Chapter 3
Two Dimensional Motion

3.1 General Two Dimensional Motion Concepts

When we take our discussion of motion to the next level, so to speak, we find that many of the concepts or ideas that applied in one dimensional motion still apply in two dimensional motion. We still have variables like Δx (displacement), v (velocity), and a (acceleration). However, things become just a little bit trickier.

In a general high school or college physics class, there are four sub-topics that make up the topic of two dimensional motion. They are general two-dimensional motion, projectile motion, circular motion, and relative velocity problems. The first two topics are very closely related and the problems are solved the same way in both topics. The last two, however, are very different.

The one main idea behind understanding general two dimensional motion and projectile motion is this:

> **When an object moves in two dimensions (for example, the x direction and the y direction), we can treat each direction independently of the other and we can use our equations of motion in each direction.**

Another way of saying this same things is to say that:

> **When an object moves in two dimensions, is has a position, velocity, and acceleration in each dimension.**

For example, if we consider the case of an object rolling at a diagonal across the floor, the object has:

	X-Direction	Y-Direction	Overall
Position	x	y	(x,y) or Δd
Velocity	V_x	V_y	V
Acceleration	a_x	a_y	a

The overall variables are related to the x and y variables in the exact same way that vectors are related to their components (this is natural, since that is what they really are).

All of our equations for one dimensional motion still work, provided we only use them in only on direction at a time. So we have:

$\Delta x = v_x t + (½) a_x t^2$	and	$\Delta y = v_y t + (½) a_y t^2$
$v_{fx} = v_{ix} + a_x t$	and	$v_{fy} = v_{iy} + a_y t$
$v_{fx}^2 = v_{ix}^2 + 2a_x \Delta x$	and	$v_{fy}^2 = v_{iy}^2 + 2a_y \Delta y$

Now, an astute student will notice that all we are really saying is that since displacement, velocity, and acceleration are all vectors, then our equations of motion can be treated as vectors. In fact, these types of problems can be done either way. You can treat them as components, or deal with the entire vectors. We will do a few examples each way, but for a beginning student you will probably find that treating the x and y individually is much easier. A few examples will demonstrate exactly what all this explanation means.

Example 3.1.1

A spaceship takes off from rest with a forward (x) acceleration of 6 m/s^2, and a vertical acceleration of 10 m/s^2.

a.) What is the displacement of the ship after 4 sec?
b.) What is the magnitude and direction of the velocity after 4 sec?
c.) What is the overall magnitude and direction of the acceleration?

Example 3.1.2

A spaceship is traveling at 50 m/s horizontally when it encounters "space turbulence" that causes it to accelerate vertically at 120 m/s^2 for 0.3 seconds.

a.) What is its new velocity?
b.) What is the overall displacement?

Example 3.1.3

A spaceship is traveling at 2000 m/s at an angle of 80^o when it encounters space turbulence which cases it to accelerate at 350 m/s^2 at an angle of 150^o for 1.3 seconds.

a.) What is it's new velocity after the acceleration period?
b.) What is its displacement during that time period?

Example 3.1.4

If a spaceship has an initial velocity of v = 200i - 320j and it accelerates according to a = 32i + 12j for 4 seconds, what is its final velocity?

Example 3.1.5

An object starts at (0,0) with a V_x = 30 m/s and a V_y = 22 m/s. At the end of a certain amount of time, it is at (22, 14) with a V_x = 12 m/s and a V_y = 36 m/s. What was the time that passed and what was the acceleration?

3.2 Projectile Motion

Once we have an understanding that the x and y motion of an object are independent, understanding projectile motion becomes very easy. The entire idea of solving projectile motion problems can be summed us as follows:

> **In the case of a projectile, the horizontal direction is treated as constant velocity, while the vertical direction is treated as free-fall (acceleration with a = g = 9.8 m/s^2).**

Once an astute student is told this, they should know how to solve any and all projectile problems on their own. However, projectile problems are very important, so we will not simply leave it at that (also, if you Physics class skipped section 3.1, as many classes do, then you won't understand what was stated above). What follows is an in-depth look at how projectiles behave and how to solve projectile problems. It begins with a conceptual approach and then moves into a mathematical approach.

Understanding Projectiles Conceptually

Imagine that you fired a rifle, held perfectly horizontal, while a friend stood along side the barrel (this is not a recommended activity to try at home) and dropped a bullet from the same height as the barrel at the same instant that your bullet emerged from the gun. Which of the two bullets would hit the ground first? If we ignore air resistance, the answer is that both bullets would hit the ground at the same instant. This somewhat surprising answer comes from the fact that was mentioned earlier: gravity causes the same acceleration in all objects.

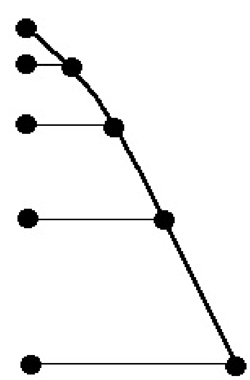
There is also another concept at work here, a very important one:

When an object moves both horizontally and vertically, the motion in one direction can be treated as occurring completely independent of the motion in the other.

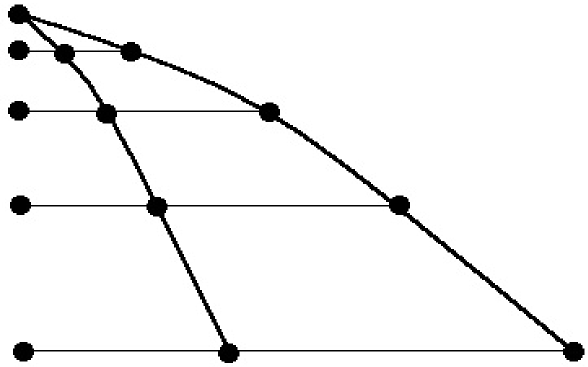
To understand this concept, we need to resolve the velocity of any object into a horizontal (x) component and a vertical (y) component. In the case given above, the dropped object has zero x-velocity and zero y-velocity (initially). The fired bullet has a very high x-velocity and zero y-velocity. For a moment, let us ignore the x components and focus on the y components. Both projectiles have zero initial velocity in the y direction. Thus vertically they will behave identically (remember, we can ignore the x direction because the horizontal and vertical directions are independent). In short, they fall together and since they have the same distance to fall, they will take the same amount of time to fall.

This can be explained graphically. Consider an object being dropped. We learned in the previous chapters that the object falls according to the squares of the time passed. Thus if we mark the position of the object after each second (or any unit of time) the diagram might look like the one below:

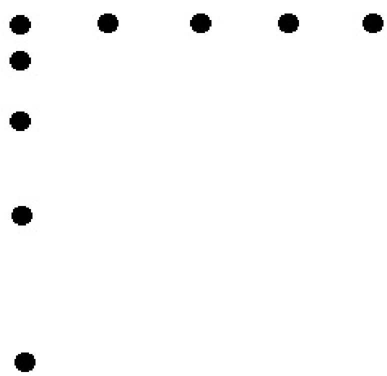
If we threw an object horizontally from the same point at the same time, it would fall the same vertical distance each second, but also move over horizontally. The resulting motion would look like this:



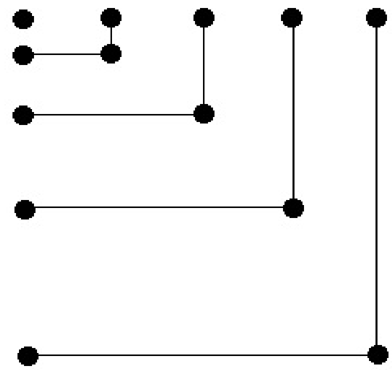
Where the darker line indicates the path of the ball and the thinner lines are drawn to show you how the balls match up in vertical position every second. If we threw three balls together, it would look like this (remember, these are all thrown perfectly horizontal, not at an angle):



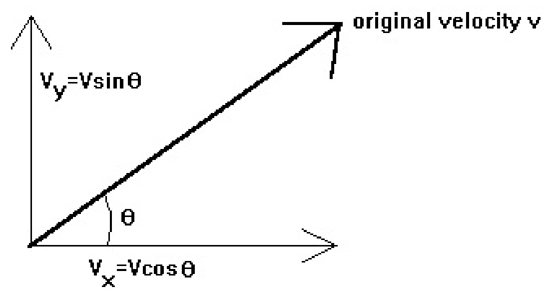
Although we have been focusing primarily on the vertical components, one very special mention should be made about the horizontal motion. There is no acceleration in the horizontal direction, thus it is motion with constant velocity. Since the horizontal velocity remains the same, the particle moves equal distances in the x direction in equal amounts of time. Since the two motions can be considered independent, a projectiles path can be seen as the combination or intersection of a falling object and an object moving horizontally. Below we have a falling object and an object moving horizontally with zero acceleration.



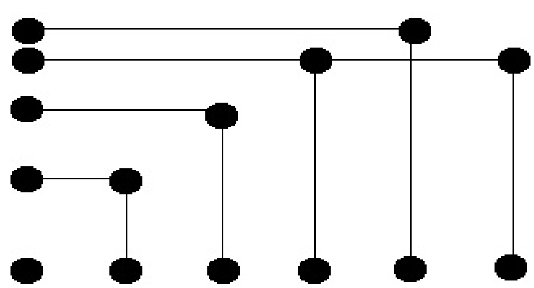
If we draw lines and connect the corresponding dots, we get the path of a projectile fired <u>horizontally</u> with a velocity equal to the velocity of the original x projectile.



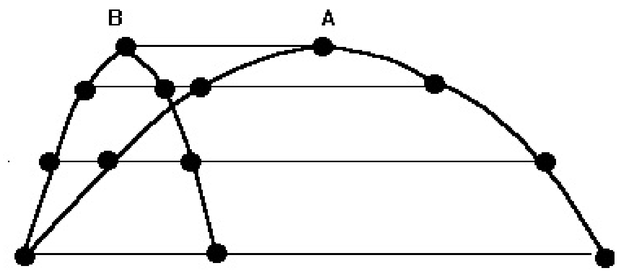
The following examples all had the disclaimer that the projectiles must be fired horizontally. But what about a projectile fired up at an angle? In order to deal with such a situation, we need to resolve the velocity into x and y components as shown below.



Once that is accomplished, we treat the x and y directions independently, using V_x as the initial velocity in the x direction (which will remain unchanged) and V_y as the initial velocity in the y direction (which will change according to the acceleration due to gravity). In this case, we have two motions, one of an object thrown upwards and the other of an object moving to the side at constant velocity. Combining these two motions gives a diagram like the one below.



Comparing the motion of two projectiles launched at different angles is a bit more complicated than our previous comparisons. A projectile launched at such an angle (θ) will land at the same time (and match the vertical motion exactly) as any other projectile launched at the same y velocity (vsinθ). For example, consider the two projectiles drawn below.



In this case, projectile A was launched with velocity v at angle θ giving it vertical velocity of vsinθ, while projectile v was launched at a lower velocity v_2 and a steeper angle θ_2, giving it vertical velocity of $v_2\sin\theta_2$. The important thing is that the statement $v\sin\theta = v_2\sin\theta_2$ must hold true. Since the projectiles have the same vertical velocity, they have the same vertical motion and thus match each other in y position along their paths. The student should also be able to conclude two other facts about projectiles launched with the same y velocity (I leave that to the student to ponder). One last comment on this example, the student should note on the diagrams that the distances traveled in the x direction are not the same for each particle, but are consistent through the motion of a single particle (in other words: $v\cos\theta \neq v_2\cos\theta_2$, but there is still no acceleration in the x direction for either particle.

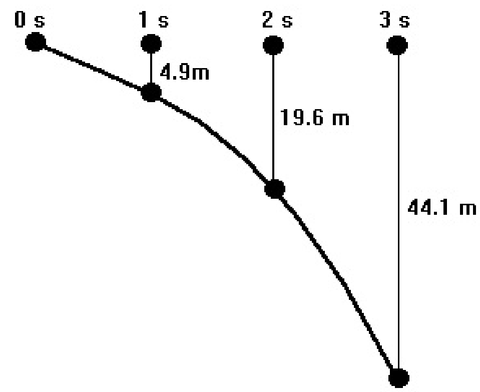
Before we proceed onto a number of numerical examples, there is one other very interesting conceptual idea to discuss. The entire above discussion can be summed up in one line.

All projectiles deviate the same vertical distance from their straight line intended path in the same amount of time.

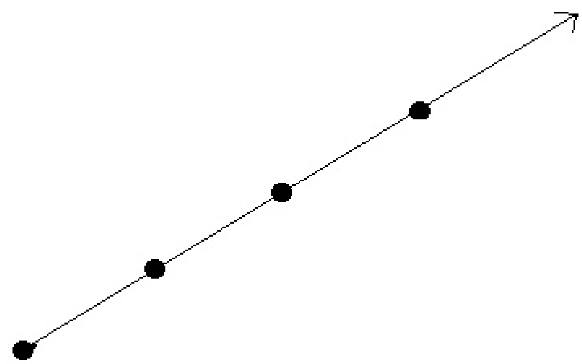
The straight line intended path mentioned is the path the particle would take in the absence of gravity (straight line, constant velocity, simply imagine the object moving at its initial velocity forever). Consider an object being dropped. We know that in the first second it is 4.9 m below the starting point, at 2 seconds it is 19.6 m below the starting point and at 3 seconds it is 44.1 m below the starting point.

Now consider an object thrown horizontally. Its straight line intended path would look like:

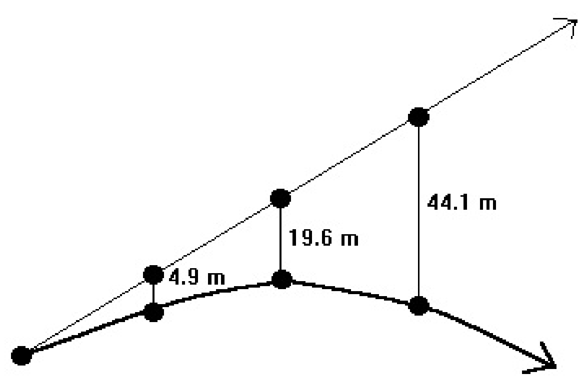
But according to the above statement, it should deviate from this path the same as a dropped object. Thus:



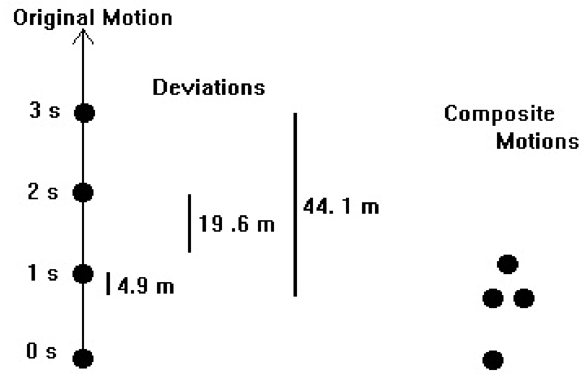
The previous example was just a restatement of what was said before. However, consider the case of a projectile fired at some angle. The intended straight line (constant velocity) path would then resemble the diagram below.



Since we know that the particle will deviate from this the same amount as the dropped object (or the horizontally thrown object), we can immediately draw the path of the projectile.



An even more interesting example can be made with an object thrown directly upwards. This, however, is very difficult to draw. The diagram below is an attempt, where the straight line path is marked with dots and each deviation is drawn <u>along side</u> the path (instead of directly on top which would be confusing). Please remember that these lines are all on top of each other in reality and the motion is along a straight line. The dots to the right are the composition of all the other dots, with the downward part of the motion drawn along side for clarity.



In this diagram we can see how the deviations become so much greater than the positive gains made by the constant velocity, that the particle begins to descend.

Although the above discussion seems complicated, it all derives from an understanding of vector addition. When you consider the equation:

$$\Delta \underline{x}=\underline{v}_i t+(1/2)\underline{a}t^2$$

We see that it is actually a vector addition equation stating that the position vector is equal to the sum of a velocity vector times a scalar (time) plus an acceleration vector times a scalar. Since the acceleration vector is the same for all projectiles (it is the acceleration due to gravity), provided the time is the same, all projectiles will fall the same distance because of gravity from their straight line path (represented by the velocity vector). Don't worry if you don't quite follow all this, it is a pretty sophisticated concept.

Projectile Motion Problems

When attempting to solve projectile problems there are a number of things to keep in mind. First, these problems must all be in free fall (no air resistance) near the surface of a planet (the earth, if we want gravity to be 9.8 m/s^2). For projectiles that go very high into the atmosphere, this method will not work. Secondly, you must be very careful about your signs. Sign mistakes are the most common errors made in these problems, so be very careful.

The concepts we have studied in the previous discussion tell us our method of solving these problems. We treat the x and y directions independently and in essence have one equation of motion in each direction.

The most common equations to use are:

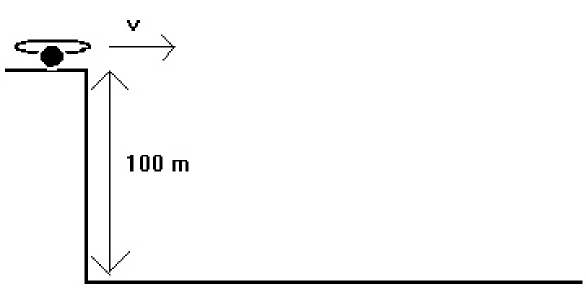
$$\Delta x = v_x t$$

for the x direction and

$$\Delta y = v_y t + (1/2) g t^2$$

for the y direction. Notice how each equation contains only the information for a specific direction. The first contains Δx (the x displacement) and v_x (the component of the initial velocity that lies in the x direction) and the second contains Δy (the y displacement), v_y (the y component of the initial velocity) and g (the acceleration in the y direction). Further note that v_x can be expressed as vcosθ and v_y as vsinθ. Information gathered in one equation can be transferred to the other (for example, the time in both directions must be the same), and this allows us to combine equations to solve for more than one variable. Without further ado, let us try our first problem.

Example 3.2.1
If a cannon is positioned horizontally to fire at 70 m/s off a 100 m cliff, how far from the cliff will the cannon ball land?



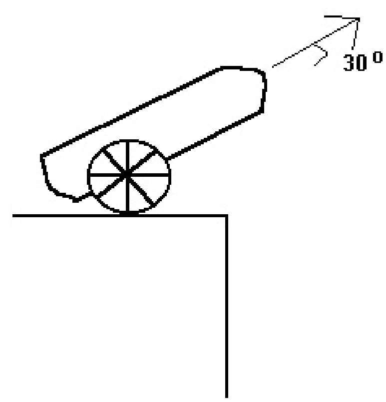
This past example should have shown the importance of superimposing a coordinate system on the motion to make it more easily understandable, as well as the importance of signs. This was also a perfect example of the method described above. When the x equation was written (since the quantity asked for was the x distance, this should have been the starting point), we noticed that there were two unknowns. In math class, you should have learned that in order to solve an equation of two unknowns, you need two equations. Thus we went to the x direction and solved for the time, and by plugging back into the y equation we narrowed it down to one unknown.

Two points of "cannon terminology" should be made clear; the velocity of a projectile out of a cannon or gun barrel is called the muzzle velocity and the horizontal (x) distance traveled is called the range.

The next example shows us how to deal with cases involving projectiles fired at an angle.

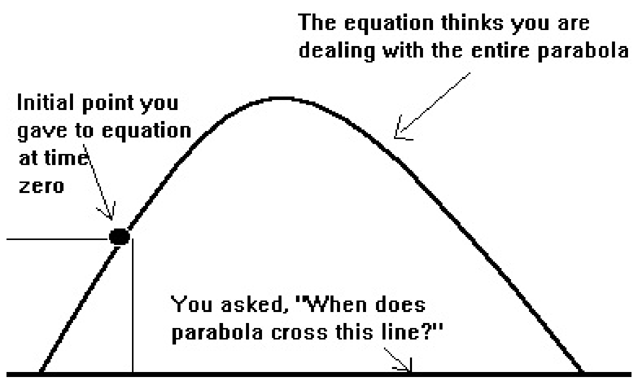
Example 3.2.2

Using the same cannon and cliff, what would the range be if the cannon was tilted at 30° to the horizontal?



In order to solve this problem, the student should be aware of the necessity of superimposing a coordinate system onto the motion. It is typical for the origin to be at the top of the cliff (or more accurately, at the point where the free-fall begins as the ball leaves the cannon). This helps us keep our signs straight and it gives readily recognizable meaning to our negative answers.

This second example also shows us two interesting results. The most obvious point of discussion revolves around the two answers given by the quadratic equation. What does the second, negative root mean? Many students are tempted to say that the answer is meaningless, since very often they are taught to throw out the answer that doesn't make sense. That is not the case here, however. Think about the physical situation involved and what you asked our "mechanistic" equation to tell you. You asked the equation to tell you when an object that was at position zero with initial velocity of 70 m/s at 30^0, reached point y=-100 m. Notice that you didn't tell the equation (really) that the particle began at 0 m with 70 m/s velocity. In reality, you said at the time we called zero, it was at this position with this velocity. The equation told you that the particle will be at y=-100 m in 9.3 seconds and was at -100 m, 2.1 seconds before the arbitrary time you called zero. To rephrase this, the path of a projectile is a parabola (a fact we will mathematically rigorously prove later). The two equations of motion (x and y), when combined, give the formula for a parabola. You are solving for one point on the parabola given another, known point (the initial conditions). The entire equation of the parabola is there, the math machine is unaware of the existence of any "cliffs" or "plateaus". The diagram below illustrates this point.

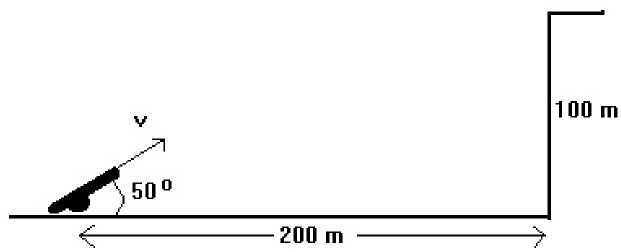


Another way to think about this mechanistic aspect of mathematical equations is that the two equations being used describe a general parabola, without beginning or end. Inputting initial conditions (initial velocity and acceleration) "pick" out a specific parabola from the infinite number of possibilities. The solution tells you the coordinates of a specific point on the parabola (for example, in the case above the equations told you that the parabola has two points where y=-100 m).

The second thing to learn from this example comes from comparing the result to the previous example (after all, it is the same cliff and cannon). We have a much greater range in the second example, but why? If we compare the velocity of the cannon ball in the x direction, we see that it was greater in the first example. Since $\Delta x=v_x t$, a greater v_x should mean a greater Δx. Obviously there is more to this question than is being presented. I leave it to the student to attempt to explain why the range was greater in the second example. The student should also think about whether or not this pattern will continue to hold for greater angles (i.e. if we continue to increase the angle further, will the range continually increase?).

Example 3.2.3

Consider the situation below. If the cannon fires at 60 m/s, will the cannon ball land on or hit the cliff?



There are numerous methods for solving this problem. Below are listed a few:

1.) Solve for the y position when x=200 m and determine if y>100 m.
2.) Solve for the x position when y=100 m and determine if it is greater than x>200 m.
3.) Solve for the maximum height and determine if the x coordinate is such that it will clear the cliff.
4.) Solve for the minimum velocity and angle that will cause the cannon ball to just arrive at x=200 m, y=100 m.

However, the student must be very careful in choosing a method for solving the problem, since a number of the methods listed above are not complete. Some of the methods listed need further proof in order to answer the question sufficiently.

The previous example was probably done in a very crude manner, so we leave it as the next exercise to demonstrate a more sophisticated method of solution.

Example 3.2.4

Solve the same problem by creating an equation for the height of the projectile as a function of horizontal position. Use this equation to justify the statement that projectiles follow a parabolic path.

The above equation is very important, since it involves only the horizontal and vertical position and not the time. In other words, it is a general equation that can be used for any projectile when you are not interested in the time in the air. It is worth writing again, in type.

$$\Delta y=(\tan\theta)\Delta x+(g/(2v_i^2\cos^2\theta))\Delta x^2$$

Please note that the student is responsible for inserting the correct signs into this equation (take special note of the sign of g).

If we consider the special case where a projectile is fired from ground level and lands on ground level, we can set $\Delta y=0$ in the above equation, giving us:

$$\tan\theta=(-g/(2v_i^2\cos^2\theta))\Delta x$$

Solving for Δx,

$$\Delta x=-2v_i^2\cos^2\theta\tan\theta/g$$

With $\tan\theta=\sin\theta/\cos\theta$ and using $2\cos\theta\sin\theta=\sin 2\theta$,

$$\mathbf{\Delta x=-(v_i^2\sin 2\theta)/g}$$

This is a very simple equation for the range of a projectile that is fired from the ground and lands at the same level.

Example 3.3.5

Use the above special case to determine the angle needed for maximum range.

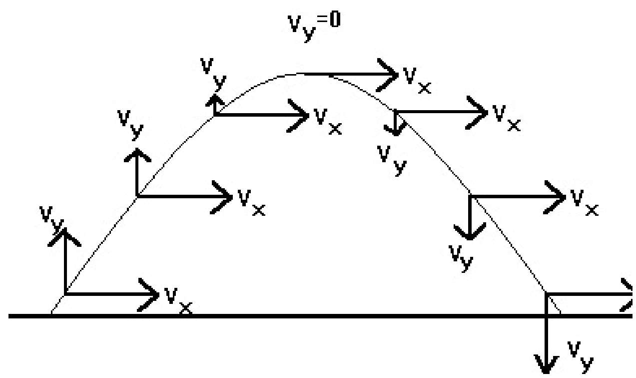
Our discussion of problem solving thus far has centered around distances and times, but of equal importance is the velocity. Below is an example of how to determine the velocity of a projectile at a given time or position.

Example 3.2.6

A cannon ball is fired from a 50 m cliff at an angle of 40° with a velocity of 70 m/sec. At what angle does it hit the ground and with what velocity?

We see from this example that the Δx and Δy equations of motion are not the only equations that we can use. We can use any of our three equations of motion, provided that we are aware that we are using them only in one direction and remember that the velocity is a vector made up of x and y components.

It is an interesting exercise to examine the x and y velocities of a projectile as it progresses along its parabolic path. We know that the horizontal velocity never changes (why?) and we know the y velocity begins at some value and gets more negative every second. One interesting thing to notice is that at the top of the path, the velocity is not zero, since there is still some x velocity (remember: the actual velocity of the object is the vector addition or combination of the x and y components).



Before we leave this section, I would like to make a list of statements about projectiles, both important and trivial. Some of the statements have already been mentioned (or alluded to)and I would encourage the student to be sure they understand all of these (as well as how to prove them).

Conclusions Regarding Projectile Motion

1.) The maximum range for a projectile occurs at 45^0 (if it lands at the same level from which it was launched).
2.) All projectiles with the same initial velocity in the y direction will spend the same time in the air.
3.) All projectiles with the same initial y velocity will rise to the same height.
4.) The student should combine number 2 and 3 to make a new rule.
5.) Projectiles launched from angles that are the same degree measure above and below 45^0 (complimentary angles) will have the same range.
6.) A projectile launched from the ground reaches the ground with the same velocity with which it started.

Example 3.2.7

An archer fires an arrow at a 40º angle at a speed of 75 m/s. How far from the archer does it land? (Assume free fall, and neglect the height of the archer in the problem.)

Example 3.2.8

A child kicks a stone off the edge of a canyon. It free-falls to the floor (80 m below). If he kicked it at 15 m/s horizontally,

a.) How much time elapses before it hits the ground?
b.) How far from the cliff does it land?
c.) At what speed and angle does it hit the ground?

Example 3.2.9

A circus cannon is mounted on a train that is traveling east at 25 m/s. It is fixed at a 60 degree angle to the horizontal. The circus wants to fire a clown from the train car and have him land in a large bucket of water. If the clown will have a muzzle velocity of 30 m/s, how early should they fire the clown before the cannon car reaches the bucket of water. Make the following assumptions - the bucket of water is right next to the train tracks, so that the cannon fires almost perfectly straight ahead, ignore the small angle needed for the clown to move to the side and reach the water. Also assume that the top of the bucket of water is the same height as the muzzle of the cannon ($\Delta y = 0$).

Example 3.2.10

A cannon ball is launched at a muzzle velocity of 60 m/s upwards towards a 150 m cliff. The cannon is 200 m from the base of the cliff and is positioned at a 70 degree angle to the horizontal. Does it land on the cliff of strike the cliff face? What is its velocity when it lands or strikes? Remember, velocity includes angle.

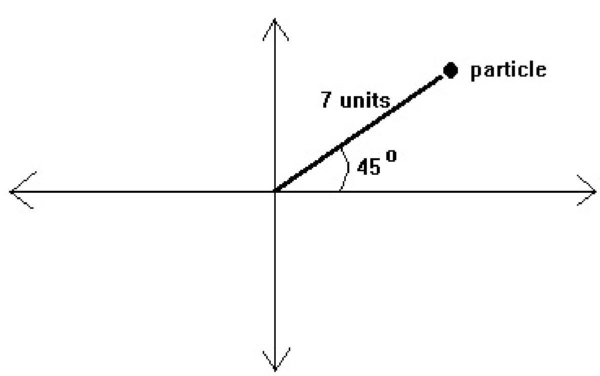
3.3 Circular and Angular Motion

The Angular Variables

We began our discussion of motion by focusing on motion in a straight line, then moved on to projectile (or two dimensional) motion. There is yet another type of motion that we should deal with: circular motion. Circular motion can be viewed as either two dimensional motion (since it occurs in a plane and not on a straight line) or one dimensional motion (where the dimension simply wraps around a circle and back onto itself). If we view it as one dimensional motion, where the circle is continuous (with 360^0 just happening to fall at the same physical location that 0^0 does) then mathematically the motion is easier to understand.

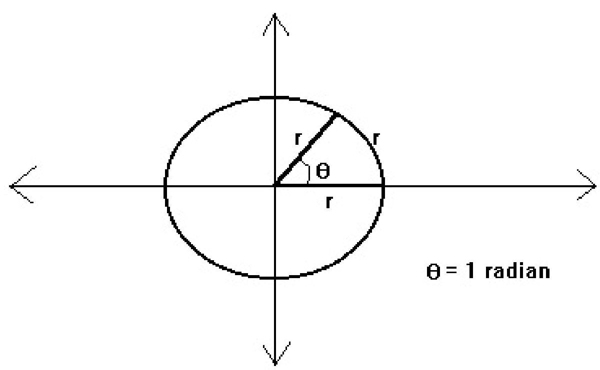
The actual definition of circular motion is motion of an object about some external point which is called the axis or center of the motion. In the cases we will be dealing with, the distance from the axis to the object will be considered a constant. To an outside observer, this motion occurs along a circular path (special note: generally we consider the object to be very small in relation to the radius, or else we will have different radii for the different parts of the object. It is best to think of the object as a point mass). It should be noted, however, that there is another type of "circular" motion that we will be dealing with. This occurs when the axis of rotation is inside or on the object. When such a situation occurs, we call it rotational motion. Note that in rotational motion, the concept of radius in general is useless. Each point on the object will have a different radius. In this case, we either talk about the object as a whole (never mentioning radius) or we discuss the motion of one point on the object and its corresponding radius.

When dealing with circular motion, it is convenient to change coordinate systems. We are used to locating a point on a plane by assigning it a name consisting of an x and y variable. This is called the Cartesian or rectangular coordinate system. For example, if we say the particle is located at (3,7), we are giving its location as 3 units to the right and 7 units up from the origin. There exist many other different coordinate systems that would have located the particle equally as well. In circular motion we use polar (sometimes called plane polar) coordinates to locate a particle. Instead of x and y, we use r and θ. When we say that the particle is at $(7, 45^0)$ we are saying that its r value (the distance from the origin) is 7 units and it is at a 45^0 angle from the +x axis (see diagram).



This may seem very similar to our vector notation, and indeed it is, however there are some very important and subtle differences. The main difference is that vector notation indicated the direction of a quantity and polar coordinates only give the location of a single point. However, your understanding of vectors should make working in polar coordinates easier.

Imagine the point on the above diagram were execute circular motion. In that case, consider your answer to the first question of motion ("Where is it?"). You would give your answer as a distance r and an angle θ. Since r is assumed constant for true circular motion in our discussion, your complete answer could just be an angle θ. In fact, the particle is only moving in one direction, the θ direction. When you give the position of the particle, you would give an angular measure, such as degrees ("The particle is at 129^0"). Although degrees are an acceptable unit to measure angles in, in math and science it is better to use radians. One radian is defined as the angle subtended by an arc equal to the radius of the circle. In other words, imagine a circle like the one below, positioned on a coordinate system. Start at the +x axis and begin going around the outside of the circle counter clockwise for a distance equal to the radius. The angle inside the circle that is underneath this arc is one radian.



One radian is equal to about 58^0. Regardless of the radius of the circle, if you followed the above procedure, there would always be the same angular measure under the arc. The mathematical definition of the measure of an angle is radians is given by:

θ=arc length/radius

Many of you may remember this from math class, where you had to find the arc length of a segment around a circle. Please remember that this formula only works if the angle is measured in radians.

The above formula shows us that in a complete circle there is a total of 2π radians (since the arc would be the circumference). This leads us to a very useful conversion method to and from radians and degrees:

$$\frac{x \text{ degrees}}{360^o} = \frac{y \text{ radians}}{2\pi}$$

Since π is such a prominent part of the definition of radians, an angle in radians is often expressed in terms of π. For example, 60^0 is often written as π/3 radians and 45^0 as π/4. Although this method of writing an angle is convenient, it sometimes causes the student to forget that the angle measure is a number. Since π is a number (3.14159...), π/3 radians is 1.0472 radians. Keep that fact in mind when you work in radian measures.

Another angular measure sometimes employed in physics is revolutions. One revolution is simply 2π radians or 360^0 (all the way around the circle).

Now that we understand how to represent the position of a particle in circular motion, we can proceed to some definitions.

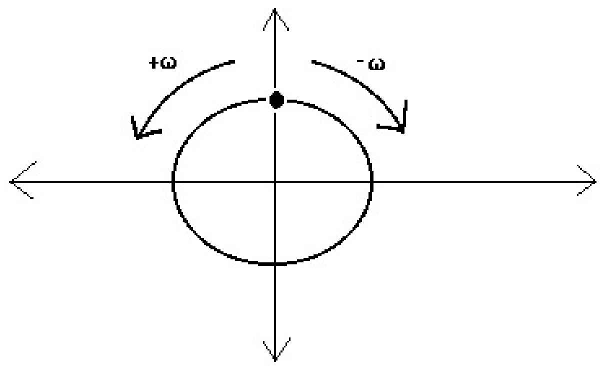
In circular motion we no longer identify the position or displacement of a particle with a measure in distance, we use an angle. Thus we have what is called angular displacement (Δθ) which is the angular change in position of the particle. If the angular displacement of the particle is 2 rad, for example, that means that over the time period considered, the particles position changed a total of 2 radians. Remember that angular displacement is a change, thus

$$\Delta\theta = \theta_f - \theta_i$$

We also might be interested in how quickly the particle moves around the circle. This is given by the angular velocity of the particle (ω), which is defined as the time rate of change of angular position. Mathematically,

$$\omega = (\theta_f - \theta_i)/t$$

The reader should note that the above definition is only good for average angular velocity, since we run into the same problem as was discussed previously for instantaneous velocity. The units of angular velocity are radians per second (rad/sec), or likewise any other angular measure per second (rev/sec, degrees/sec, or even revolutions per minute). I believe the reader understands the meaning of angular velocity from their previous experience with velocity, an angular velocity of 8 rad/sec means the particle covered an average distance of 8 rad in one second (or some ratio equal to that). It should be noted, as an aside, that angular velocity is a vector and thus has a direction. For our purposes, we should consider the direction to be according to the diagram below, where counter clockwise is positive and clockwise is negative.



You may have noticed that I said "for our purposes", which means that this is not really the case. It is a simple and direct way to assign a direction, but in reality the angular velocity of an object in circular motion has a direction perpendicular to the plane of the motion (don't even ask why). For now, though, we can just refer to the direction as CCW or CW (+ or -).

An astute observer will have already guessed where we are headed next with this discussion, to a definition of angular acceleration. Angular acceleration (α) is the time rate of change of angular velocity, thus:

$$\alpha = (\omega_f - \omega_i)/t$$

Angular acceleration answers the same question as regular acceleration does, "how fast is it getting faster?" It tells you how quickly the angular velocity of an object is changing. If was say an object has an angular acceleration of 3 rad/s^2, we mean that every second that goes by, the velocity changes by 3 rad/sec.

Having an understanding of the angular variables, we can now go immediately to using them mathematically. Our equations of motion are the same as they were in the one dimensional motion case, with a simple substitution of variables:

$$\omega_f = \omega_i + \alpha\Delta t$$

$$\Delta\theta = \omega_i t + (1/2)\alpha\Delta t^2$$

$$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$$

Without further ado, let us begin the examples.

Example 3.3.1

An object in a circular path has an initial angular velocity of 6 rad/sec and experiences an angular acceleration of -0.3 rad/sec^2. How long does it take to come to a stop?

From this example we see how easy it is to use these equations once one has a grasp of motion in general. The next example brings up the point of units in angular problems. The units used for angular measure are arbitrary, you can use radians, degrees or revolutions, as long as you are consistent in your usage.

Example 3.3.2

An object in circular motion begins at rest and angularly accelerates at 3 rad/sec^2. How long does it take to complete 4 revolutions?

The last example could have been done either completely in revolutions or in radians, as long as all the units matched in the problem. One last example to make sure you have not gotten rusty with these equations.

Example 3.3.3

An object in rotary motion begins with a velocity of 25 rad/sec and accelerates at 2 rad/sec^2. How long does it take to complete 45 revolutions?

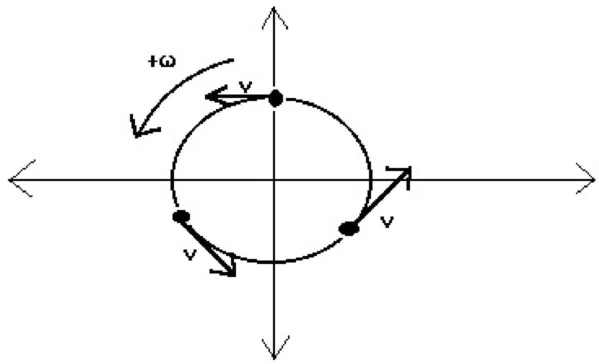
The Tangential Variables

We have been discussing how an object moves in a circle by using the angular variables to describe its motion. However, as anyone who has rounded a turn in a car can tell you, objects moving in a circular motion also have a regular velocity, measurable in meters per second. The example below shows you how to calculate that velocity.

Example 3.3.4

With what speed does the earth orbit the sun? (distance from earth to sun=1.5x10^{11} m)

This type of speed is generally referred to as tangential velocity when it is used to describe a particle in circular motion. It is termed tangential because of its direction. The velocity vector continually points tangential to the circle as the particle moves, as illustrated below.



The particle can be seen as having two velocities, one angular and one tangential. This concept, of applying "regular" velocity to an object in circular motion can be extended to our two other descriptive terms, displacement and acceleration. When we discuss the distance traveled, we are really referring to arc length covered during the motion of the object. Along the same lines, we could also discuss the tangential acceleration, which would be the acceleration the object would have if it were traveling in a straight line (in m/s^2). As you may have figured out, the tangential velocity and the angular velocity are related, in fact all three pairs are related in the same way. The relation is a simple one, the tangential variables are equal to the angular variables times the radius of the circle.

$$\text{arc length} = \Delta\theta r$$

$$v_t = \omega r$$

$$a_t = \alpha r$$

The equations above are very important, simple relationships and the student should commit them to memory. They come, however, with one caveat: all angles must be in RADIANS for these equations to work. This is just another example of why radians are a more “natural” way to measure angles, since they fit the equations easily.

Very often, the "regular" or tangential variables are referred to as the linear variables. An astute student, after viewing the above equations, might ask: "How can those equations be true when the units don't work out? On one side of the velocity equation you have m/s and on the other you have m.rad/sec. What happened to the radians?" The answer is that radians are not true units and can be discarded and incorporated whenever convenient. If we recall the definition of a radian;

radian = arc length/radius

we see that a radian is a length over a length and is thus a unitless pure number. Disposing of radians and causing them magically to appear when needed is a mathematical slight of hand (seems like a trick, but there is a logical explanation for it) that you must be accustomed to preforming.

Before we move on, it should be noted that when discussing linear variables related to an object in circular motion, we must be a little careful. For example, if the object in undergoing angular acceleration, then it will not have a constant angular velocity and therefor it will not have an constant tangential velocity. In such cases we can only refer to instantaneous tangential velocity. Also, if ω is constant, the only way we can be absolutely sure that v_t is constant is if the radius remains constant. In the case of rotational motion, tangential velocity only makes sense in regards to one particular point on the object. It should also be noted that we have referred to the linear acceleration as tangential acceleration. This is not the complete acceleration of the object, it is only the part of the acceleration that points along a tangent to the circle. It turns out (and will be discussed in detail later) that the object will also have a component of the acceleration that points along the radius of the circle.

Another interesting aspect about the linear variables is that it allows us to do combination problems were we have a system of objects connected and one of them is undergoing circular motion. This method can be summed up by one statement:

If an object undergoes linear motion and is connected to an object undergoing circular motion by a string (or other similar means) wrapped around it, the linear variables of the object in straight line motion are the same as the linear variables of the object undergoing circular motion.

The above statement is simple common sense, provided the string does not slip (there is another condition that must be met, and I leave it to the student to figure it out). An example will illustrated this concept better than words.

Example 3.3.6

A winch (radius of 10 cm) on a tow truck is attached to a car and imparts an acceleration of 0.04 m/s^2 to the car over a time of 15 seconds (starting from rest). Through what angle has the barrel of the winch rotated during this time?

The previous example was the first example of containing a case of rotary motion. Notice how the point in question was on the outside of the circular winch and thus a particular point and radius were inferred. Let us try another, similar example.

Example 3.3.7

A child is fishing off a dock, when a fish bites the hook and takes off across the surface of the lake. The fishing line is wrapped around a large spool r = 5 cm), and in 3 seconds the spool revolves 110 times as the fish pulls out the line. Assuming that the fish swam in a straight line, and ignoring the angle the line makes between the pole and the fish, find the distance the fish traveled and the acceleration (assume constant) of the fish.

Example 3.3.8

A record player spins at 33.3 rev/min. What is the angular velocity and tangential velocity of points at 4, 10, and 20 cm from the center?

We see then that if an object is in rotary motion, all points on the object have the same angular velocity, but different linear velocities depending on their distance from the center. This important fact shows us the importance of angular variables, since they can completely describe a rotating object with one term.

Centripetal Acceleration

It was mentioned previously that the tangential acceleration on an object moving in a circle might not be the only acceleration on the object. This is indeed the case. If we remember back to the definition of acceleration, we will recall that it measured the time rate of change of velocity. In other words, to accelerate meant to change velocity. Now consider the motion of an object that is moving in a circle at a steady speed. Is this object accelerating? Yes, indeed it is. Even though the object maintains a steady speed, its velocity is changing directions constantly. If the velocity is changing, it is accelerating. Thus, any object moving in a circle is actually accelerating.

But what is the direction this particular acceleration? We will examine the how and why in a later chapter, but the short answer is that by vectors, using a very small time period, we find that the acceleration is directed exactly towards the center of the circle. The value of this acceleration can be found to be:

$$a = V_T^2/r = \omega^2 r$$

This is a very important fact about objects moving in circles:

Every object moving in a circle, no matter under what conditions, is accelerating at exactly V_T^2/r towards the center of the circle.

This acceleration is called centripetal acceleration or radial acceleration (since it is along the r-axis). So, it turns out that every object moving in a circle will have an acceleration in the r-direction, and it may or may not have an acceleration in the θ direction (that is the angular or tangential acceleration). The overall acceleration of the object would be vector sum of the two accelerations.

Normally, in first year physics classes, there are not problems that would require you to use both of the accelerations. Notice that the previous problems in this chapter did not require the use of the centripetal acceleration at all. It usually works the same way in reverse - meaning that the problems that involve centripetal acceleration do not usually require you to consider the tangential acceleration.

Part of the reason for this is because the centripetal acceleration involves the tangential velocity. If the object was tangentially accelerating, then the tangential velocity would increase, and thus the centripetal acceleration would increase. Or put another way:

If an object has constant angular velocity (moving in a circle at a steady speed) - it has no tangential acceleration but has constant centripetal acceleration.

If an object has constant angular acceleration (speeding up around a circle) - it has changing centripetal acceleration

Let us see if we can do a few sample problems with these ideas:

Example 3.3.9

If an object moves in a circle of radius 1. 4 m at 4 rad/sec with no tangential acceleration, what is the centripetal acceleration?

Example 3.3.10

An object moves in a circle of radius 2 m, and completes 5 revolutions in 2 seconds. What is the centripetal acceleration of the object?

Example 3.3.11

An object starts from rest and angularly accelerates in rotary motion at 3 rad/sec^2 for 10 seconds. Considering a point on the end of the object (20 cm from the center of rotation), what is the TOTAL acceleration (magnitude only) at that point after a.) 2 seconds, b.) 5 seconds, c.) 10 seconds?

Example 3.3.12 - Graphical Analysis

An experiment is carried out where an accelerometer is attached to a point 30 cm from the center of an object, set to measure centripetal acceleration only. The object begins tangentially accelerating and the following data is collected. Graphically determine the angular acceleration of the object. Note, since this is a lab experiment, there may be some errors in the data.

Time (seconds)	Centripetal Acceleration (m/s^2)
1	2.9
2	11.3
3	23.3
4	46.2
5	70.5
6	91.2
7	138.3
8	175.8
9	220.7

3.4 Relative Velocities

There remains one type of two dimensional motion problem for us yet to discuss - a relative velocity problem. This is a very limited, but important, aspect to two dimensional motion, and it involves knowing the velocity of one object in one reference frame and wanting to know the velocity of that object as measured in a different reference frame. We have been neglecting reference frames since we first introduced them in Chapter 2, when we stated that we would consider the default reference frame of all problems to be the reference frame of the earth or the ground. However, we do know that motion appears very different when we switch reference frames. Learning how to take a velocity and transpose it to a different frame is an important skill in physics. Although the process is very straightforward, it takes some practice getting used to. Instead of telling you exactly how to do this, if we begin with a very simple problem and then analyze it, you should be able to figure out the process for yourself. Consider the example below.

> Example 3.4.1
>
> A swimmer swims at 2 m/sec North relative to the water. If the current is 1 m/sec North, relative to the bank, what is the swimmers speed relative to the shore?

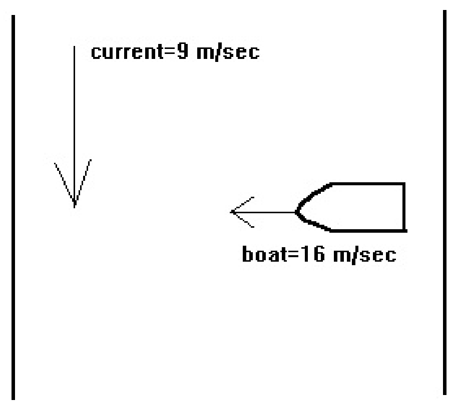
This example, although simple to do, deserves some analyzing. Consider what we were given: the velocity of one object relative to another and the velocity of the second object relative to a third. We were then asked for the velocity of the first object relative to the third. If we think of how we did the problem, we will find that we added the two velocities together as vectors to arrive at our answer. Mathematically we see that:

$$\underline{\mathbf{V}}_{\mathbf{ac}} = \underline{\mathbf{V}}_{\mathbf{ab}} + \underline{\mathbf{V}}_{\mathbf{bc}}$$

Where we have use the notation $\underline{V}_{ab}$ to mean the velocity of object A relative to object B (the same pattern applying for the other velocities). There is, however, one assumption made in putting this equation forth. This will only hold true for reference frames that are "inertial". Inertial reference frames are frames that are not themselves changing velocities. In other words, your car traveling 55 mph down a highway is an inertial frames, while your car taking off from a light is not. The term "inertial frame" will come back to visit us in later chapters and it is important to be able to identify when a frame is inertial and when it is not. With that understood, we can say that this equation holds true for a number of instances. Consider:

Example 3.4.2

Suppose a boat moves at 16 m/sec west relative to the water in a river with a current of 9 m/sec relative to the bank. What is the boats velocity relative to the bank?



This type of problem is important for handling situations where one object is moving relative to another object that is itself moving (especially cases where an object is moving in a moving fluid such as water or air). I should make a number of notes that might be helpful for the student to remember at this time. First, the order of the subscripts is very important. It must read velocity of a to b plus velocity of b to c plus equals velocity of a to c (notice how b is in the middle and then eliminated). Oftentimes the problem will not give the velocities in the correct order so students should take care to sort them out. Also, sometimes you will need the velocity of b to c in a problem but will only be given c to b. In this case, all you need to do to transform $\underline{V}_{cb}$ to $\underline{V}_{bc}$ is to make $\underline{V}_{cb}$ negative (as was discussed in the vector chapter). This procedure makes sense if you give it a little thought and perhaps use physical examples instead of variables. The final note is that occasionally you will be given $\underline{V}_{ab}$ and $\underline{V}_{ac}$ and asked to find $\underline{V}_{bc}$ (for example, you might be given the velocity of the boat relative to the water and the boat relative to the shore and asked to find the current). In this case take our original equation and subtract $\underline{V}_{ab}$ form both sides to isolate $\underline{V}_{bc}$. On a final note, (for some reasons students seem to have trouble figuring this out for themselves) the current of a river is the velocity of the water relative to the shore, the wind is the velocity of the air relative to the ground, and the windspeed of a plane is the velocity of the plane relative to the wind.

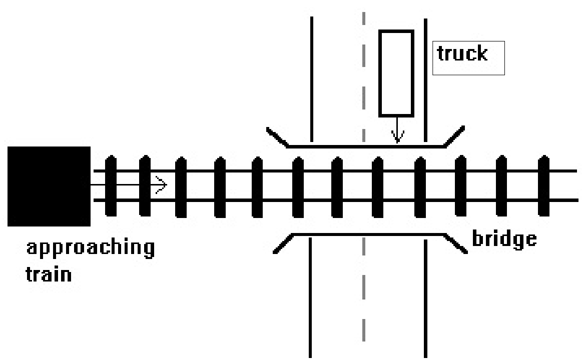
After these lengthy notes, let us do two last problems:

Example 3.4.3

If a plane is flying at 15° N of E at 200 mi/hr and the wind is 30 mi/hr at 60° N of E, what is the plane's velocity relative to the ground?

Example 3.4.4

Imagine that a train travels at 100 k/hr along a track eastward and a van is driving down a highway at 60 k/hr southward. What is the velocity of the van as seen from someone on the train?



In this second example, we see and example of a common mistake that some students make - don't just get in the habit of adding whatever two vectors are given to you in the problem and then reporting that as your answer - sometime you will need to reverse one of the vectors, or sometimes you will have to subtract the vectors. To avoid this mistake, always write your equation down first, using the subscripts given, so that you know what it is you will need to do to find the answer.

There is one other type of problem that should be discussed before we proceed, and that is that occasionally you will be given a relative velocity problem with "mixed unknowns". What is mean by this is as follows - considering that a standard relative velocity problem involves 3 vectors, each with a magnitude and a direction, that would mean that there are 6 possible variables (3 magnitudes, and 3 directions). In order to solve any of these problems, you need to have at least 4 of these variables given and you will generally be asked to provide the missing two variables. In all of our previous examples, you were asked to find the magnitude and direction of the same, missing vector. It is possible, however, to be given one full vector, and then also given the magnitude of a second and direction of a third. This would leave you to find the magnitude of the third and the direction of the second. Perhaps a chart will make this clearer.

Standard Problem		Variant Problem	
Vector 1 Magnitude	Given	Vector 1 Magnitude	Given
Vector 1 Direction	Given	Vector 1 Direction	Given
Vector 2 Magnitude	Given	Vector 2 Magnitude	Given
Vector 2 Direction	Given	Vector 2 Direction	Unknown
Vector 3 Magnitude	Unknown	Vector 3 Magnitude	Unknown
Vector 3 Direction	Unknown	Vector 3 Direction	Given

<u>When doing these types of problems, it is essential that you draw a picture to help you visualize the situation</u>.

An example this type of problem is given below:

> Example 3.4.5
>
> A boat attempts to travel across a river by pointing itself directly towards the other bank. Its engine can produce enough thrust to travel at 12 m/s in still water. A person on the bank measures the speed of the boat to be 12.63 m/s. What is the speed of the current and what is the direction downriver that the boat actually traveled?

Two other examples are presented to here for further practice.

> Example 3.4.6
>
> An airplane has a windspeed of 350 mph NE and the wind is blowing 40 mph at 20° North of West. What is the speed of the airplane according to someone on the ground?

Example 3.4.7

Two people pass each other on escalators in a department store. On is going down at 3 m/s and the other is going up at 3 m/s. Each escalator is at a 40 degree angle to the horizontal. What is the velocity of the person going up according to the person going down?

Homework Assignments

Homework 3.1 - Two Dimensional Motion

1.) If an object starts at rest and accelerates at 3 m/s^2 in the x-direction, and 5 m/s^2 in the y-direction for 8 seconds. What is its velocity and displacement in each direction at the end of that time? What is its overall velocity and displacement?

2.) A child is sledding across a frozen lake (a magic frozen lake, with no friction) at 4 m/s directly east. A friend comes up from south and pushes the sled with an acceleration of 7 m/s^2 directly north for 0.7 seconds. What is the new velocity of the sled?

3.) A space ship is traveling at 3000 m/s at 20 degrees north of east, and it fires its thrusters for 10 seconds. It has two thrusters, one pointed down at 270 degrees and one pointed at 180 degrees (thus the acceleration they cause would be opposite to the way they point). Each thruster causes an acceleration of 40 m/s^2. What is the new velocity of the space ship?

4.) A boat is traveling at 10 m/s east when it is knocked off course by an acceleration of 22 m/s^2 at 330° for 0.5 seconds. What is the new velocity? If it travels for another 1000 m in that direction, exactly how far off course (in meters) is it from where it would have been had it traveled 1000 m directly east?

5.) The velocity of an object is given by v = (22 m/s)i - (16 m/s)j. If it undergoes an acceleration of a = 15 m/s^2 at 40 degrees for 1.3 seconds, what is its new velocity?

6.) A ship pulls into harbor with a velocity of (18 m/s)i + (30 m/s)j. A tug boat begins to push the ship for two minutes, leaving it with a velocity of (18 m/s)i + (4 m/s)j. What was the acceleration caused by the tugboat?

Homework 3.2 - Projectiles

1.) A person jumps over a hurdle that is 1 m high. She starts 1.3 m before the hurdle and lands 1.3 m after the hurdle. How long was she in the air?

2.) During a police chase, a burglar attempts to jump from one building to another. If he is running at 4 m/s and the next building is 6 m away and 3 m down, will he make the jump? Assume he is jumping straight off the roof horizontally.

3.) A persons flicks a penny off a desk (horizontally) and it lands 1.4 meters away. If the “flicking speed” was 2.6 m/s, how high was the desk?

4.) At what angle must you fire a cannon with a muzzle velocity of 300 m/sec in order to hit a target 8000 m away? (P11)

5.) If a hose is squirting water at a 50 degree angle and it is landing at the same height 8 m away, what is the speed of the water leaving the hose?

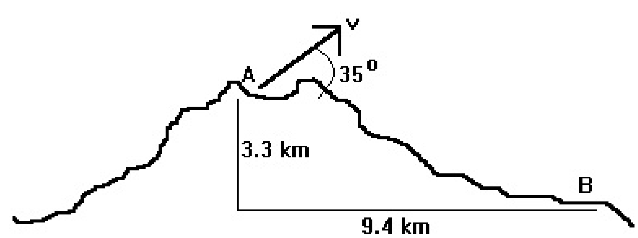
6.) During a game of darts, you aim directly at the bull's eye, and throw your dart with an initial speed of 9 m/s. It hits the board 0.20 sec later, directly below the bull's eye. Find a.) The distance from landing point to bull's eye and b.) The distance away from the board you are standing.

7.) An object is projected up at 30 m/s at a 45° angle. What is the horizontal and vertical displacements at 2 sec, and at 4 sec?

8.) Draw an accurate picture (using graph paper) of the following projectile: A 60 kg cannon ball is fired at a 70° angle from a point 150 m from the base of a 150 m cliff. The firing speed is 75 m/sec.

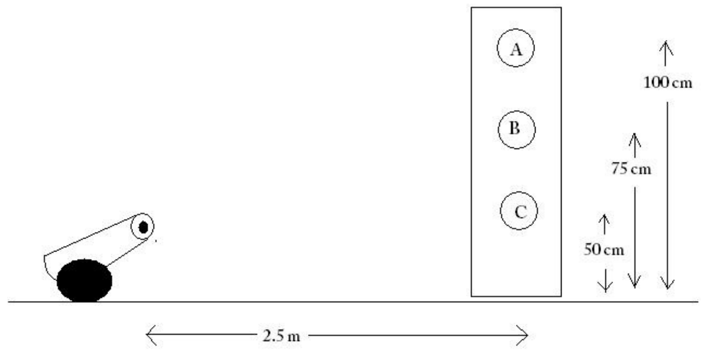
a.) determine the velocity (speed and angle) of the ball when it hits the ground on top of the cliff.
b.) If the ball was fired from on top of the cliff from the same point where it landed with the velocity found in a.), would the drawing look the same ? Why or why not ? (P5)

9.) Consider the volcano below. If a large rock is shot from the opening and lands at point B,
A.) What was it's initial speed (v) if it was launched at a 35 degree angle?
B.) How long did it take to reach B?
C.) What was its speed when it hit point B?



10.) If an bullet is fired at 400 m/s at a 40 degree angle, and lands on top of a cliff 50 m higher than the level from which it was launched, how much time did it spend in the air? How far away (horizontally) did it land?

11.) In the game shown below, a mini cannon is used to fire a small marble at the targets shown to the right. If the firing speed is fixed at 20 m/s, and the angle is fixed at 60 degrees, find the horizontal position from the target needed to land in each hole. (Ignore the 2.5 m in the diagram).



12.) A warship is sighted 4000 m from the coast of Zanzibar. The ship is traveling at 20 m/sec south and is directly east of the Zanzibarian cannons when it is sighted. If the cannon has a muzzle velocity of 300 m/sec and the cannon is fixed at 45^0 (up from the horizontal), (a.) where should the Zanzibzrians aim their cannon to hit the ship and (b.) how long should they wait to fire? Extra credit: determining the velocity of the cannon ball relative to the ship is a three dimensional vector problem. Can you determine it? (P10)

13.) We have learned that a particle on a projectile path traces out a parabolic curve because the horizontal displacement is the same over each period of time while the vertical displacement is the same as a freely falling object. Now consider a projectile subject to air resistance only in the x-direction. What would path would such a projectile trace out? On graph paper, draw an accurate graph of a normal, frictionless projectile and then superimpose (using a different color) the path of a projectile with friction only in the horizontal direction.

14.) Consider a projectile with air resistance (both in the vertical and horizontal direction). Would such a projectile spend the same amount of time going up as coming down? Would it spend more or less? Would it hit the ground with the same speed as it was thrown? Would it hit at the same angle? Defend your answers completely.

15.) Galileo states that if you fire two projectiles with the same velocity at angles that differ from 45° by the same amount, they will land in the same spots (ie: 35 degrees and 55 degrees). Prove this statement.

16.) Decipher: "Trust should never be extended to a Hellenic inhabitant carrying items destined to serve as personal favors." (DNCTHWG)

Homework 3.3 - Angular Motion

1.) If a record player slows down from 45 rpm to 33 1/3 rpms in 2 sec. what was its angular acceleration (in rad/sec^2).

2.) What angle in radians is subtended from the center of a circle with radius of 1.5 m by an arc of length 2.0 m (express in radians and degrees)?

3.) What is the angular speed of a car rounding a circular turn of radius 105 m at 60 km/hr ? (C6)

4.) The sun is 2.3 x 10^4 light years from the center of the Milky Way galaxy and is moving in a circle with a speed of 250 km/sec.

a.) How long does it take our sun to make one revolution around the center of the galaxy?
b.) If the sun is 4.5x10^9 years old, how many revolutions has it completed?

5.) What is the angular speed in radians/seconds of (a.) the minute hand, (b.) the second hand, and (c.) the hour hand of a clock?

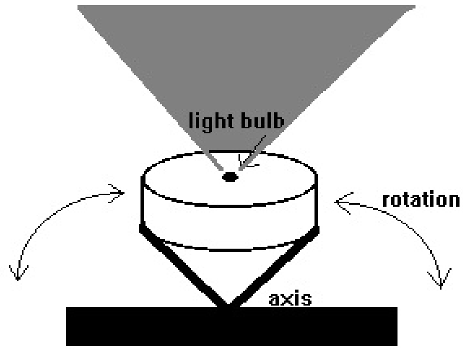
6.) If a grinding wheel changes speed from 10 m/sec to 30 m/sec in 6 seconds. What is the average angular acceleration during this period (radius = 15 cm)?

7.) If a disk (r = 30 cm) begins revolving from rest with an angular acceleration of 2.5 rad/sec^2 and accelerates for 5 seconds, (a.) how many radians has a point on the rim moved ? (b.) how far did the point travel in meters ? (C14)

8.) In the frame of reference of your car, what is the angular velocity of one tire (radius = 33 cm) when your car is traveling at 55 mph?

9.) Consider one of those enormous search lights that stores often rent for their grand openings. Imagine that the light executes rotational motion around some point 50 cm from the bulb (for half a turn as it sweeps from horizon to horizon). If it takes the casing 6 sec to make the sweep (see diagram - next page),

a.) How fast if the bulb moving (both angularly and tangentially)?
b.) How fast is the spot of light seen on the clouds over head (at approximately 25,000 ft) moving (both angularly and tangentially)?
c.) Angularly, how far behind the light bulb is the spot on the cloud seen (the speed of light is 3 x 10^8 m/s)?
d.) How high would the clouds have to be for the spot to be traveling at the speed of light?



10.) Imagine that you grab one end of a roll of toilet paper and run at 2 m/sec for 10 seconds. Through how many radians has the roll of paper rotated after this time? (consider the roll to have a radius of 6 cm and consider the loss of toilet paper to be negligible compared to the thickness of the roll)

11.) One end of a large roll of string (radius = 20 cm)is tied to a weight which is dropped off a cliff. What is a.) the angular acceleration, b.) the angular velocity and c.) the angular displacement of the roll after 5 seconds. Assume the loss of string to be negligible to the entire roll and assume there is not enough friction in the roll to interfere with the free falling weight.

12.) On a beautiful spring day, you find yourself flying a kite. When you go to roll the string in to bring the kite down, you count the number of revolutions that is required to retrieve it. If it takes 500 revolutions and the roll of string has a radius of 6 cm (assume it to be one of those jumbo rolls and imagine that the change of radius is negligible as the string is rolled in) how high was your kite off the ground if the string originally made a 60^0 angle with the horizontal? Another assumption must be made to answer this problem. What is that assumption and will it cause your answer to be less than or greater than the actual answer?

13.) A car completes a U-turn of radius 100 m in 10 seconds. What is the centripetal acceleration of the car?

14.) A child rides a horse on a Merry-Go-Round, at a distance of 6 m from the center. The ride starts from a stop, and angularly accelerates at 0.1 rev/sec^2 for 5 seconds before leveling off at a constant angular velocity. What is the acceleration of the child a.) a moment before 5 seconds (magnitude only), and b.) a moment after 5 seconds (magnitude and direction)?

15.) As you stand on the earth at the equator, you are rotating with it.
A.) What is your tangential velocity?
B.) What is your angular acceleration?
C.) What is your centripetal acceleration?

D.) What percent is the centripetal acceleration compared to the acceleration due to gravity on the surface?

16.) Imagine that you were standing on the earth and someone suddenly tried to stop the rotation of the earth (an evil villain). How long would it take them to bring the earth to a complete stop in such a manner that your total acceleration (meaning gravity + centripetal pointed down, plus your tangential acceleration) never was more than 12 m/s^2?

17.) Decipher: "Neophyte's serendipity." (DNCTHWG)

Homework 3.4 - Relative Velocities

1.) If a plane is flying with an airspeed of 400 m/s at 20 degrees N of E and the wind is 20 m/s SW, what would be the velocity of the plane relative to the ground?

2.) Planes A and B pass each other while flying. Plane A measures the velocity of plane B to be 700 m/s east, relative to plane A. Plane A’s airspeed is 500 m/s at 25 degrees N of W. What is the airspeed of plane B?

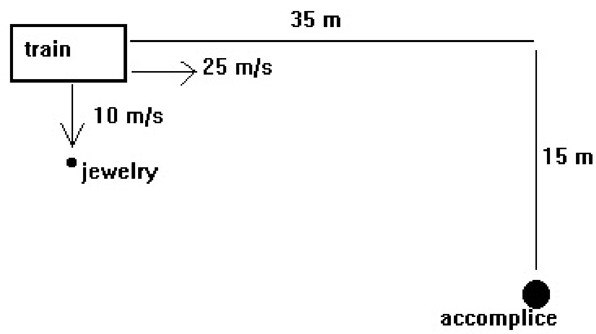
3.) If a car and a plane have the following velocities, what is the velocity of the car relative to the plane?
Car relative to ground: 30 m/s NW
Plane relative to ground: 400 m/s S

4.) A boat is traveling at 8 m/s relative to the water, directly east. If the current is flowing at 2.3 m/s toward 20 degrees North of East (relative to the shore), what is the velocity of the boat relative to the shore?

5.) A car traveling east along a highway passes another car that is on a overpass traveling north. If both cars are traveling at 25 m/s, what is the velocity of each car relative to the other? Give both magnitude and direction for each.

6.) A car traveling directly north along a highway (car A) spots another car (car B) trying to enter the highway along an entrance ramp. Both cars are traveling at 30 mph relative to the ground. The velocity of car A relative to car B (note the order) is at an angle of 60 degrees north of west. Find the angle of the entrance ramp and the velocity of car A relative to car B.

7.) A train robber on a train going 25 m/sec attempts to throw a load of stolen jewelry to his accomplice who stands 15 m away from the passing train. If the robber throws the jewelry straight out the window at 10 m/sec when his window is 35 m from his accomplice, (a.) what is the velocity of the jewelry relative to his accomplice. (b.) Suppose the jewelry lands directly to the right (east) of the accomplice. How far must he walk to pick it up? (see diagram)



5.) A boat can maintain a speed of 6 m/s in still water. The Captain of the boat want to travel straight across a river with a current of 3 m/s (downstream, obviously).

A.) At what direction must the Captain point the boat?

B.) What will be the magnitude of the velocity relative to the shore? (M8)

7.) Decipher: "Members of an avian species of identical plumage congregate." (DNCTHWG)

Labs and Activities

Activity 3.1 - Projectile Launching

In this activity, you will launch two different projectiles (a tennis ball and a golf ball) and measure the time in the air and the horizontal distance traveled. From this you will calculate the other significant quantities of motion and compare your results to the results obtained factoring in air resistance.

Procedure:

1.) Using a tennis ball, throw it as hard as you can at about a 45° angle into an open field. Have a friend measure the time in the air and then measure the horizontal distance traveled.

2.) Record this information and repeat this four more times (you will not be averaging these, so you do not need to be precise about the angle or speed).

3.) Repeat the above procedure for a golf ball.

4.) Using the information collected, determine the initial velocity and angle for each throw.

5.) Compare your results and draw some conclusions. Also compare velocities with others in your class to see who has the fastest throw.

			without	air res.	with	air res.
Trial	Air Time	H. Dist.	Velocity	Angle	Velocity	Angle
Tennis	Ball					
1						
2						
3						
4						
5						
Golf	Ball					
1						
2						
3						
4						
5						

Lab 3.1 - Projectile Motion

In this lab you will attempt to predict the landing spot of a projectile launched at an angle and determine the accuracy of your predictions.

Materials: Dart gun, metal ball, meter stick, ruler, protractor.

Part I: Determining the muzzle velocity of the gun.

In order to predict the landing point of a projectile, the initial velocity must be known. In this section you will determine the velocity of the metal ball by firing the gun horizontally off a table and noticing where the ball lands.

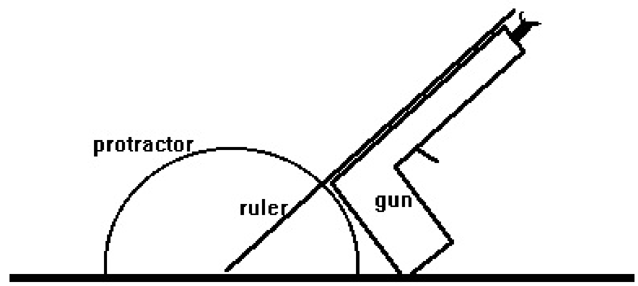
1.) Position the dart gun so that it is resting horizontally on a table, with the dart just over the edge. Place the metal ball in the suction cup on the dart. Your projectile will be the ball, not the dart (why?).
2.) Fire the gun, noticing exactly where the ball landed.
3.) Measure the height of the table (Δy) and the distance the ball moved horizontally (Δx).
4.) Repeat the above procedure five times and average the different Δxs for a single value.
5.) Using the height of the table and the average range calculated, determine the initial (horizontal) velocity of the projectile.

Trial #	Δx (m)
1	
2	
3	
4	
5	
average Δx	
table height (Δy)	
muzzle velocity	

Part II: Predicting Landing Positions.

In this section you will use the protractor and ruler to position the gun at a specified angle for which you have already predicted the landing position of the ball.

1.) Using the muzzle velocity found in part I, fill in the chart below by predicting where the ball will land for each of the given angles.
2.) Using the protractor and ruler as pictured below, position the gun to the best of your abilities at the appropriate angle with the ball in the suction cup.
3.) Fire the gun and notice where the ball lands.
4.) Repeat the above two steps twenty times for each angle and average the results.



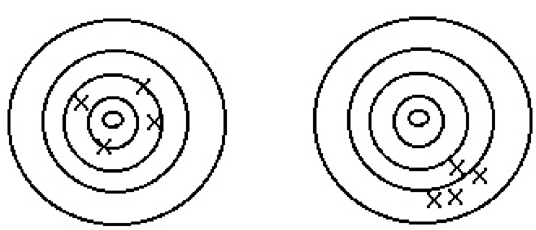
Angle	25°	45°	65°	80°	-20°	-45°
Predicted Landing Point						
Trial 1						
Trial 2						
Trial 3						
Trial 4						
Trial 5						
Trial 6						
Trial 7						
Trial 8						
Trial 9						
Trial 10						
Trial 11						
Trial 12						

Trial 13						
Trial 14						
Trial 15						
Trial 16						
Trial 17						
Trial 18						
Trial 19						
Trial 20						
Average Δx						

<u>Notes on Conclusions:</u> When you make your conclusions, be sure to evaluate the percent error for each angle and explain what may have contributed to this. Also notice if any patterns arise in the error or in any of your other data and explain why these patterns are there. Suggest improvements for the lab, to make it more accurate.

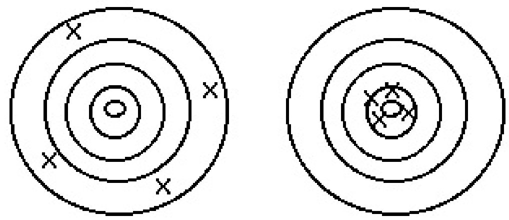
<u>Lab Skill Extension.</u>

In this extension, we will examine the concepts of accuracy and precision in labs. Short definitions are in order. <u>Accuracy</u> is how close you actually come to the correct answer. <u>Precision</u> is how repeatable (or reliable) your lab set up actually is. The best illustration of the difference between the two is taken from a chemistry text book. Consider throwing darts at a dart board. If you are aiming for the bulls-eye, and you get a set a set up as shown in picture A, you are very accurate (close to the mark), but not very precise, since the darts are spread out everywhere. Picture B shows an example of precision without accuracy (very consistent, but not near the mark).



Picture A   Picture B

Picture C shows both inaccuracy and imprecision and picture D shows both accuracy and precision.



While most student understand that accuracy is important in a scientific laboratory investigation, what many don't realize is that precision is just as important. Getting the correct answer by accident is not a good thing. Being able to get the correct answer over and over again is (recall in the very first chapter that repeatability is the mark of a scientific claim). In a school setting, precision is just as important as accuracy, because very often the precision with which a lab is done is a measure of the care the student takes in executing the lab (it is also a measure of the equipment itself, but in a school setting that aspect is within the student's control).

Percent error is a common method of evaluating the accuracy of a lab, however, very often students are not taught how to evaluate the precision of a lab. In the extension below, a method of evaluating precision is outlined, which is actually a statistical technique that is commonly used. It is called the deviation.

<u>The deviation</u> of a set of numbers tells you the average of the distance of the points from the average value of the data. A simple example will explain things perhaps more clearly than words. If our data set was {2,2,6,6} then our average for the data would be 4. Our deviation is 2, since all of our points are 2 units away from the average. Consider how we would calculate something like this. We would first subtract each data point from the average and take the absolute value. We would then average (add together and divide by the number of numbers) the results of these subtractions. This is exactly the method we employ for calculating the deviation. Unfortunately, the standard symbol for deviation is one which we already employ, the delta (Δ). Because of this, we will use the small Greek letter delta (δ) to mean change and take the place of the upper case letter. The student should remain aware that this is our own substitution, and although δ is used in math and science for "change in", it is not the commonly accepted symbol for deviation.

Now to define our shorthand symbols:

$\bar{x}$ = average value of x (it is standard notation in math for a line above the value to signify the average of that value)

$\delta x_i = x_i - \bar{x}$ = the difference between the value of the ith data point and the average

$\delta\bar{x} = \sum|\delta x_i|/n$ = the deviation, the sum of the absolute value of the difference between the data points and the average divided by the number of data points.

Before we calculate the deviation for our lab experiment, we should take a few moments to consider what a deviation means. If we were given a set of data points with an average of 30 and a deviation of 20, would that have been a very precise experiment? No, since the points were an average of 20 units away from the average. However, if the average was 10,000 and the deviation was 20, we would probably call that a fairly precise experiment. So you see, the deviation only makes sense in terms of the average value. It should also be stated that deviation can be used in conjunction with expected error (a measure that tells you how precise you can expect your apparatus to be) to tell you if your execution of the lab affected the outcome.

Now, with the help of the table below, calculate the deviation for the lab (you only have to do it for the angles given) and included it in your conclusions. Comment on the accuracy and precision of the experiment.

Angle	25^0	45^0	80^0	-25^0
Average value of Δx				
δx				
Trial 1				
Trial 2				
Trial 3				
Trial 4				
Trial 5				
Trial 6				
Trial 7				
Trial 8				
Trial 9				
Trial 10				
Trial 11				
Trial 12				
Trial 13				

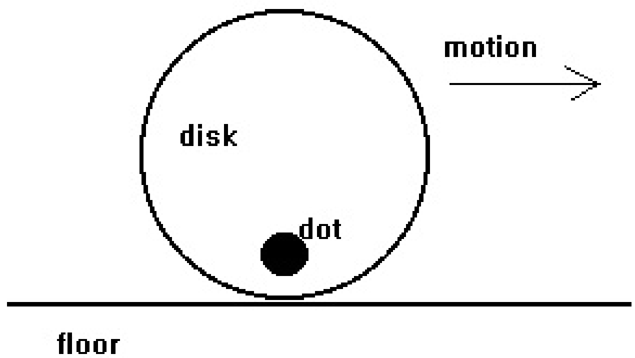
Trial 14				
Trial 15				
Trial 16				
Trial 17				
Trial 18				
Trial 19				
Trial 20				
$\delta\bar{x}$				

Activity 3.2 - Rotational and Linear Motion

In this lab, you will attempt to determine a relationship between the angular motion of an object and the linear motion of its center. You will do this by rolling two different disks each at four different speeds. You will determine the angular and linear velocity of the disks and then graph them to find a relation.

Procedure:

1.) Roll one large disk with a dot marked near its edge across the floor. Position the dot so that you begin the rotation with the dot at the floor.



2.) One person should time the event, while another observes the dot. When the dot reaches the floor for the tenth time, that person should note where the disk was and call time.

3.) The distance the disk rolled and the time it took should be recorded.

4.) This procedure is repeated three more times at different speeds and then the entire process is repeated for a disk of different radius (both radii should be recorded).

	Disk 1 Radius:		Disk 2 Radius:	
Trial	Distance	Time	Distance	Time

5.) Using this information, determine the velocity of the center of the disk for each trial and then the angular velocity of the disk for each trial.

6.) Make a graph of velocity versus angular velocity for each disk. From that graph determine a relationship.

Conclusions: Be sure to discuss the success or failure of the lab and what errors might have occurred. Consider the following questions: What was the relationship between velocity of the disk's center and the angular velocity of the disk? Was there any physical meaning to the slope of your graph? Was this relationship a surprise? Does this relationship match anything in the notes? Be careful with this last question, since it really requires some thought to understand this issue - what might the differences be? What conditions must be met for your relationship to hold true? Think about the tangential velocity of a point on the edge of the disk. What would be the velocity relative to an observer on the ground of a point on the end of the disk when that point was at the top (12 o'clock)? at the bottom (6 o'clock)? at a midpoint (3 or 9 o'clock)? Note: an observer on the ground would see the result of two different kinds of motion that the disk in undergoing - answering this question takes some serious thought.

Activity 3.2 - Relative Velocities

Purpose: In this activity you will use the computer interface to measure relative velocities. Doing so will help you identify the variables involved and give physical meaning to the equations.

Procedure: To carry out this activity, you will need two motion sensors and two carts. Identify one cart as cart #1 and the other as cart #2. Your goal will be to measure the velocity of each cart according to the ground (V_{1g},V_{2g}) and then also measure the velocity of cart 1 relative to cart 2 (V_{12}). The setup will involve the two carts approaching each other from opposite ends of the lab table. Since you only have two sensors and need to measure three values, it will be done in two steps. The propulsion of cart #1 will be done by the spring loaded mechanism on the cart and the second cart will be propelled by hand. To measure the velocity of the carts, have the computer graph velocity versus time for each cart, select an appropriate area on the graph and take an average value.

1.) Set cart #1 on the table with the spring mechanism set against a brick. Attach a large sail to the cart (this will later be used as the reflector for the second sensor).

2.) Set up a motion sensor to measure the velocity of cart 1 (V_{1g}) and release the spring. Since this is not done by hand, we are going to assume that this will be constant throughout the experiment.

3.) Set cart 2 on the opposite end of the table and place one motion sensor to measure its velocity (V_{2g}).

4.) Set the second sensor on top of cart 2, facing cart 1.

5.) Be sure that both carts pass close to each other and do a few trial runs to make sure that the wires from the sensor on cart 2 do not interfere with its motion.

6.) Have one person push cart 2 while a second person releases cart 1. Using the computer, measure V_{2g} and V_{12}. This may take a few attempts to be sure that the sensors are not interfering with each other.

7.) Verify that your results match the equation for relative velocities: $V_{12} = V_{1g} + V_{g2}$ (be sure to remember that velocities are vectors and therefore have a direction).

Lab 3.2 - Linear and Angular Variables

Purpose: In this lab a set up involving linear and angular acceleration will be investigated and the linear and angular variables will be compared.

Materials: Computer interface with a photogate, large spool, string, hanging mass, extra pulleys.

Special set up note: This lab requires data to be taken at the top of a balcony. Thus either remote, self standing interfaces or a clever combination of pulleys are needed to gather the information.

Procedure:

1.) With a sufficient quantity of string wrapped around the spool, set it up so that the hanging mass (attached at one end), hangs off the balcony. The goal of the lab will be to have the mass fall and measure its linear acceleration. This will be compared to the angular acceleration of the spool.

2.) The linear acceleration of the weight will be determined by measuring its time and distance traveled and using our equations of motion. Be sure that the mass is not too great (about 30-50 g), otherwise it will fall at near "g". We want an acceleration of about 2 m/s^2.

3.) The angular acceleration of the spool will be found by using a computer interface and a photogate. A photogate is a precise timing device that measures how long it's light beam is interrupted.

4.) Attach a small (1 cm) piece of cardboard to the outer edge of the spool in such a manner that when the spool rotates, it will interrupt the photogate once each rotation.

5.) When the mass is dropped, the computer will record how long each pass lasted and when it occurred. The computer will then graph velocity versus time for the spool.

6.) From this graph, students must use the radius (be sure the radius is all the way out to the flag) to construct a graph of angular velocity versus time.

7.) From this graph, the angular acceleration of the spool can be found.

8.) This can be compared (via percent error) to the linear acceleration of the mass, if the radius is used (but now the radius is only measured out to where the string comes off the spool - this may be different from the previous radius).

Information to be recorded

Radius of Spool:
Radius of String:

Width of cardboard marker:

Distance mass has fallen:

Time it took mass to fall:

Also remember to get a print out of the photogate measurements, including time and time the gate was interrupted. This is best done by printing a table and a graph.