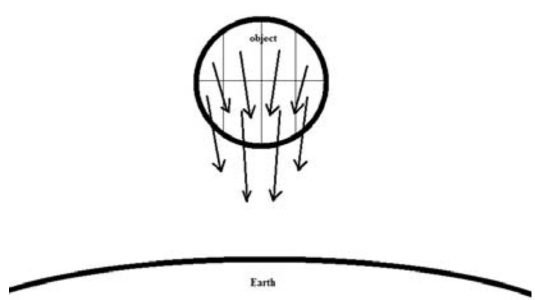
Understanding the connections between energy and gravity and being able to read a problem like the one above and determine what is being asked, then determining how to solve it are very important physics skills.

The Shell Theorem

Before we close this section, let me make a rather extended comment on one assumption that has gone (probably) unnoticed throughout this discussion. When we discussed gravity, we have been looking at it as seen in the diagram below:



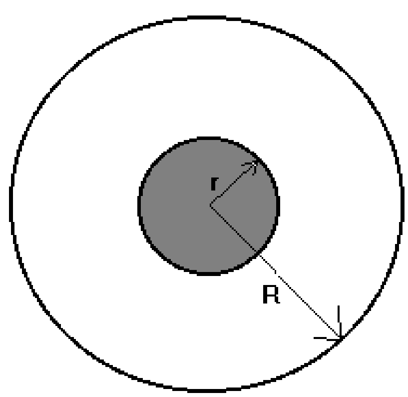
When in reality, we know that this is not the case. The object is made up of many tiny masses, all of which pull at each other and all of which are pulled by the earth in a different direction and at a different value. The diagram below shows what is meant by this in a simplified way.



Here, the object is drawn as 8 small objects and each part is being pulled towards the center of the earth. Notice how they all pull slightly off from straight down, because the center of the earth is not straight down from every point! Also notice that the forces on the top are weaker because they are further away from the center of the earth. What we have been doing in this section is to simplify this by saying that all the mass of the object is concentrated at the center of the object (the center of gravity). In other words we have been using point masses (an infinitely small object that contains all the mass) instead of evaluating the entire object.

But is that an acceptable assumption? Is it correct to say that all the forces acting at the center is exactly the same as the (almost) infinite number of forces acting on an object? This original assumption bothered Newton quite a bit as he was formulating his theory of gravity. Eventually he proved that it was accurate (provided that the object was a perfectly homogeneous sphere) by proving what is now called the "Shell Theorem".

The shell theorem gave us two important and interesting results. The first is that a uniform sphere behaves exactly like a point mass. The second is that the gravity <u>inside</u> a uniform sphere is governed only by the mass between the point of consideration and the center. The mass "above" it has no effect.
This effect can be best seen by use of a diagram:



Suppose you wanted to calculate the force of gravity on an object inside a planet at radius r. The shell theorem tells us that standing <u>inside</u> a planet at radius r is the same as standing <u>on</u> a planet with radius r (provided the densities are the same). The mass between r and R has no effect on an object located at r. Actually it does have effects, but they all cancel out.

The student should ponder the two consequences of the shell theorem and see if they make good logical sense (don't bother to try to prove them mathematically, just see if you can explain why they are true).

Why Bother With Fields at All?

Before we close the chapter, some time should be spent articulating the reasons that we bother with the concept of fields at all. Fields are important for a number of reasons, and many of the problems in more advanced physics deal with fields. The reason is that the field is the simplest concept that can lead to complete information about a situation. If we know the field, then it should be easy to determine the force on another object(F = mg). Once we have the force, the acceleration can be found (F = ma). Once the acceleration is found, the motion of the object can be determined. Since a field describes the gravity around an object, it can be used to determine the motion of any other object placed in the area. Although this sounds direct, it is often difficult since fields themselves can be complex. However, it does allow for a method to arrive at an answer.

The student should remember that a field describes two things: 1.) the "gravity" of one single object and 2.) what the effect would be on an object placed at that location (without ever having to actually go and place an object there to determine it). Although sometimes hard to understand, fields are essential and useful concepts.

A Short Summary of Gravitational Concepts

	Involves	Equation (for spherical objects)	What it means
Gravitational Force	Two Objects	$F = Gm_1m_2/r^2$	Force between any two objects
Gravitational Field	One Object	$g = Gm_1/r^2$ or $g=F/m$	Gravity of one object - the gravity that would be there if an object was placed there
Gravitational Potential Energy	Two Objects	$\Delta U = Gm_1m_2/r$	The energy in the bond between the two objects - the energy required to separate the two objects completely from one another

Practice Problems

Example 8.2.9

Two cars pass each other on the street, one with a mass of 2000 kg and the other with a mass of 1500 kg. At their closest point their centers of mass are 4 m apart. What is the gravitational force between them?

Example 8.2.10

A 20,000 kg space ship passes a planet of mass $7x10^{25}$ kg and needs a force of 42,000 N to keep it in its intended path. How far away from the center of mass is the ship?

Example 8.2.11

The cavendish experiment is carried out and the following measurements are made:

Mass of small balls = 10 kg
Mass of large balls = 300 kg
Mass of rod = negligible
Distance between small and large centers at equilibrium: 0.45 m
Length of rod = 2 m
Thickness of wire = 2 mm
Tension measurement of wire: 0.0002 N per degree of twist, with force measured at edge

With this information, calculate the degree twist of the wire, and the distance that the small balls moved toward the larger balls once they were put in place.

Example 8.2.12

Determine the gravitational field strength at a point 1 m away from a 100 kg object.

Example 8.2.13

A 20 kg object is placed in a gravitational field of 56 m/s^2. What is the force acting on the object?

Example 8.2.14

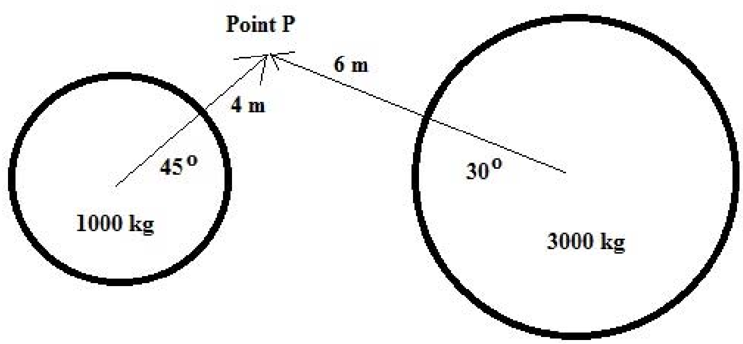
A deep space probe discovers a strange area of space where the gravity appears to following the following equation:

$g = (16\ s^{-2})r - (38\ m^2/s^2)/r$

where r is the distance from the probe to the point in question. What would be the force on a 10 kg object at 6 m and 12 m?

Example 8.2.15

What is the gravitational field strength at point P in the diagram below?



Example 8.2.16

Two large, twin stars are in orbit around each other at a distance of $6x10^{11}$ m. If both stars have a mass of $6x10^{29}$ kg, what is the energy in the gravitational bond between them?

Example 8.2.17

If we wanted, for some strange reason, to move the moon to a higher orbit, 1.5 times its present orbit, how much energy would it take?

Example 8.2.18

Consider a 50 kg object at the center of a coordinate grid, with all distances marked in meters. Find the following:

a.) The force on a 6 kg object placed at (3,5)
b.) The gravitational field at (3,5)
c.) The gravitational field at (4,4)
d.) The energy required to completely remove the 6 kg object from position (3,5)
e.) The energy required to move the 6 kg object from (3,5) to (4,4)

Example 8.2.19

A space ship is in orbit 100 km above the surface of the moon when a clumsy astronaut drops a wrench, which falls to the surface. What is the speed of the wrench on impact?

8.3 - Electric Charges and Forces

Since we now understand gravity, it is time to apply this new found knowledge to the second of the four forces; the electromagnetic. The electromagnetic (EM) force manifests itself in two fashions; as the electric force in some circumstances and as the magnetic force in others (occasionally it will be both simultaneously). As the electric force, which we will discuss first, it is the most important force in the universe (a debatable statement). It is true that each force is necessary for the universe to exist at all, however, when I refer to the em force as the most important, I am saying this because it is the force that <u>most directly</u> affects our lives. The em force is responsible for all chemical reactions (including our own digestion), it is responsible for giving matter its solidity, and for holding the electrons in their orbits.

If the student will permit, I would like to take a side track and discuss how the em force affects our perceptions of matter itself. Firstly, consider trying to touch an object. Are you really in contact with that object? The answer is no. When you touch something, you are actually bringing the atoms in your hand close to the atoms in that object. Around the atoms swirl electrons. As the electrons in your hand get close to the electrons in the object, they repel. When you hold your pencil, there is still space between your fingers and the pencil. If you try to push harder, the electrons get closer and push apart harder. You can never make contact. You are not really in contact with the chair you are sitting in. You are actually floating on a thin vacuum layer between your atoms and the atoms in the chair. Dr. Richard Feynman, a famous modern day physicist, was once asked if scientists would ever invent an anti-gravity machine. He said there is one, called the electromagnetic force which causes everything to float on everything else. The second point I wish to make is about the illusion of solidity. We know that the nucleus is a very, very tiny percentage of the atom, and the electrons are even smaller. Thus most of the atom is empty space. In fact, if we were to take all the empty space out of the earth, it would only be about 6 miles wide. Why then do objects look solid? The answer lies in the electromagnetic force. We see things as solid because of the way light (which is an electromagnetic phenomena) interacts with the electrons on the outside of the material. Matter is not solid, it just looks that way. I give you these examples to think about because they show how important the em force is to us in the world around us. They also show that the effects of the em force are numerous but not obvious to the untrained eye. Besides those reasons, they also show us that there are marvels unbelievable in nature if we only know how to look for them.

Electric Charges and Coulomb's Law

But let us begin at the beginning. The electrical force is the force of attraction or repulsion between two charged objects. The charge of an object is a physical property that describes how the object reacts to the electric force. Charge is very similar to gravitational mass, in fact charge is the "electrical mass" of an object (we really don't know what it is, but we can measure its effect and give it a name and that makes us think we are smart). All matter has charge, although it is possible to have a charge of zero. You have probably learned that electric charges are either positive or negative (a description that was devised by none other than Benjamin Franklin), and that protons have a charge of +1 while electrons have a charge of -1. The SI unit for measuring charge is the Coulomb (C) and its actual definition is rather confusing. A Coulomb is not a fundamental unit, it is actually a derived unit. At this point in time the student would not be able to understand the derivation, since it involves units that have not yet been introduced. One Coulomb is a very large amount of charge, and so a smaller unit is often used. This other unit is called an elementary charge, usually abbreviated with an e. The conversion between the two is very important and will be used often.

$$1\ e = 1.60 \times 10^{-19}\ C.$$

The elementary charge is the charge on one proton or the negative of the charge on one electron. We can see from the above conversion that an elementary charge is very small in relation to a Coulomb.

You probably also remember that it is charge difference in an object that matters, not the number of charges. What this means is the difference between the positive and negative charges in an object determines its overall charge. An object with 50 positive charges and 45 negative charges is the same as an object with five positives and no negatives.

Another reason why the elementary charge is so important is that charge appears to be quantized. When something is quantized, it means that it exists in discrete bundles. The fact that charge is quantized means that we can find an object with a charge of 5e or 30001e, but never with 2.6e or 0.4e. Charge must exist in integer multiples of the elementary charge (as another example of a similar system that is quantized, consider money: the smallest (quantized) unit is a penny). Since the elementary charge is so small, however, it would almost appear as if any value of charge were possible. The quantization of charge only holds to the particle level (electrons, neutrons and protons). Inside the protons and neutrons are what are called quarks which seem to have fractional charge (2/3, or 1/3 of an e).

Another important aspect about charges is that in any physical situation, charge must be conserved (just as mass, energy and momentum). Charges can then be neither created nor destroyed, which is almost a matter of common sense. If you charge an object (like shuffling your feet across the carpet) you do not create charges, you simply rearrange them. In nature, the conservation of charge can be seen in the strange happenings of particle and nuclear physics. For example, it is possible for a gamma ray (a particle of light) to spontaneously turn into an electron and an anti-electron. Notice how charge is conserved. The gamma had no charge and the electron and anti-electron cancel each other out (-e and +e charges, respectively). It would be impossible for a gamma to change into two electrons, since that would violate the conservation of electric charge. Another example is the nuclear reaction of beta-decay mentioned earlier. In β^- decay, an element moves up one atomic number while keeping its mass number the same (ex: Tc-100 can change into Ru-100). For this to happen, a neutron must disappear and a proton must take its place. However, conservation of charge prohibits this unless an electron (or other negative particle) is also formed. In β^- decay, an electron (also called a beta particle) comes flying out of the nucleus.

Electric charges attract and repel each other just as two masses attract each other due to gravity. However, instead of the mass determining the strength of the force, the charges of the two objects determine the strength and direction of the force. It is important to note that charge and mass are parallel concepts. In fact, the charge could be called the "electrical" mass of the object, or the (gravitational) mass could be called the gravitational charge of the object. The electrical force is determined quantitatively by using a law that resembles NULG:

$$F_E = Cq_1q_2/r^2$$

Where F_E is the electric force, q_1 and q_2 are the charges of the two objects in question and r is the distance between the two charges. The above formula is the "modern" version of what is called Coulomb's Law, named after Charles Coulomb. Coulomb devised this law by using a "torsion balance" which was almost identical to Cavendish's experiment to measure G, except that this time charged objects were used. Historically, Coulomb's Law was written as:

$$F_E = \frac{1}{4\pi\varepsilon_o}\frac{q_1q_2}{r^2}$$

Comparing the two versions, we see that:

$$C = \frac{1}{4\pi\varepsilon_o} = 8.99 \times 10^9 \text{ N.m}^2/\text{C}^2$$

Where

ε_o = the permativity of free space

$\varepsilon_o = 8.85 \times 10^{-12}$ $\text{C}^2/\text{N.m}^2$

Obviously, the two versions of the equation are equivalent, however, some books and some professors use the older version. The reason for this is that while the new version looks very much like NULG and is easier to remember, the older version fits in more nicely with some advanced concepts that link electricity and magnetism. The constant factor (ε_o), is an electrical property of a vacuum. The beginning student need not concern themselves with its actual meaning.

Coulomb's law is used in precisely the same manner as NULG, so let us begin some problems to practice using it. As you will see, these problems require that you remember some basic concepts from chemistry.

> Example 8.3.1
>
> In an average lightning flash, about 10 C of charge are exchanged between the ground and the clouds. How many electrons are exchanged?

> Example 8.3.2
>
> Compare the number of electrons in the lightning flash with the number of electrons in a human hair. Assume the hair has a mass of $1\text{x}10^{-4}$ g and is composed primarily of carbon.

Besides giving you practice, this past example (although a crude approximation) should give you some inkling about the phenomenal number of charges present in everyday objects.

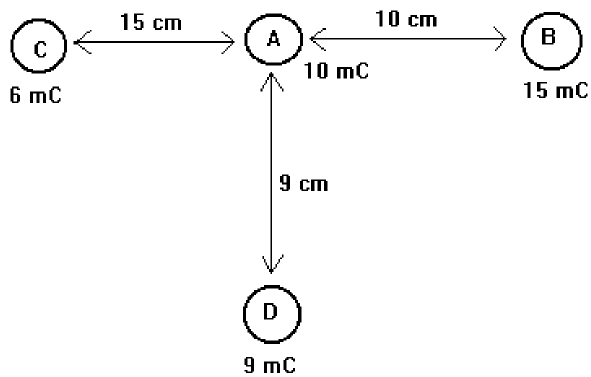
> Example 8.3.3
>
> Imagine you could take all the electrons from the hair in the last example and pull them all to one end of the hair. If all the remaining positive charges were pulled to the other end, what force would be needed to hold these two charge bundles apart? Assume the hair to be 12 cm long.

Although what was asked in the above problem is ridiculous and impossible, it was meant to demonstrate the incredible about of force, power and energy that is locked up in normal, everyday objects. The electrical force is present everywhere, it is just unusual for us to notice its effect if we don't look carefully.

The next example demonstrates that when many electrical forces are present, we need to recall the vector nature of forces.

Example 8.3.4

Calculate the net force experienced by charge A in the set up below.



Practice Problems

Example 8.3.5

If one mole of hydrogen were completely ionized, what would be the charge?

Example 8.3.6

What is the force of attraction between one proton and one electron separated by a distance of 2 mm?

Example 8.3.7

How many Coulombs of charge are contained in the electrons of 10 g of copper?

Example 8.3.8

If a 6 C charge is placed at the origin, what is the force on a 2 C charge placed at (2 m, 6 m)?

> Example 8.3.9
>
> A 5 C charge is placed at the origin, a 3 C charge is placed at (2m, 0) and a 4 C charge is placed at (0, 4 m). What is the force (magnitude and direction) on a 2 C charge placed at (3 m, 3 m). Hint: answering in vector component notation would make things easier.

Basic Electrical Knowledge Regarding Materials

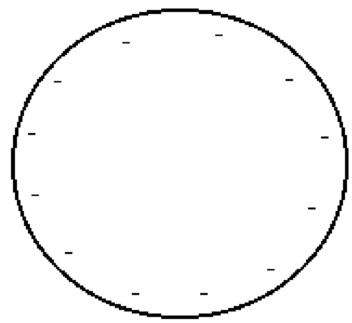
Before we move on to discussing the electric field, there are a number of interesting and basic concepts regarding the behavior of charges and charged objects that the student should be familiar with. The topics are generally covered in introductory science courses, but it might be helpful to quickly refresh your memory regarding these concepts.

Materials are generally classified as conductors or insulators (although there is no distinct line between the two, rather there is a continuous spectrum of materials running from extreme conductors to extreme insulators, with most materials falling somewhere in between). A conductor is a material that allows charged particles to move freely inside of it and an insulator is a material that hinders the free movement of charged particles. When we discuss moving charges in a material, we usually discuss electrons, since it is rare to have a free proton floating around a material. Thus conductors and insulators are often described in terms of how they allow electrons to move.

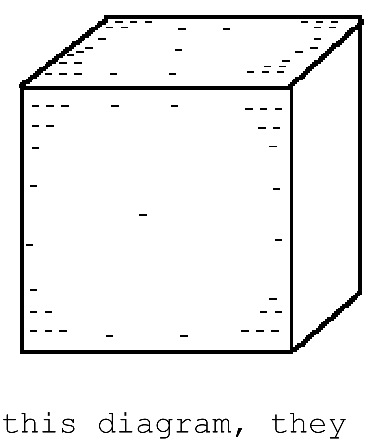
Since conductors allow charges to move easily, once we put a charge in a conductor, it will simply float around until it reaches an equilibrium point. Because of this, charges can easily leave a conductor if conditions permit. Once a conductor is charged, for example, the charge can leak off into the air. For this reason, conductors loose their charges easily while insulators tend to hold their charges for longer period of time. A good example of these principles is the case of a typical, electrical wire in your home. The inside is metal, allowing charges to flow freely and quickly, while the outside is plastic, which does allow some charge to flow through, but so slowly and in such a restricted manner that it would never pose a danger to someone or something in contact with the outside.

The terms conductor and insulator have nothing to do with the ability to <u>have</u> a charge, only with the behavior of the charge once it is on the material. A common misconception is that insulators cannot be charged. It is possible to have an electrically charged piece of rubber. However, because the charges do not move freely, it is more difficult to charge the rubber than it would be to charge, say aluminum. Once a charge is on the rubber, it would retain that charge better than a metal since the charges would have difficulty leaking off. This can be seen in many of the common electrical demonstrations done in a physics class. When a teacher wants to demonstrate charges, they often use a rubber or glass rod, so the charges do not leak off before the demo is over. When using a metal object, the object is often kept continually charged (as in the case of a Van DeGraff generator) or is given a very large charge so that if some leaks off, the demonstration will still work. The humidity in the room is often a big factor in the amount of time an object will retain a charge, since humid air is a good conductor, thus the charges can easily enter the air on a humid day. Humidity has been the cause of many failed demos in a physics class.

Because conductors allow for free passage of charges, on good conductors we get an interesting effect. When a charge is placed on a conductor, it will move so that all the charges reside on the outer surfaces. Consider pumping millions of electrons into a solid metal sphere. They will all repel each other and try to get as far away from each other as they can, as shown below.



On a regular, symmetric sphere, the charges will spread out evenly on the surface (and will even be pushed off if possible, as they try to get away from each other). However, on other objects, such as a metal cube, their attempt to get away from each other will result in a non-uniform distribution of charges on the surface. It turns out that charges will migrate to the furthers extremities and get as far away from the center as possible. This means that charges will generally conglomerate at points, with fewer charges in other areas. The diagram below illustrates this, but charges were not drawn on the side for clarity.



When students first see this diagram, they often wonder why the charges would gather so close together at the corners. The reason is that although this puts them very close to each other, it allows them to get as far away as possible from as many other charges as possible.

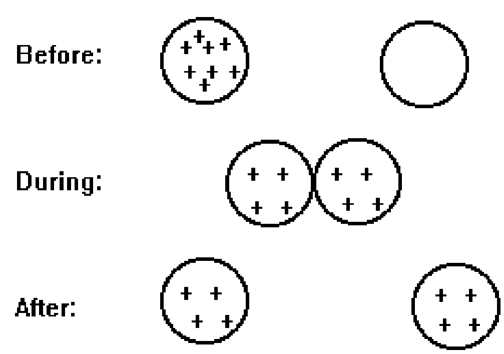
The student should keep this behavior in mind in later discussions of lightning and electric fields.

Before we move on, it should be mentioned that although we have been talking about putting a negative charge on these objects, it is also possible to have the same effect with a positive charge. Generally, charging an object positively requires not putting positive charges on the object, but rather removing a certain amount of negative charges. Thus, in the example of the sphere, if we remove negatives we leave positive "holes" and the electrons in the material will rush to fill these holes, leaving gaps elsewhere. In the sphere, electrons will rush from the surface to fill in the gaps in the middle, leaving a uniform positive charge on the surface.

We have been talking about charged objects during this chapter, but have yet to discuss how an object can become charged. It is important to remember that all objects have charges (the electrons and protons in their atoms), however if the number of positive and negative charges are the same, the object is neutral. This is the normal state of most objects. When we discuss charging an object, we are really discussing creating a charge imbalance between its positive and negative charges. Obviously, a positively charged object has more positive than negative charges and visa-versa. Creating this imbalance is (in general) a matter of either taking electrons away from an object (leaving it positive) or adding extra electrons (and making it negative). There are three ways of charging an object: friction, conduction and induction.

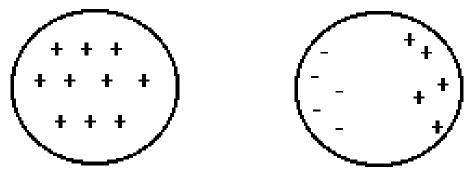
Charging by friction is what occurs when two objects are rubbed together. It is possible, depending on the materials and their ability to lose or gain electrons, to rub two objects together and have the electrons from one be transferred to the other. This is the explanation for that annoying shock you receive after you shuffle your feet across a carpet.

Conduction refers to the process of charging an initially neutral object by bringing another charged object in contact with it (or close enough for the charges to "jump" from one to the other). Since the charges are repelling each other on the charged object, they will move to the neutral object to escape each others forces. Once there, they will establish a new equilibrium. The diagram below shows a before and after sequence of charging by conduction.



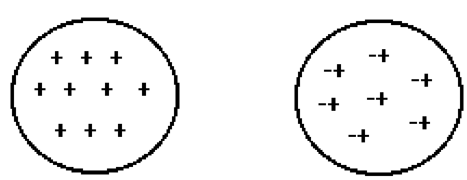
Of importance to note is that after the two objects are separated, both are still charged. Also, conduction applies to any situation where there is a transfer of charges. It is not necessary to actually have the two objects contact each other, they simply need to be brought close enough together so that the charges can leave one object and "jump" on to the other.

As a charged object is brought close to a neutral object, the charges affect the electrons in the uncharged object even before conduction occurs. If objects are brought close, but not close enough for the charges to make the jump from one to the other, it is called induction. Let us consider inducing a charge in a conductor and in an insulator separately. If we bring a charged object near an uncharged conductor, we will get a situation as shown below:



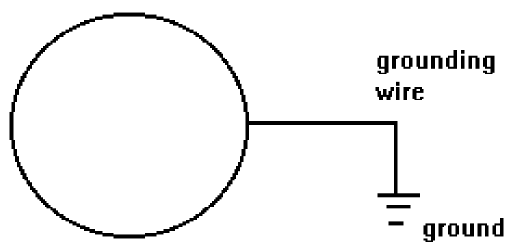
Here, the positive charges have "chased" the positive in the other object over to the far side, while attracting the negatives to the side closest to the charged object. Thus, while the second object actually has no charge (it is still neutral), each end of it appears to have a different charge. It is said that this object is <u>polarized</u> (a situation where one end of the object is predominately positive and the other end predominately negative). If the charged object is removed, the polarization disappears.

Although a similar situation occurs in an insulator, the charges are not free to move around as much. What happens then is that each atom becomes polarized in its own spot. This results in a polarization that is much weaker than that of a conductor. The diagram below shows the situation.

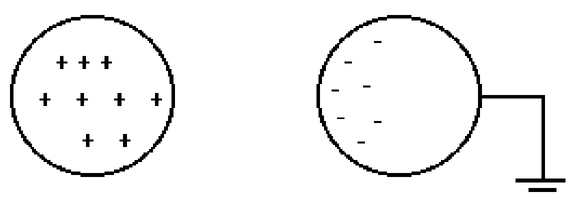


Once again, the change is only temporary.

There is yet another type of induction, usually associated with conducting materials. This involves grounding an object. Grounding is an electrical term that involves attaching a wire to an object that connects it to the earth. The actual definition of grounding, however, it connecting the object to a source or sink of electrons. A source is an unlimited supply of electrons that neither has an effect on nor is affected by the object. A sink is a place that can accept an unlimited supply of electrons with out being affected. In other word, grounding an object allows the object to "pull up" as many electrons as it needs or dispose of as many as it wants. The symbol for grounding an object is shown below:



Consider what happens if you attempt to induce a charge in a grounded object. You will get a situation as shown below.



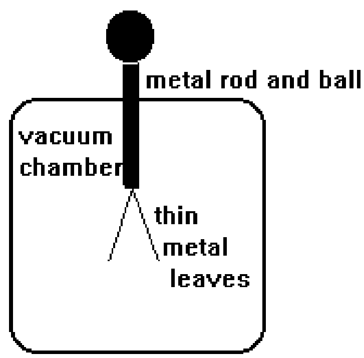
The extra negatives were pulled up out of the ground because of the attraction of the positive charges (or alternately, the positive were "chased" out of the object and down the ground wire). Notice now that the grounded object is charged, not just polarized. Consider what would occur if the charged object was moved away.

Also take a moment to consider what would happen if the ground wire was disconnected before the charged object was moved away. The student should take a moment to see if they can explain what would happen if the original charged object was negative.

So we see that each of the three methods of charging an object is different. The chart below summarizes the differences.

Method of Charging	Start with...	End with...
Friction	two uncharged objects	two oppositely charged objects
Conduction	one charged, one uncharged object	two similarly charged objects
Induction (cutting ground)	one charged, one uncharged object	two oppositely charged objects

A good exercise to review these concepts is to discuss the electroscope. The electroscope is a device that has a metal rod suspended into a vacuum chamber where two very thin and very light metal strips (called leaves) are suspended. On top of the rod is placed a metal ball. This object is used to determine or measure the charge on an object, by observing its behavior when the object in question is brought near or in contact with it. It is also used to detect certain types of radioactivity, since charged particles (like β-particles) are often given off. Below is a diagram and a few questions about the behavior of the electroscope in certain situations.

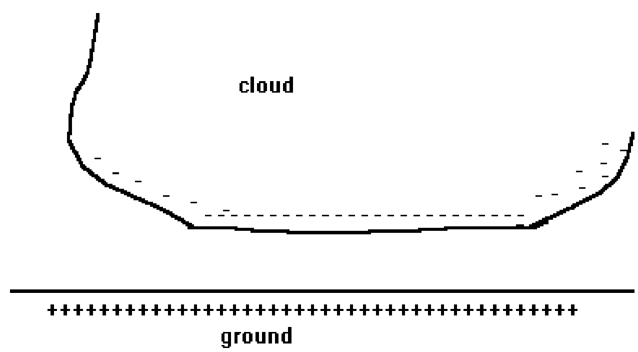


What happens to the electroscope if...

a.) The ball is touched by a positive object?
b.) A positively charged object is brought near the ball and then moved away?
c.) The ball is touched by a positive object and then a negative object is brought near?
d.) The ball is touched by a negative object and then a positive object is brought near?
e.) The ball is touched by a negative object and then grounded?
f.) The ball is grounded, a positive object is brought near, the ground is broken and the positive object is taken away?

There are some other "real world" examples that are worth discussing before we close out the chapter. In the past, you may have noticed that some tractor-trailer trucks carrying flammable cargos will have chains dragging along the road a few feet behind them. The reason for this is to ground the truck in case of static build-up from rolling along the highway. By dragging the chains, the charge is allowed to gradually leak from the truck instead of building up and causing a spark that might ignite the cargo. The practice is fairly rare today, since they have begun to manufacture truck tires that allow the charges to flow through more easily. However, tire manufacturers very recently have made new tires that last much longer than their predecessors. These tires are made of new combinations of rubbers and engineers have discovered that a longer lasting tire does not allow charges to flow as easily as one that wears out quickly. Thus we may see chains added again in the near future.

Another example of these concepts is the phenomena of lightning. Some aspects of lightning are well understood, but many pieces of the explanation simply are not complete. Lightning begins when clouds become polarized (why they do so is still not understood) and we end up with negative charges along the bases of the clouds. This highly charged base can then induce a charge in the ground below.



As the charge in the cloud builds, the attraction between the ground and the cloud increases. Finally the pull gets strong enough for charges from the ground to begin to rise up and charges from the cloud to begin to descend. When these two "streamers" meet, lightning occurs. The electrons rush from the cloud down to the ground, superheating the air and causing it to form a brief plasma, which glows (the lightning). The hot air rushing out from the bolt causes the clap of thunder. This explanation should tell you why lightning tends to hit the highest object and why your hair will stand on end just prior to a lightning strike in your immediate area. It should also explain why lightning rods work so well. Remember back to our discussion of charges in conductors? We said that charges gather at extremities, such as points and can leak easily from there. Thus a long, thin metal rod pointed skyward will be the most likely place for an upwards streamer to begin.

Let us close this section with one final example from your childhood.

> **Example 8.3.10**
>
> Explain why a balloon, after being rubbed on your hair will stick to the wall and then fall off after a certain amount of time.

In terms of first year physics classes, or the AP Physics test, the main concepts you should know about are the following:

- Know the difference between a conductor and an insulator
- Know that charges on conductors reside only the surface and gather near points.
- Know the three methods of charging an object, and know an example for each one.

8.4 - Electric Fields

As I am sure that you noticed in the previous section, the electric force is very similar to the gravitational force. The both have many aspects in common: they are both inverse square relationships in regards to distance, they both have one quantity that determines the objects reaction to the force (mass or charge) which is first order in the equations, and they both have a constant of proportionality. However, there are also some glaring differences, the primary one being that gravity only attracts, never repels. With all of these similarities, one would expect that what we did for gravity we will repeat now with the electric force. We will take the force, create a concept of a field from that force, expand to determine how the field is related to energy, and then try to write an equation for all of those.

However, there is one aspect that will become very clear, very quickly when dealing with the electric force. The easiest forces to deal with are, of course, forces that are constant (uniform) regardless of where you are. The next easiest set of forces to deal with is a spherically symmetric force. Most of gravity deals with spherically symmetric forces simply because most of time we notice gravity it is coming from a planet, which is spherical. The electric force, however, has many different forms and shapes, because charged objects can exist in almost any form. This will make electric fields more difficult to work with than gravity.

The Electric Field

Just as we did when discussing gravity, we can also construct a field equation for the electric force. The reasons are the same: we want to be able to discuss the electricity caused by one single object instead of always dealing with the force between two charged objects. The procedure and concept is essentially the same as it was when dealing with gravity. To remind you, the field was measured by placing a test object at the location, measuring the force on that object and dividing by the factor on the test object that determined the force. For electricity, that factor would be the charge on the test object. Mathematically,

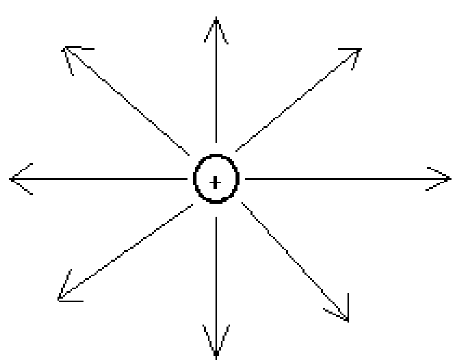
$$\underline{E}_1 = \underline{F}_{12}/q_2$$

Where E_1 is the field caused by object 1, F_{12} is the force between objects and one and two, and q_2 is the charge on the test object. Once again, we encounter the concept of the test object. In this case the test object must have a small enough charge so as not to disturb the field that is being measured. It is also important to notice that the test particle is defined as positive, therefore the field will always be given in terms of its effect on a positive charge. The above equation shows us the units of an electric field,

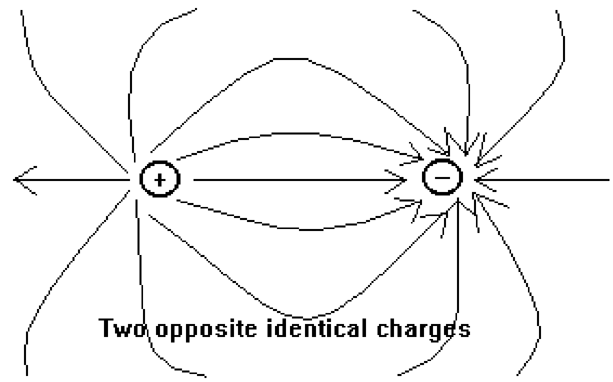
$$E = F/q = \text{Newtons/Coulombs} = \text{N/C}.$$

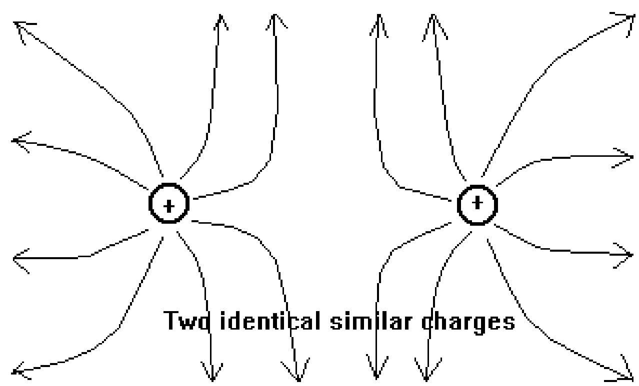
Notice how now the field is not in units of acceleration, as was the case with gravity.

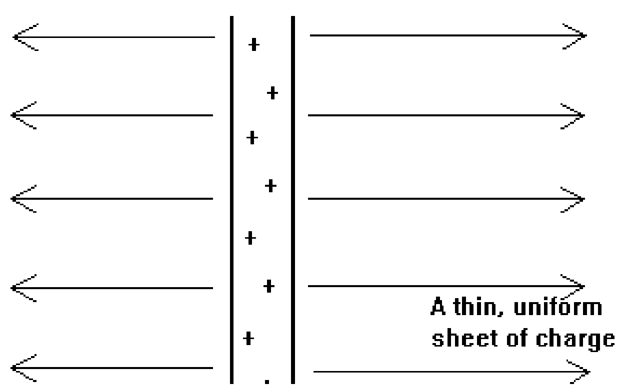
Because the concept of a field is often hard to understand, Michael Faraday came up with a conceptual method for visualizing a field. Faraday was a phenomenal physicist and chemist in the 1800's who was not only entirely self taught, but hated mathematics. He accomplished all his work by experimentation and conceptual explanations. Even without (or because of) his lack of mathematics he made some of the most important discoveries in physics. It was his lack of mathematical knowledge that propelled him to invent the concept of the lines of force. Faraday's line of force are lines drawn in the following manner. Imagine that we wanted to draw the lines of force around a single, positive charge. We would place a test charge at a location around the positive charge and draw an arrow in the direction the force acts on the test charge. We would then move the test charge to the head of that arrow and then we would draw another arrow in the direction of the force at that point. After doing this many times around the positive charge, we would end up with a pattern of lines that represent the field direction around the object, as shown below.



Notice how the lines point away from the positive charge, since the test charge is defined as positive. In the case of the field around a negative charge, they would point inward. The above diagram is very simplistic and probably doesn't shed too much light on the concept of forces. Faraday's lines of force are actually more useful when dealing with complicated fields. Consider the following diagrams which represent field lines for different situations.







Hopefully, this concept will help you visualize what exactly a field is. There are a couple of extra notes on Faraday's Lines of force that are listed below.

1.) Do not forget that although we draw these lines on two dimensional paper, a field is actually three dimensional.

2.) The line tangent to a line of force gives the direction of the field at that point.

3.) If the field lines are drawn correctly, the number of lines in a given area is proportional to the field strength in that area (when lines are close together the field is strong).

4.) The lines are always drawn using a positive test charge, a negative charge would feel the opposite force.

5.) Lines must either begin (or end) at a charge and start (or end) at infinity. Lines may not end or begin in the middle of a field.

6.) Lines cannot cross (why?).

7.) Lines cannot be drawn through an equilibrium point (why?).

8.) These are lines of force direction, not lines showing the motion of a particle. A particle released at the beginning of a line of force will not necessarily follow that line.

9.) Dipoles align themselves along lines of force (we will discuss dipoles later).

10.) Lines emanating from a charge on the surface of a conductor will always leave at 90^0 to the surface of the conductor.

Example 8.4.1

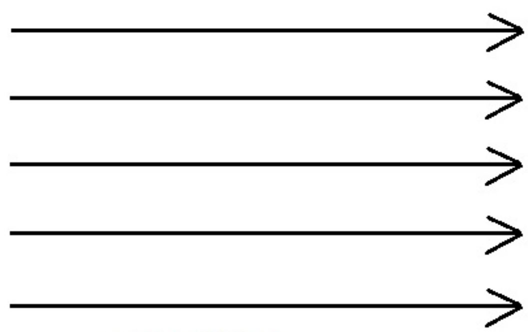
Draw the field lines around a set of 3 equal point charges arranged on the corners of an equilateral triangle.

Example 8.4.2

Draw the field lines around a set of 12 negative charges placed in a circle around a positive charge in the middle (assume the positive charge is significantly higher in value than the negative charges.

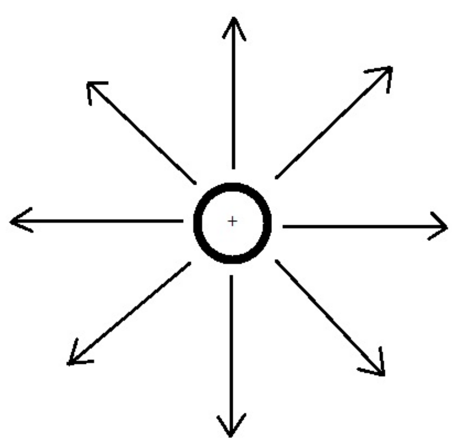
Being able to draw the field lines can give you a great, visual sense of what the field looks like, but beyond that we will need to rely on the mathematics (as usual). In order to learn how to mathematically deal with fields, it is easiest if we break our discussion into four categories. Whenever you are working with a field problem, it is very important that you can recognize which category of field you are dealing with. This will allow you to more easily pick out your equations and solve the problem.

Uniform Fields: These are the easiest fields to work with. Being uniform, they have one value at every point in the field. A vast majority of field problems in first year physics courses fall into this category. You can recognize a uniform field because all the field lines are parallel. Spherical fields can be created by long uniform sheets of charge, but often in a uniform field problem it won't even tell you the cause, it might just say something like, "a uniform field of 8 N/C exists..."

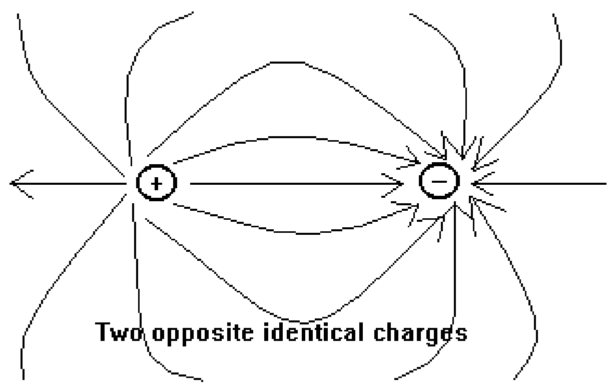


uniform field lines

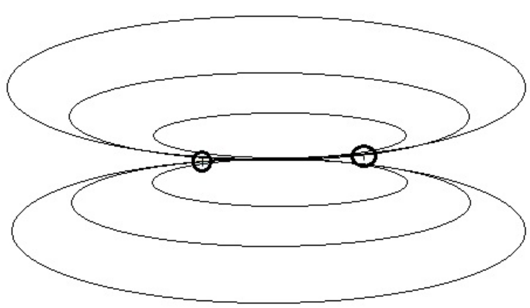
Spherical Fields: Spherical fields are also very common in first year physics classes. While not as easy to work with as uniform fields, spherical fields have nice, easy equations to give you the value at any point around the object. Even if the field itself is not spherical, there is a good chance that it might be a combination of spherical fields put together. You can recognize a spherical field because it will be caused by a spherical object (for example: a point charge) and it will have field lines radiating away from a central point.



Dipole Fields: One of the common fields that is created by combining spherical fields is a dipole field. A dipole is an arrangement of one positive and one negative charge (assume equal value unless otherwise stated) that are separated by a distance and held apart at that distance. It is an important field to be familiar with, since it is an integral part of magnetism and plays an important part in many electric phenomena. The problem with dipole fields is that it is easy to write a vector equation for the field, but it is hard to understand what that equation mean or simplify it in any way. Mathematically, dipole fields are just very difficult to deal with. We will do a little math with them later, but only for special cases and only approximations. A dipole field looks like the picture shown below, and all students should be able to recognize a dipole field by sight.



As seen from a distance, a dipole field looks like concentric ovals, as shown below.



Complex Fields: The term complex field is simply a catch-all term that applies to any field that is not uniform, spherical, or a dipole. When fields are more complicated, their equations and field line drawings get more complicated and that makes them difficult to work with. The one exception, of course, is a complex field that is caused by two (or more) simple fields combined together. In that case, you can simply add the results of each field as vectors.

Let us now look at each of these types of fields individually in more detail.

Uniform Fields

We now turn our attention to mathematically handling fields and field equations. The most important fact is the simplest fact: a charged particle in an external field will experience a force. Thus we can always find the force on a particle if we know the field the particle is in. Things get more complicated if we wish to go further than that, however. For example, if the field is uniform, F = ma can be used to find the acceleration. If the force is constant, the acceleration is constant, and our one dimensional motions equations apply. However, in a spherical or complex field, F = ma will give us the force, but since F is not constant, as soon as the particle moves slightly, the forces will change and thus the acceleration. Deriving the actual path and determining the velocity of an object in such a field is often a very complex procedure. Let us attempt to apply this knowledge to a few examples. The first type of field, the uniform field, is very easy to deal with. We already know which equations apply:

$$E = F/q$$

and

$$F = qE.$$

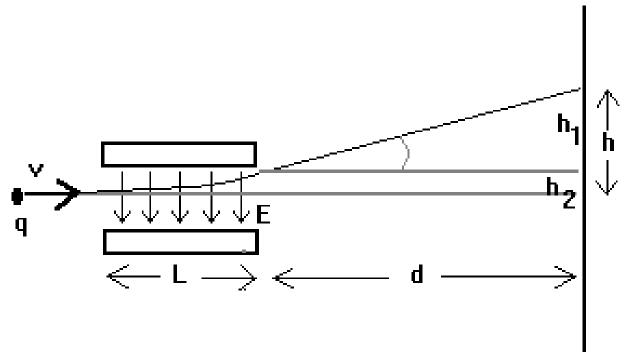
> **Example 8.4.3**
>
> A 3 mC charge is placed in an external, uniform field and experiences a force of 4.5 N. What is the field strength?

> **Example 8.4.4**
>
> A charge of 6.4 μC is placed in a uniform electric field of strength 2 N/C. If the charge has a mass of 65 mg, how fast is the charge moving after 3 sec?

As we can see, the equations couldn't be much easier. Before we move onto spherical fields, let us take a moment now and try to use this concept in a problem that involves many of our past discussions.

Example 8.4.5

In the diagram below, a charged particle q is launched into the electric field E as shown, deflected and sent to a screen. Determine the height above center for the impact of the particle in terms of the other givens (E, v, L, d, m, and q).



The above example shows us exactly how a cathode ray tube works (a tube like the kind used in hospitals to monitor heart rates, or an old fashioned TV screen or a computer monitor). The "screen" is coated with phosphorous which glows when an electron hits it. A beam of electrons is shot out of a cathode and passed through two electric fields (one vertical and one horizontal). By controlling the electric field, the glowing dot can be made to move across the screen. Since the phosphorous will glow for a while after the electrons cease hitting it, it will leave a line across the screen. If the electric field is hooked up to the pumping of a heart, the dot will dance according to the beat.

The same principle is applied to ink jet (or sometimes called bubble jet) printing from computers. Each letter is fashioned by up to 100 dots of ink (a 10x10 matrix). Each little tiny dot is shot out of the printer at the paper with exactly the correct charge to hit the appropriate spot on the paper. Dots in the matrix that are not needed are shot out uncharged and gather in a gutter located directly across from the gun.

Spherically Symmetric Fields

Now let us turn our attention to spherically symmetric fields. As the name implies, a spherically symmetric field is one that is based solely on the distance from the source, not on the location around the source. To put it in mathematical terms, the field has only one variable, radius, and does not contain anything in relation to the theta direction. These fields, while not as simple as a uniform field, are easy to work with since all the equations are equations of one variable. Let us begin by deriving a simple law for spherically symmetric fields from Coulomb's Law. Remembering that:

$$E_1 = F_{12}/q_2$$

we can substitute in Coulomb's Law for the force:

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2 q_2}$$

then,

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r^2}$$

or

$$E_1 = \frac{Cq_1}{r^2}$$

This formula above will give the value of the electric field at some distance r from a single point charge or spherically symmetric charge distribution. We should also remember that this is a vector equation, thus fields can be added or subtracted as vectors. Let us do some sample problems involving field calculations.

> Example 8.4.6
>
> Find the field value at a distance of 2 m from a charge of 5 C. Where would the field value be one half of this value?

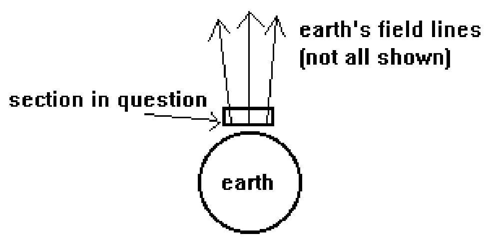
> Example 8.4.7
>
> Imagine a 6 C charge placed at the origin, and a 3 C charge placed at (4 m, 3 m). Find the following:
>
> a.) The field of the 6 C charge at (3 m, 0)
> b.) The field of the 3 C charge at (3 m, 0)
> c.) The force on a 2 C charge placed at (3 m, 0)
> d.) The field of the 6 C charge at the location of the 3 C charge.
> e.) The field of the 3 C charge at the location of the 6 C charge.
> f.) The force on the 3 C charge from the 6 C charge.

This next example shows how to use a spherical field as a rough approximation technique for a natural phenomena.

Example 8.4.8

An atom of uranium has 92 protons in its nucleus with a radius of 6.8 fm ($6.8x10^{-15}$ m). As a rough approximation, consider the fact that each proton is repulsed by every other proton in the nucleus at a distance that averages out to be approximately r/2. (a.) What is the field that the one proton experiences? (b.) What force does this proton feel?

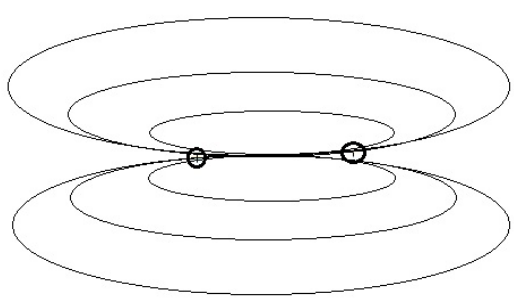
At this point, the astute student might be asking something like: "The earth produces a spherical field, yet we used a uniform field as an approximation. Because the two are so very different, why did our approximation work?" The answer is as follows. A uniform field can be used to approximate a spherical one as long as the area under question is small compared to the overall field. There are two ways for justifying this. First, if we look at just the first mile above the earth's surface, the field changes very little, thus it appears uniform. A second way of thinking about this is to imagine the field of the earth as a spherical field. If we focus in on one small section, as shown in the diagram below:



we can see that inside the rectangle, the field lines appear to be roughly parallel. As stated previously, parallel field lines indicate a uniform field.

Dipole Fields

As mentioned earlier, a dipole field is the field produced around two equal but opposite charges that are held a certain distance apart. It is a very important field, but not an easy one to handle mathematically. The field lines look (approximately) like the previous picture:

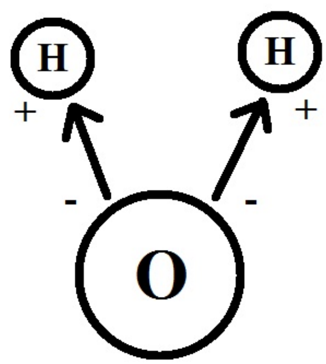


In order to understand exactly how the dipole field is mathematically complicated, let us start by doing the two problems below.

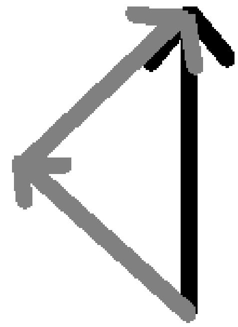
> Example 8.4.9
>
> A dipole made up of two 3 C charges is placed on an axis so that the positive charge is at (0,1) and the negative charge is at (0,-1). Find the field value, including direction at the following points: (1,0), (0,2), (2,2), and (2,1). The coordinate system is marked in meters.

> Example 8.4.10
>
> A dipole is made of two 4 C charges, spaced 25 cm apart. A -2 C charge is placed 2 m from the positive charge in a straight line through the dipole. Find the force on the 2 C charge.

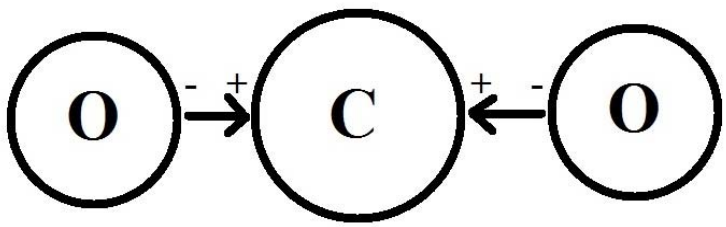
Dipoles play an important role in physics and in chemistry. Consider a molecule of water. In such a molecule the electrons in the covalent bonds spend more time around the hydrogen atoms than they do around the oxygen atom. Because of this, each hydrogen-oxygen bond forms something called a dipole moment (a vector, pointing from negative to positive - these will be discussed in detail later) as shown in the diagram below.



Because of the bent shape of the molecule, the two moments add as vectors into one total dipole moment for the water particle (as shown below).



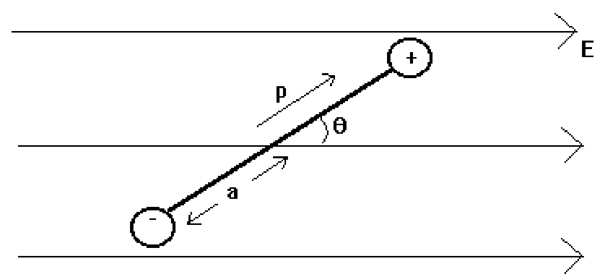
Thus each and every water particle acts like a tiny dipole and although it is electrically neutral, it does have a field and does react to external fields. This is what is commonly meant by "polar" molecules. An example of a non-polar particle would be CO_2, where the dipole moments cancel each other out because they point in opposite directions (due to its straight line form).



A few random notes should be made before we move on. First, notice how in the example of the water particle, it had a dipole moment without having a net charge. This shows that the dipole moment of an object is simply another inherent property, like (but separate from) mass and charge. Also, in the water diagram, the lone pairs of electrons were left off for clarity.

Example 8.4.11

Describe qualitatively, but using quantitative arguments, what would happen to a dipole placed in the uniform E field below.

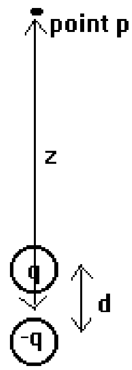


This example shows you an important property of a dipole, it will always align itself along the field lines of an external field. This explains compasses and the little magnetic fillings that are often sprinkled over a magnet to show the magnetic field. As a question to ponder, why are compasses always encased in water or some other fluid?

Dipole Approximations (optional)

Because dipoles are not easy to work with mathematically, one technique that is often used is to do approximations for special cases. Approximation techniques are often used in higher level physics, as a way of getting a grasp of a complicated situation without getting lost in the mathematics. As first year physics students, it is often difficult to grasp when an approximation would be appropriate, and how to carry it out successfully. We have done some approximations, in previous chapters (remember the $m_1 << m_2$ situation where we had two masses, one on each side of a pulley?). But the dipole approximation is usually the first serious approximation that first year physics students encounter. We go through it here as a learning exercise.

The first step is to choose a special situation where we think the math might be easier to work with. Our special case will be using the dipole below, and only dealing with points along the main axis.



As a second stipulation, we are going to only work with points far away from the dipole, or mathematically: z>>d. So, what we are really asking is: Approximately, what does the field of a dipole look like far from the dipole along the main axis?

The field can be written as the combination of the two (spherical) fields caused by the charges.

$$E = E_+ + E_-$$

$$E = Cq/r_+^2 - Cq/r_-^2$$

$$E = Cq\{r_+^{-2} - r_-^{-2}\}$$

$$E = Cq\{(z-d)^{-2} - (z+d)^{-2}\}$$

While this equation is not terrible complicated, we might be able to reduce it further if we could do something with those pesky terms in the main parenthesis. In order to do that, we will expand those terms out using the binomial theorem, which you may remember from your high school algebra class. We need to take a short side trip at this point to review the theorem and its uses.

The Binomial Theorem is a method of expanding a set of variables or numbers raised to a power. The theorem states that:

$$(1\pm x)^n = 1 \pm \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} \pm \ldots$$

or

$$(1\pm x)^{-n} = 1 \quad \frac{nx}{1!} + \frac{n(n+1)x^2}{2!} \quad \ldots$$

These are used when expanding a polynomial and can be used to approximate answers to otherwise unsolvable problems. The way we approximate is to simply throw out terms that are very small. We label approximations by "order" - meaning the exponent of the last set of variables that we keep. For example: a first order approximation is one where we only keep the term containing x to the first power, and discard all the other terms. A second order would contain terms with x and x^2, and a third order would contain x, x^2, and x^3. But students often ask, "How can we simply ignore the other terms?" The answer is that we can only do this if the terms get progressively smaller - each extra term is simply a very slight "tweak" to the answer, so what we are saying by throwing out the smaller terms is, "This is close enough." Let us see how this works with actual numbers before we move on to the algebra term.

> Example 8.4.12
>
> Approximate the value of $(1+0.25)^3$ to the first, second and third order.

The above example shows not only the definition of "to the first order", but also how the binomial theorem can be used for an expansion. The problem itself is useless, since it could have been easily plugged into a calculator in its original form, but the next example shows how to use this to solve an otherwise unsolvable problem.

> Example 8.4.13
>
> Solve $(14+y)^6 + (14+y)^3 = 16781312$

We were able to get a reasonable answer just by doing a first order approximation. However, one thing should be mentioned. We can only discuss first order approximations if the higher order terms are negligible. In other words, each consecutive term must be significantly less than the term before it. This will only occur if the variable in the parenthesis (the x in the $(1+x)^n$ term) is much less than one. <u>Approximations using the binomial theorem are only valid if x is less than one.</u>

Now we return to the dipole approximation.

> Example 8.4.14
>
> Use the binomial expansion to expand the equation of the field of a dipole at great distances along the axis to the third order.
>
> $E = Cq\{(z-d)^{-2} - (z+d)^{-2}\}$

This gives us a much simpler equation. Also notice that this equation is good to the fourth order, since the even terms were dropping out anyway.

At this point, physicists often step in and define a new quantity called the dipole moment. A dipole moment is defined as p = qd, and is actually a vector that points from the negative to the positive charge. With that substitution our equation becomes:

$$E = \frac{p}{2\pi\varepsilon_o z^3}$$

Which is an approximation for the field due to a dipole at distances along its axis much greater than the distance separating the charges. p is an interesting quantity called the dipole moment of the charge distribution. The dipole moment is an inherent property of the set-up, much like charge or mass. In other words, in many objects we would describe their electrical properties in terms of dipole moments, just as we would describe their mass or charge. A dipole is defined as any charge set up that has properties where one side is primarily positive and the other is primarily negative. Interestingly, at large distances from a dipole, it is impossible to measure the charge of the set up, one can only measure the dipole moment. Notice how the moment will remain the same if the charge is halved and the distances between them doubled. Also notice that the field goes according to $1/z^3$, instead of the usual $1/z^2$ that we would expect from an electric field. Dipole fields drop off more quickly because the negative and positive charges effectively cancel each other out, leaving only a component of the field.

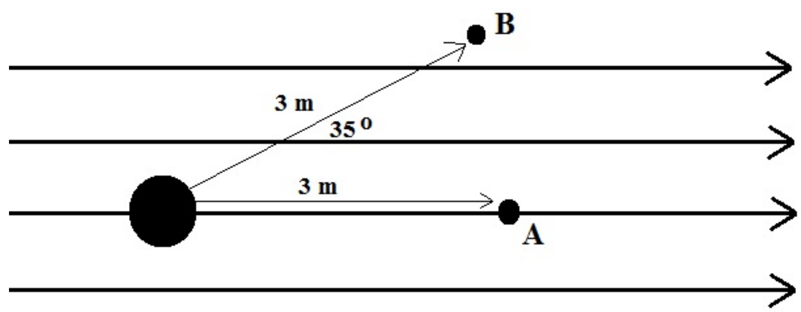
Before we leave dipoles for the moment (ha ha!) I would like to stress the importance of this latest concept. Dipoles are essential to electricity and even more essential to magnetism, so it is good to understand them fully. Consider this: If you were asked to solve a physics problem that involved gravity, what is the first thing you would look for in the problem? The mass of the object, so that you could use mg, or Gm_1m_2/r^2. What if you encountered an electric force problem? You would immediately look for the charge of the object so that you could use F = qE or Cq_1q_2/r^2. Likewise, if you were describing a situation to a physicist involving objects, and you failed to mention their masses, that would be a big mistake. Or if you were discussing electrical objects and failed to mention their charges, it would be obvious you were leaving things out. For dipoles, we always think first about their moment. Since a dipole moment defines the dipole, any time we have discuss a situation that involves a dipole, the first thing that should come to mind is the moment. You will notice this when we start our discussion of magnetism, where everything is a dipole.

Complex Fields

In first year physics courses, complex fields usually only show up as combinations of the fields we have already covered. The important thing to remember is that all you are doing is evaluating each field individually, and then adding the fields as vectors. Since there is little more to say, let us try a few for practice.

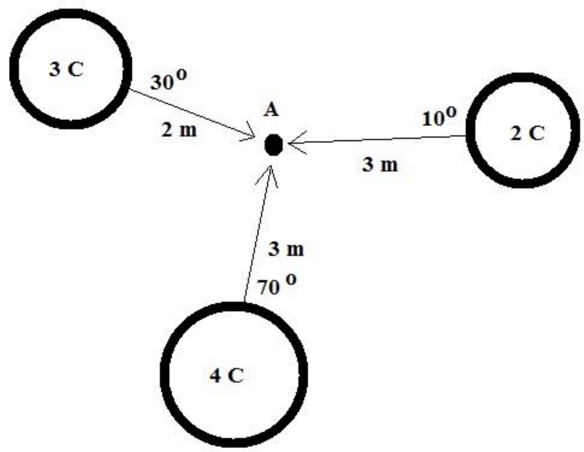
Example 8.4.15

A 2 C charge is placed in an uniform field with a value of 8 N/C as shown below. Find the field (magnitude and direction) at the two points labeled A and B (all distances are center to center).



Example 8.4.16

Find the field at point A in the diagram below. All distances are from center to center and all angles are from the horizontal.



Complex Fields - Gauss's Law

There is another way to derive field equations, other than simply using Coulombs Law over and over again in combination. This other method is called Gauss's Law and it is a calculus based method. It is covered in some first year physics class, and skipped over in others. In most of the classes in which it is covered, the majority of the problems can be done without calculus at all, if you understand the ideas behind the law.

In order to use Gauss's Law, we need to discuss two main ideas first: how to draw a field from symmetry and how to calculate the flux through a surface.

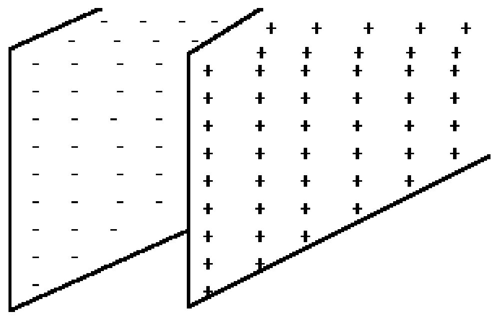
Drawing fields from symmetry is an exercise in logic. We simply look at a field situation and ask ourselves: "what is the most logical way for the field to look? We also ask: "Is there any reason it would not look like this?" We also need to realize what the different regions in an area would look like. In some cases the field will change abruptly as it passes through a region where situations change.

Another thing to consider when drawing fields is our rules from earlier in the chapter regarding field lines - for example: lines can only start and end on charges or go to infinity, and lines cannot cross, and cannot pass through an equilibrium point.

The easiest way to learn this concept is by example. Let us try to draw, describe and graph the following situations:

Example 8.4.17

Draw, describe, and graph the field between and on either side of two equal, opposite and infinite sheets of charge, a distance d apart.

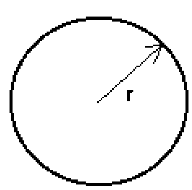


Example 8.4.18

Draw, describe, and graph the field between and on either side of two equal, positive infinite sheets of charge, a distance d apart.

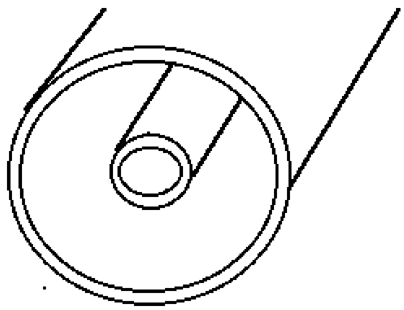
Example 8.4.19

Imagine a solid, uniform ball of charge of radius r. Draw, describe and graph the field created by this charge from r = 0 to infinity.



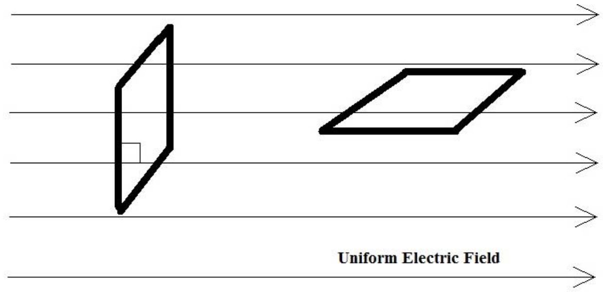
Example 8.4.20

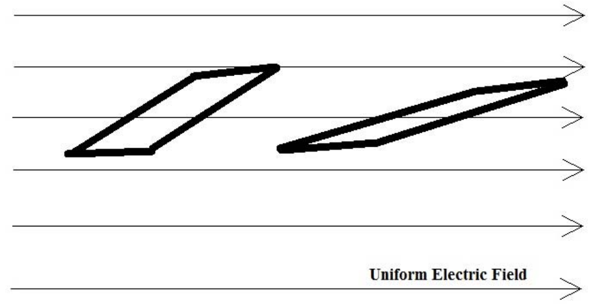
Draw, describe and graph the field shown below. It consists of a metal pipe with charges on it inside another, larger charged pipe.



The second concept to discuss before we can apply Gauss's Law is the idea of flux. Flux is a concept that is used in many areas of physics, from fluid dynamics to electromagnetism. The flux in any situation is the amount of something that passes through an area. As an example, imagine if you wanted to install solar panels on your roof. You might need to know how much sunlight would land on the panels. This would be based on the size of the panels and also their angle. For example, it the panels were installed perpendicular to the roof (a silly idea) they would receive very little sunlight. If they were installed flat, depending on the angle of the roof, they would receive more. There is probably an optimal angle for the installation. That angle would give the maximum flux – it would have the most sunlight striking the panels and thus generate the most electricity.

The flux of an electric field is calculated by finding out how much of the field passes through a given area. Consider the diagram below. A square loop, 2 m by 2m, is placed in a uniform electric field in four different orientations. The first has the loop placed so that the field lines cut through the plane of the loop at 90°. The second has the loop lying flat, and no field lines are cutting through the loop. The third and fourth have the loop angled so that the field lines are at some angle to the plane created by the loop.

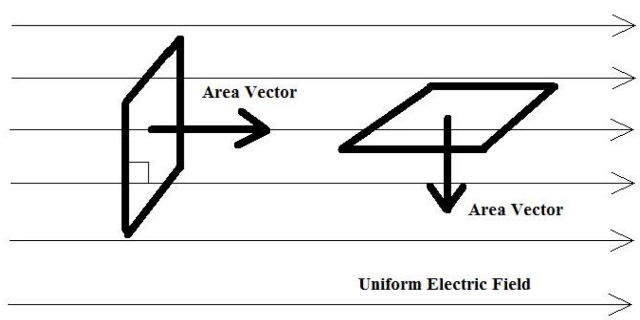




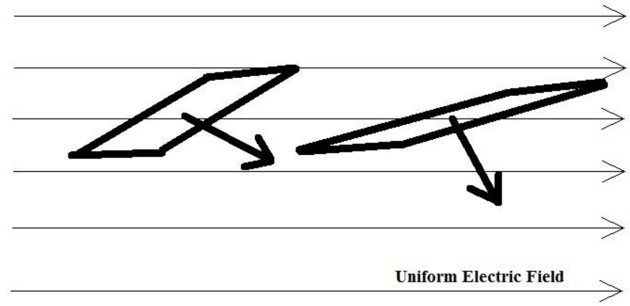
Which of those situations has the most electric field passing through the loop? Obviously, the first situation. But what if we had to calculate the amount of flux in each case? How can we put a number on this idea? We do so by calculating the flux – and we find it by multiplying the area of the loop times the electric field, and then adjusting based on the angle. Mathematically:

$\Phi = EA\sin\theta$

Notice that Φ is the Greek letter phi, and is used to represent flux. E is the electric field, A is the area of the loop, and θ is the angle between the electric field and the area. Now, an astute student might immediately protest that areas don't have directions, so how can we find the angle between the electric field and the area? The answer is that when dealing with vectors, we assign a direction to the area of a surface. The direction is always normal to the surface, or perpendicular to the plane of the area. In this situation, the area vector will always point 90° to the plane of the loop. The four diagrams are repeated again, this time showing the vector of the area in each case, and the angle used in the equation above.



This formula, or idea, might look familiar. The reason it does is because in actuality, the flux is a vector dot product between the field and the area. Recall that dot products take two vectors and produce a scalar, and that they are based on how parallel the two vectors are: maximum value when the two are parallel (as in the first picture above), zero value when the two are perpendicular (as in the second picture). In formula form:

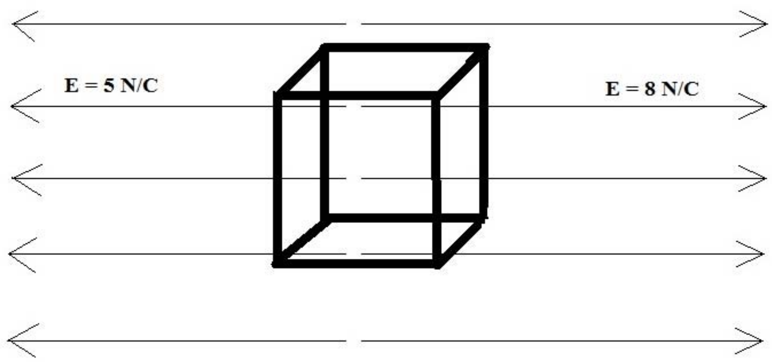


$\Phi = E\ A$

Notice how the concept of flux is very simple when the area is a nice, rectangular shape, and the electric field is uniform. Things get much trickier if those two conditions are not met. Let us try two quick examples of calculating flux

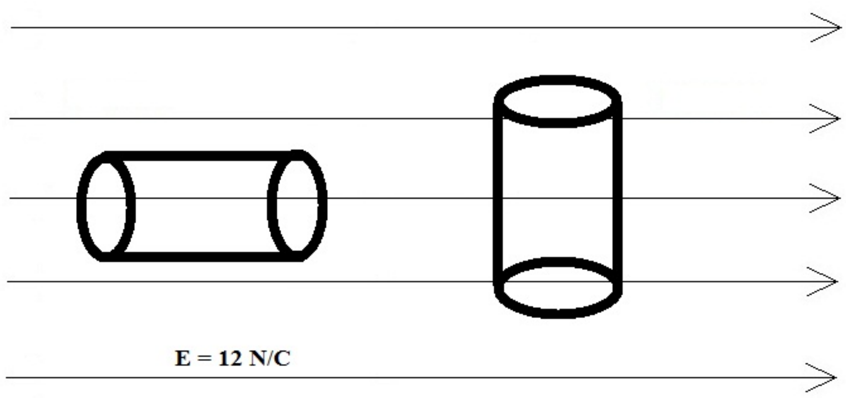
Example 8.4.21

A cube, with sides of 20 cm each is placed in an electric field as shown below (don't worry about what is causing it, that won't enter into this problem.) Find the electric flux through the cube.



Example 8.4.22

A cylinder, with radius = 10 cm, and a length of 50 cm, is placed in a uniform electric field in two different positions as shown below. In each situation, find the flux through one end cap, and the total flux through the cylinder.



This last example shows us an important and interesting aspect to flux – even when the shapes are complicated it will often be easy to use symmetry to determine the flux.

Now that we have a basic understanding of flux and drawing fields, we can begin to examine Gauss' Law. Gauss' Law is a calculus based method of writing a field equation – keep that in mind. Your goal in every situation is to finish the problem by writing an equation for the electric field (except, of course, for those problems that run backwards…that will become clearer in later examples). Although Gauss' Law is calculus based, the majority of uses, especially in first year physics classes is very limited and it is usually only used in situations with a great deal of symmetry and ones where the calculus can be skipped. Because of this, we will present a simplified Gauss' Law here, one that is easier to use and yet still gives the student the understanding of how to apply the law.

$\Phi = q_{enc}/\varepsilon_o$

In the above equation, we recognize Φ as the flux through any random closed surface, q_{enc} is the sum of the charges enclosed by the surface, and we have see ε_o before, as the permittivity of free space. The equation above is a simplified form, there is also a calculus based form of Gauss' Law, which of course, means the exact same thing:

$E\ dA = q/\varepsilon_o$

In this format, the law says that you would evaluate the flux by finding the closed surface integral of the electric field dotted with the differential surface vector (an infinitely small surface which is summed up through the integral).

In words, what Gauss' Law states is that the flux through any closed shape in an electric field is related directly to the charge enclosed by it.

As was mentioned before, Gauss' Law has very limited used in first year physics classes, but let us go over some of the most common ones.

To use Gauss' Law in a given situation, the steps are simple:

1.) Using symmetry and logic, draw out the electric field lines in the situation.
2.) Draw a closed shape (square, cylinder, sphere, etc) that will match the symmetry around some or all of the charges. What you are trying to do is to find a shape that will make it easy to calculate the flux through its surfaces.
3.) Calculate the flux through all the surfaces and add them together to find the total flux.
4.) Set that equal to the charge inside the shape, divided by the permittivity of free space.
5.) This equation should allow you to find a formula for the electric field.

Gauss' Law - Simple Applications.

We start by using Gauss' Law in the most symmetric and simple of applications. However, before we jump into the problems, let us either review or introduce a few variables that will be used often in this section.

We will often discuss charge densities when dealing with electric fields. When we use the term density, we are usually referring to the amount of mass per volume of an object. But the idea of density can be carried further to apply to the amount of anything per something else. In electricity, we will often make reference to charge densities. This is the amount of charge per unit volume, area, or length. The three variables most often used are:

ρ = volume charge density - in C/m^3
σ = surface charge density - in C/m^2
λ = linear charge density - in C/m

So, for example, if we were talking about charging the surface of a table, we would say it has a surface charge density of 6 C/m^2 or something like that, instead of simply giving the total charge on the table. Likewise, the charge on a wire could be given in terms of how many Coulombs per meter of wire were there.

Example 8.4.23

Use Gauss' Law to determine the electric field around a point charge, and show that the result matches with Coulomb's Law.

Example 8.4.24

Use Gauss' law to determine the electric field both inside and outside a solid sphere with a uniform volume charge density of $+\rho$.

Example 8.4.25

Use Gauss' Law to determine the electric field around a infinite sheet of charge with a uniform surface charge density of $+\sigma$

Example 8.4.26

Use Gauss' Law to determine the electric field around a very long wire with a linear charge density of $-\lambda$

Gauss' Law - More Sophisticated Applications

As we move into more sophisticated examples of using Gauss' Law, there are two important conceptual ideas from a previous section to keep in mind:

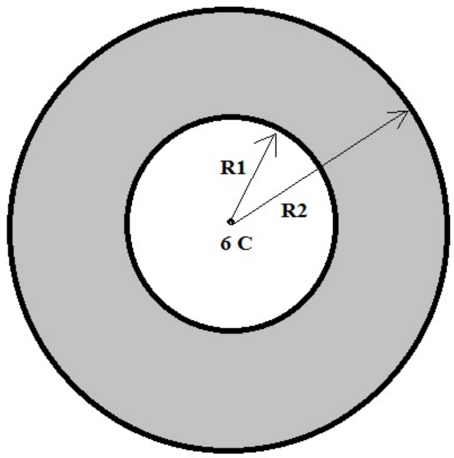
1.) Regardless of the shape or circumstances, all of the charges on a conductor reside on the surface. Think about what this means for Gauss' Law - if there is no charge inside a conductor, then there can be no electric field. The electric field inside a conductor is always zero. (The one exception is that this only works when charges have been given time to settle out - if you put a large quantity of charges onto a piece of conducting material, there will be a very short time during which the charges are moving to their equilibrium points.)

2.) Because of the nature of conductors, the electric field lines always leave the surface of the conductor at 90°. Once they leave the surface they might diverge, but they always leave at 90°.

Now let's try some more complicated examples.

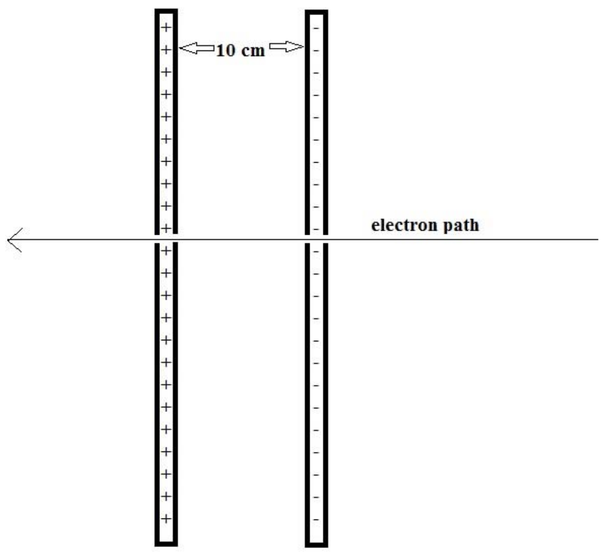
Example 8.4.27

A thick walled, metal, uncharged conducting sphere is shown below. A 6 C point charge is placed in the center of the sphere. Write an equation for the field inside the cavity, inside the metal walls, and outside the sphere. Also determine the surface charge density on both the inside and outside surfaces.



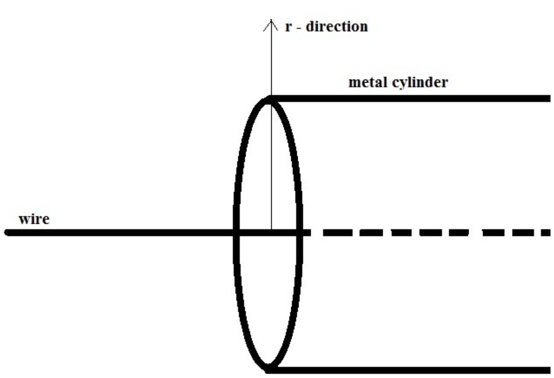
Example 8.4.28

Two large, insulator plates are placed 10 cm apart and the one on the left charged positively and the one on the right is charged negatively. The surface charge density is 2 C/m^2 on both plates. A small hole is drilled through the center of both plates and an electron is fired at the hole from 2 m away on the right. The plan is to have the electron leave the second hole with as little speed as possible. What should the firing speed of the electron be?



Example 8.4.29

A very thin wire is place coaxial with a metal cylinder as shown below. The field in all areas is symmetric and based only on the distance r from the center of the wire. Inside the cylinder the equation becomes $E_1 = 2.6/r$ and outside it is $E_2 = 1.7/r$. Find the charge densities of both the wire and the cylinder.



<u>Electric Fields and Conductors</u>

We will take a minute at this point to discuss electric fields around and inside conductors. Conductors, you will recall, are materials that allow electrons to flow through them easily. If a conductor is charged, two interesting things occur in relation to the field.

1.) The electric field will always radiate out from the conductor at right angles to the surface. No matter how irregular the object is, the field near the surface will always be at right angles to the conductor. Thinking about this a little will lead an astute student to the conclusion that very near the surface of the conductor the field is...

2.) The electric field inside a charged conductor is always zero. This interesting fact will lead to a number of conclusions, which are worth discussing in depth.

Recall that we discussed that electric charges will always reside on the outer surface of a conductor. We said that they do so because they are pushing each other apart and want to get as far from each other as possible. Another way of saying the same thing is to say that <u>charges on a conductor will arrange themselves around the surface so that the electric field inside is always zero</u>. If the electric field were not zero, then a charge placed inside would accelerate and charges would tend to move in one direction. We know, from experiments, that charges in a conductor are free to move in any random direction. The student should be reminded that this phenomena applies to any conductor, regardless of shape or solidity. It works inside a hollow sphere as well as a solid sphere. It would work on a metal box or a thin, solid wire.

The result that shows that the electric field inside a conductor is always zero is the reason that you are safe from lightning inside of a car. Often people think that you are safe because you are insulated from the ground by your tires, but that is not the case. Because a car (or airplane) is basically surrounded by a sheet of conducting material (metal), when lightning strikes it, the electrons move around such that the field inside the object is zero. Humans feel no effect. It is of interest to note that as new, lighter plastics are developed and used for car body parts and parts of airplanes, this shielding effect will no longer protect the inhabitants of the vehicles.

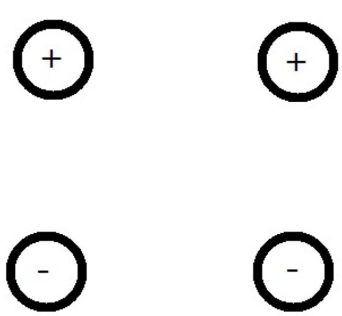
This effect is also used in some "high tech" situations where equipment must be protected. In these cases, scientists use what is called a Faraday's Cage. It is basically a box made of metal. Instruments can be placed in the box, with the wires coming out, and then the instruments are protected from electrical interference. A situation where such protection might be necessary is in an electrical lab where large sparks might be common. Computers, for example, are instruments that can be destroyed by even small, stray electrical sparks contacting their inner chips. In such a lab, the equipment (and personnel) are usually located in a special room that is lined with copper. In such a Faraday's cage, no matter how big the spark is on the outside, noting inside will be affected. We can see this happen on a smaller level if you place a radio inside a metal box (this affect is somewhat evident inside buildings). Since radio waves are electrical phenomena, they cannot reach the antenna and you will get no reception.

Before we leave this discussion, two points should be made to remind the student of the limitations of this effect. First, it only occurs in conductors. It is impossible to set up an electric field inside a solid (or hollow) conductor, but it is possible to do so inside an insulating material. Secondly, this situation only applies in equilibrium, after the charges have settled moving around. If you dump a huge amount of charge on a conductor in a short amount of time, there can and will be an instantaneous field set up, but this field will vary and change quickly until the charges have settled and the field goes to zero.

Practice Problems

Example 8.4.30

Draw the field lines created by four equal charges arranged in the pattern of a square, as shown below.



Example 8.4.31

A charge of 6 μC is released into a uniform electric field of 2000 N/C. If the mass of the charge is 0.003 g, what is the final kinetic energy of the particle after it has moved 10 cm?

Example 8.4.32

An object with a mass of 250 g and a charge of 1.5 C is dropped 5 m through an area where the only force acting on is gravity. After that area, it reaches an electric field with a value of 10 N/C acting upwards. If the field is designed to bring the particle to a complete stop and then automatically shut off, what is the minimum thickness of the field required?

Example 8.4.33

At a distance of 3 m from a point charge, the field value is measured by placing a 0.002 C charge at that point and measuring the force to be 0.05 N. What is the value of the field at that point, and what is the value of the point charge?

Example 8.4.34

If a 0.4 mC charge is positioned at the origin and 0.5 mC charge is placed at (3 cm, 0), what are the following measurements:

a.) The field at (3 cm,0)
b.) The force the 0.5 mC charge experiences
c.) The field at (0,0)
d.) The field at (2 cm, 2 cm)
e.) The force on a 0.1 mC particle placed at (2 cm,2 cm)

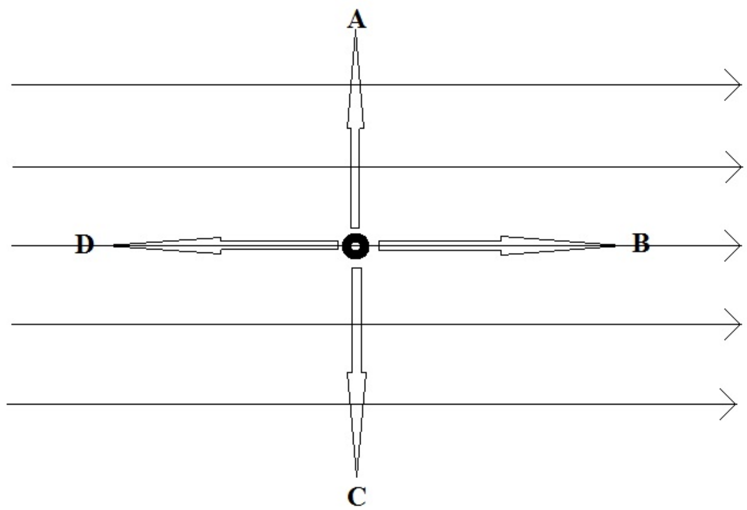
Example 8.4.35

A dipole is created by taking 2 0.8 mC charges and placing them 10 cm apart. If we imagine that the center of the dipole is placed at the origin, and the positive charge at 5 cm. Find the value of the force on a particle charged to -0.022 C and placed at (3 cm, 3 cm).

Example 8.4.36

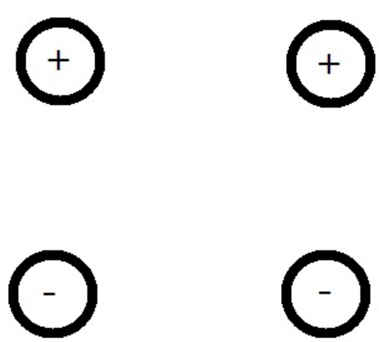
A spherical point charge of 4 C is placed in an external field with a value of 8 N/C as shown below. Find the following:

a.) The force on the 4 C charge.
b.) The field value at points A, B, C, D (each of which are 1.5 m away from the point charge)



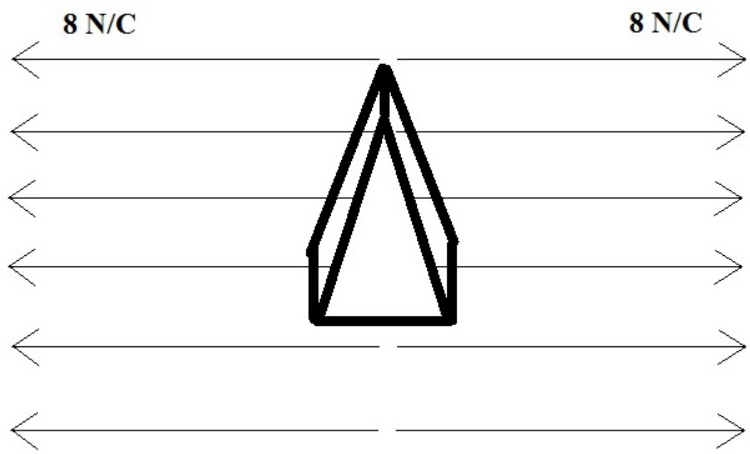
Example 8.4.37

Four charges of equal value (2 positive, 2 negative) are arranged in the square as shown below. Each side of the square if 30 cm. A 0.12 C charge is placed in the exact middle of the square and it feels a force of 1.75 N directed straight upwards. Find the value of one of the four charges in the square.



Example 8.4.38

A prism shaped box is placed in the electric field shown below. The front faces of the prism are triangles, with a base of 6 cm and a apex angle of 30 degrees. The length of the prism is 15 cm. Find the flux of the electric field through the prism.



Example 8.4.39

A solid, insulating sphere of radius = 12 cm is charged with 2 C worth of charge which eventually spreads itself uniformly through the sphere. Find the volumetric charge density and use Gauss' Law to find the field at r = 6 cm and r = 10 cm.

Example 8.4.40

A small, 0.04 mC charge is placed 10 cm away from a long, charged wire and experiences a force of 0.3 N. What is the linear charge density of the wire?

Example 8.4.41

Three identical charges are placed randomly on a plane. A box is then placed around the charges which is rectangular and is 10x12x20 cm. The flux through the box is 32 Nm^2/C. What is the value of one of the charges?

Example 8.4.42

A point charge of 1.6 C is suspended in the center of a hollow conducting sphere of inner radius 20 cm and outer radius 24 cm. What is the surface charge density on the inner shell? What is the charge density on the outer shell?

Example 8.4.43

An electron is placed 5 cm above a large sheet of charge with a surface charge density of -1.3 C/m^2. What is the velocity of the charge after it has traveled 10 cm? Why can't this same type of problem be done with a solid sphere of charge instead of a sheet?

Homework Assignments

Homework 8.2 - Review of Gravity

1.) What is (a.) the force of attraction between the Sun and Saturn? (b.) the value of the Sun's gravitational field at Saturn, (c.) the value of Saturn's gravitational field at the Sun, and (d.) the acceleration due to gravity on the surface of Saturn?

2.) Suppose you had two cars (assume a mass of 1500 kg each) and wanted to position them so that the gravity between them was measurable (say 0.1 N). How far apart would you have to place them? Comment on this distance.(G9)

3.) At some point between the earth and the moon the combined gravitational field between them is zero. How far away from the earth is this? Express the answer in meters and in percent of the total distance. Draw a scale diagram showing this location.(G1)

4.) Which exerts a stronger combined pull on the earth from the sun and moon, a solar eclipse or a lunar eclipse? How much stronger is the strongest of the two? Express you answer in terms of percent (the difference divided by the weaker).

5.) Imagine that an object was released from a point half way between the earth an the moon and allowed to fall to earth.

a.) What would be its speed on impact, neglecting air resistance?
b.) If it arrived at the surface with only half of the speed found in part a, how high would its temperature have risen provided it was made of iron and had a mass of 1000 kg? (You must also assume that all the energy went into heating the object and very little into heating the air.)

6.) Using formulae, show how the gravity on the surface of a planet would change if the radius was increased and the density was kept constant.

7.) How far above the surface of the earth must you go for your weight to be one half of its value on the surface? Answer both numerically and in terms of the earths radius r_e.

8.) It is possible, using the sum of the forces and Newton's Universal Law of Gravitation, to derive Kepler's Third Law of Planetary Motion. Do so.

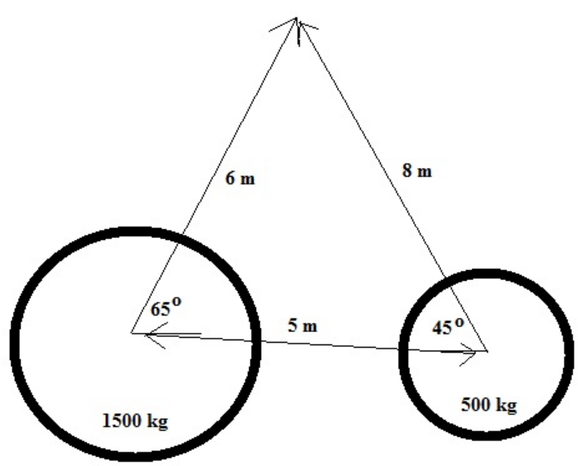
9.) Using the results of the shell theorem, prove that the gravitational force on an object inside a planet of constant density is proportional to r, the distance from the center.

10.) If you stand on the earth, and consider yourself as stationary, you have not only gravity acting on you, but also a centrifugal force from the rotation of the earth. What percent of your weight is the centrifugal force acting on a person at the equator? (G13)

11.) Consider the earth-moon system.

a.) What is the force of gravity from the earth acting on the moon?
b.) What is the force of gravity from the moon acting on the earth?
c.) What is the value of the gravitational field of the earth at the position of the moon?
d.) What is the value of the gravitational field of the moon at the position of the earth?
e.) What energy is stored in the bond between the earth and the moon?
f.) How much energy would it take to increase the moon's distance from the earth by 100 km?

12.) What is the gravitational field strength at point P in the diagram below. How much energy would it take to move the 500 kg mass to point P?



13. The cavendish experiment is carried out and the following measurements are made:

Mass of small balls = 10 kg
Mass of large balls = 300 kg
Mass of rod = negligible
Distance between small and large centers at equilibrium: 0.45 m
Angle of twist of the wire = 1.23°
Length of rod = 2 m
Thickness of wire = 2 mm
Tension measurement of wire: 0.0008 N per degree of twist, with force measured at edge

With this information, determine how far the light would have moved on a screen 6 m away from the center, and determine what value of "G" would have been measured.

Homework 8.3 - Electric Charges

1.) How many Coulombs of charge are present in the electrons of one ounce of gold?

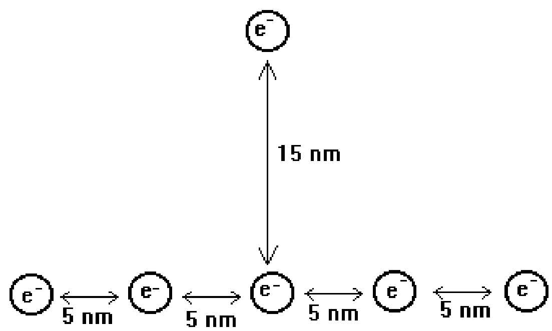
2.) If an 8 C charge is placed at the origin and a 6 C charge is placed at (3 m, 3 m), what would be the force between them?

3.) If a 3 C charge were placed at the origin, and a 5 C charge placed on the x-axis, 2 m away, there is a spot between them where you place a random charge and it would feel no electric force. What is the coordinate of that spot? Now suppose the larger charge was a negative charge (same value). Where would the spot be now?

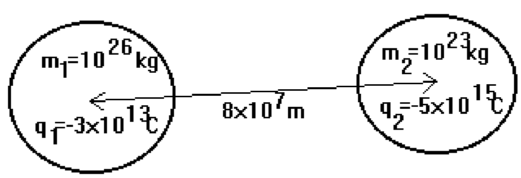
4.) Consider a penny to be made of copper (GAW = 63.5 g/mole, atomic number = 29, mass of penny = 3.11 g. Through some fluke of nature, imagine that the charge on a proton were slightly higher than the charge on an electron, by 0.00001%. What would be the repellant force of two pennies place in your pocket (2 cm apart)?

5.) Suppose an evil villain came up with a plan for pushing the moon out of its orbit. He (or she) decided to take regular water and remove all the electrons. He (or she) would then place half of the electrons on the moon and half on the earth so the two planets would repel each other. How much water would he (or she) need? Answer in kilograms and volume (i.e. a cube X m on each side). The gram molecular weight of water is 18 g/mole.

6.) Consider the set-up below. Calculate the net force on the single charge above the line of charges if all the particles are electrons. Consider symmetry, it may save you some time. (EL17*)



7.) Imagine the two planets below. If the planets not only have mass, but are also charged, what is the force between them?



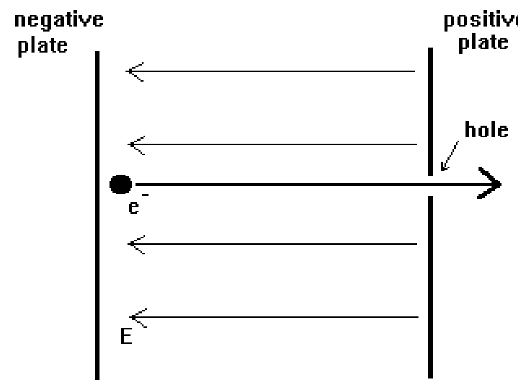
8.) Consider a 5 C charge placed at the origin. If a 3 C charge was placed at (0, 2 m), and a 1 C charge was placed at (3 m, 0), what would be the force on a 4 C object placed at (2 m, 2 m)?

Homework 8.4 - Electric Fields

1.) A charge of 14e is placed in a uniform electric field with a strength of 11 N/C. What force does it feel ? (EL20)

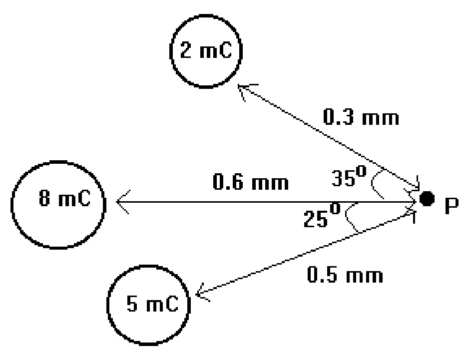
2.) Consider the electron "gun" shown below. The electron leaves the filament, and is accelerated towards the plate by a field of

200,000 N/C between the plates. It then shoots out of the tiny hole in the positive plate. If the plate are 4 cm apart, with what velocity will the electron leave the gun? (E1*)



3.) Three electric charges are arranged in a line and have charges of 5 C, 7 C and 12 C (respectively from left to right). If the 7 C charge is 0.35 m to the right of the 5 C charge and 0.60 m to the left of the 12 C charge, what is a.) the force on the 7 C charge and b.) the value of the electric field at that point caused by the other charges ? (EL19)

4.) What is the electric field caused by the three charges at point P? What force would a 4 mC charge feel if placed at that point? (EL34)



5.) On a coordinate grid, marked in centimeters, a 0.5 μC charge (A) is placed at (5,0) and a 0.8 μC charge (B) is placed at (0,3). Find the following quantities:

a.) The force on charge A from charge B
b.) The field created by A at the location of B
c.) The field created by B at the location of A
d.) The field at the origin
e.) The force on a 1 μC charge placed at the origin

6.) A single point charge is placed in a uniform field directed to the left at 6 N/C. A second point charge is placed at a point 2 m directly to the right of the first and it experiences no force. Find the value of the first point charge.

7.) A Gaussian surface in the shape of a cube is centered over a single point charge of 2 C. Each side of the sphere is 1 m long. Draw a sketch of what the flux of one side of the cube would look like in this situation. Even though the it is difficult to calculate the flux directly, Gauss' Law and symmetry arguments can be used to find the average electric field cutting through each side of the square. How does this average value compare to the actual value of the field at the center of each side, as calculated by Coulomb's Law?

8.) A bizarre electric field is found in a deep region of space that obeys the following equation:

$$E = (800\ N/C.km)y + (12\ N/C.km^2)y^2$$

Where y is the distance in kilometers from an imaginary x- axis. The field appears to completely symmetric along the z-axis. How much charge is contained in a one kilometer cubed area starting at the origin? How much charge is contained in a similar cube directly above this one (in the y-direction)?

9.) A 2 C charge is suspended inside a hollow, conducting sphere with inner radius of 20 cm and outer radius of 30 cm. 1.5 C of charge is then placed on the sphere. Calculate the following:

a.) Surface charge density on inner sphere
b.) Surface charge density on outer sphere
c.) The field equation for the inner cavity
d.) The field equation for the outer cavity
e.) The field equation for the area outside of the sphere

10.) Two, large flat plates are placed parallel to each other 8 cm apart and each is given an equal surface charge density of 2.3 C/m^2, although one is charged positive and the other is charged negative. An electron is released at the negative side and accelerates towards the positive side. What is the kinetic energy of electron when it reaches the positive side.

11.) A wire withe a charge of 3 C/m is suspended coaxially inside a hollow, conducting tube with inner radius of 10 cm and outer radius of 30 cm. 3 C of charge for every meter is then placed on the tube. Calculate the following:

a.) Surface charge density on inside of the tube

b.) Surface charge density on outside of the tube

c.) The field equation for the area between the wire and the inside of the tube

d.) The field equation for the area inside the conducting tube (radius between 10-30 cm)

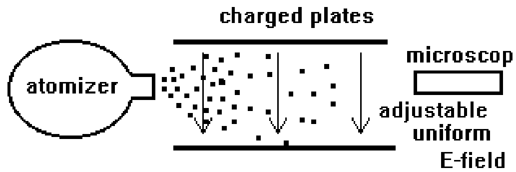
e.) The field equation for the area outside of the tube

12.) A test particle is immersed in an electric field created by a charge positioned somewhere on the x-axis of a coordinate system. The test particle feels the forces listed below at the corresponding positions on the x-axis. By graphing the data below, determine the position of the charge creating the field and the product of the two charges involved. Hints: Consult known equations in order to determine which graph to draw, use x as your dependent variable, and use $x-x_o$ as the distance from one charge to the other, where x_o is the position of the charge creating the field.

X	Force
6 cm	5.620×10^{10} N
6.5	3.596×10^{10}
7	2.497×10^{10}
7.5	1.830×10^{10}
8	1.400×10^{10}
10	6.240×10^{9}
12	3.510×10^{9}

(EL32)

13.) One of the most important experiments that determined that charge is quantized and measured the elementary charge was Millikan's Oil Drop experiment. A simplified version and explanation follows. The apparatus (below) consisted of an atomizer (basically a spray bottle) that sprayed tiny drops of oil into the field between the two charged plates. Millikan spotted one tiny drop in a microscope and then adjusted the electric field until the spot was balanced in mid-air between the force of gravity and the electric force. If he then knew the weight of the drop and the strength of the field that balanced the drop he could determine the charge on the drop. The charge on the drop must then be some integer multiple of the elementary charge.



Consider all the drops to have a radii of 1.64 micrometers and take the oil to have a density of 0.851 g/cm^3. Suppose Millikan found that the six drops were balanced by the electric fields listed below. What would he have concluded for the value of the elementary charge ?

E_1 = 163482 N/C
E_2 = 98089 N/C
E_3 = 75453 N/C
E_4 = 65393 N/C
E_5 = 51626 N/C
E_6 = 42647 N/C

(E9*)

13.) Decipher: "It is futile to indoctrinate a superannuated canine with innovative maneuvers." (DNCTHWG)

Labs and Activities

Activity 8.1 - The Electroscope

In this activity, you will use an electroscope (as discussed in the chapter) to investigate different electrical phenomena. For each different investigation, you should go beyond what is written and attempt to compare the charges produced in one instance with charges produced in another (for both strength and sign).

Hints: If the electroscope is too highly charged, instead of touching it directly, charge an isolated conducting sphere and touch the sphere to the electroscope. When grounding the object, use a wire connected either to a metal plumbing fixture or to the third slot in a three prong outlet.

Procedure:

1.) Bring the electroscope near the Van DeGraff generator as instructed by your teacher. Observe the effects.

2.) Bring the electroscope in contact with the Van DeGraff generator (please don't be surprised and drop the electroscope when the sparks fly).

3.) Ground the electroscope, bring it near the Van DeGraff and then remove the ground wire.

4.) With the electroscope charged, wave a magnet around the casing and observe the effects.

4.) Discharge the electroscope (by grounding it) and touch it with the magnet.

5.) Using a 9 V battery, touch one terminal to the electroscope. Discharge the electroscope and touch the other terminal to the electroscope. Observe the effects and discharge the electroscope.

6.) With a piece of fur, rub a plastic rod about thirty times. Touch it to the electroscope. Observe and discharge.

7.) With a piece of silk, rub a glass rod thirty times. Touch it to the electroscope, observe and discharge.

8.) Compare the effects on the electroscope between the charged plastic and glass rods by charging it first with one and then the other without discharging it in between. Discharge when finished.

9.) Get a low-level radioactive source from your instructor and touch it to the electroscope, holding it in place for a few minutes. Repeat with the other sources provided.

Draw conclusions about each situation. Remember: an electroscope determines if a charge is present and gives some clues about the strength of the charge.