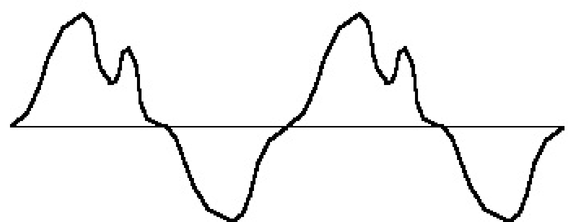
Chapter 32: Music

Human Hearing and Sound Waves

In this chapter, we will take a look at that special category of sound waves that we call music and try to understand both musical instruments and musical scales and notes in a physical way while evaluating their relationships to waves and wave behavior. Before we begin, however, we should take a closer look at hearing to understand how the ear responds to sound waves.

Human hearing roughly stretches the spectrum of sound waves from 20 Hz to 20,000 Hz (20 kHz). Sounds with frequencies below this range are called infrasonic and sounds above this range are called ultrasonic. Naturally, this range varies from person to person and will change as the subject ages. As was mentioned earlier, the range of decibels that each person can detect (or endure without pain) is also frequency dependent, but for an average person, the chart below would apply. This chart shows the range of sounds in decibels and by frequency that can be detected by the human ear. When dealing with music or sounds that are heard, we often refer to the frequency of a sound as its "pitch". The two terms are interchangeable.

How then, the astute student might ask, can we hear multiple sounds at once? The answer is both simple and complex. All of the sounds that occur at the same time form together to produce one composite wave and that is the wave that is detected by your ears. To explain more fully, a simple sound (one that is composed of only one frequency wave) is called a pure tone. These sounds are relatively rare in nature, but they are easily produced by a tuning fork. The wave they produce is a simple sine wave and the vibrations of the air particles cause your ear drum to vibrate according to the same pattern. Your brain interprets these vibrations as sound. Most sounds are not pure tones, but rather composite waves formed of many pure tones put together. If you sound two tuning forks, their waves combine to produce one composite wave that in turns vibrates your ear drum. The vibrations this time, however, are not simple sine wave vibrations but actually very sophisticated and complicated patterns (see diagram below). One of the most amazing things, in my opinion, about the human body is that the brain can take these strange patterns and "de-construct" them. While the ear drum hears the composite wave, your brain sorts out each sound and interprets its separately.



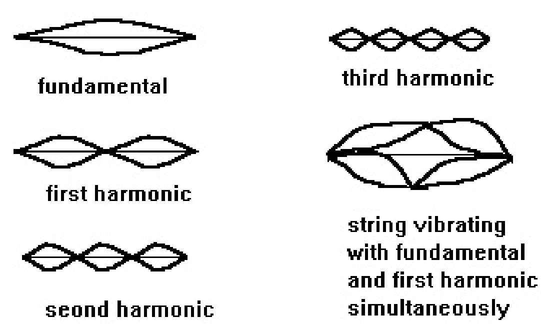
An example of a composite wave produced by two pure tones sounded together

Thus, if you hear an entire orchestra playing together, all to the sounds being produced form one complicated composite wave and that vibrates your ear drum and is deconstructed by your brain into all of the various instruments. In fact, simple speech patterns or words are actually fairly complicated composite waves that are formed by your vocal chords, they are not pure tones (with the exception of a few vowels, can you guess which ones?). This also explains how music can be recorded and played back over a speaker. When a band records an album, many different sounds (tracks) are mixed together, forming one composite wave. All that is saved on a tape or a compact disk is one single wave. It is unnecessary to save a separate wave for the guitar and the drums, since your brain will deconstruct them from the composite wave. The wave that is saved on the disk is the same wave pattern that would be vibrating your ear drum if you had been in the room with the band. The job of a stereo is to take that one composite wave and reproduce it into pressure waves in the air. This is how a speaker works. The wave is sent from the CD or tape in the form of an electrical pulse that matches the pattern of the sound wave. In turn, this pulse causes the speaker to vibrate in the same manner, compressing and rarefatcing the air in front of it.

Musical Quality

Each musical note sounded on an instrument represents a different frequency of sound. for example, a note of middle "C" is a sound with a frequency of 264 Hz. But yet the note of C on a piano has a distinctly different sound compared to the same note on a trumpet. Since they are both sounds of 264 Hz, what then is the difference? The answer lies in what is called musical quality (or timbre). Although both instruments produce a sound of 264 Hz, they differ in quality.

To understand what quality is, we must first understand overtones or harmonics. Overtones are extra notes that are played along with the original note that is produced. For example, consider a string in a piano. We know that the string will vibrate at its resonant frequency when struck. However, we also know that the string has many different resonant frequencies. The lowest of these is called the fundamental frequency and it represents the note that is being played. On a piano, however, the string vibrates in a relatively complicated manner that allows not only the fundamental frequency to be heard, but also the first harmonic. The first harmonic is the next lowest frequency that produces standing waves. In the case of a piano, this pattern continues and you not only hear the fundamental and first harmonic, but you also hear the second, third, fourth, etc. Each instrument plays a certain pattern of harmonics. For example, the french horn plays the fundamental and only the first harmonic, while a piano might play as many as ten different harmonics with each note. The quality of the instrument is defined as the number, combination and intensity of the harmonics played. All of these notes are sounded together and produce a composite wave that is detected by our ears. We are used to hearing a "rich" sound from a piano produced because so many notes are playing at once. The diagram below shows the string patterns for the different harmonics:



Do not be misled by the fact that the harmonics in the above diagram all have roughly the same amplitude. In fact, part of musical quality has to do with the respective amplitudes of each harmonic. For example, two instruments can both contain the same harmonics, but at different levels. It is illustrative to look at the chart below, which shows a number of theoretical instruments and describes the harmonics present in each one. The amplitude of each harmonic is shown, not as an actual number with units but instead as a relative amplitude, compared to the fundamental frequency, which is stated as a 10 (thus a harmonic with an amplitude of 5 has half the amplitude of the fundamental).

Instrument	A	B	C	D
Fundamental	10	10	10	10
1st Har.	6	8	4	0
2nd Har.	0	4	0	0
3rd Har.	0	8	0	0
4th Har.	3	3	7	3

In this chart, instruments A and C contain the same harmonics, but at different level, thus they have different musical quality. Instrument B has the greatest quality and instrument D has the least. It should be noted, however, that it is often difficult to determine the quality of sound just by looking at these numbers. In the case of instruments A and C, we know they have different qualities, but without hearing them, it would be hard to determine which actually has the greatest quality.

Looking at quality in terms of actual wave drawings is interesting and shows how the waves represent the quality or richness of the sound that is heard. Below are two examples where the fundamentals and harmonics are shown separately and then the composite wave is shown (the actual wave pattern that comes from the instrument. It is important not to get this confused with the vibration of the string, which is another matter altogether). Notice the complexity of the waves.

Quality is affected not only by the instruments themselves, but also by the construction and materials used in the instruments. For example, a Stradivarrius is a famous type of violin that produces a fantastically rich sound. For some reason, the materials used and the fine workmanship that went into their construction causes them to have a few more harmonics than an ordinary violin (actually it is more complicated than that, since it changes all the harmonics slightly, adds a few and in general changes the composite wave in a very complicated and mysterious manner). The same goes for a Steinway piano. Instrument making is a very sophisticated art, and one about which some general rules are known, but there is much still not understood. For example, each different type of wood used in the construction will change the harmonics that are produced, but exactly how they will change is often unknown until the instrument is constructed. Even such a small factor as the amount of glue used to join the wood can change the quality. Since metal is relatively uniform material and easily controlled, most metal instruments are very uniform in their quality, and in general not all that interesting. Instruments made of wood, which is not often completely uniform are often the ones that produce the richest sounds.

A knowledge of harmonics explains how electronic keyboards and synthesizers are able to replicate the sounds of many different instruments. For example, if you wanted your keyboard to sound like a trumpet, all you would have to do is to replicate the harmonics that a trumpet usually produces. This also explains why keyboards can never really produce the sound exactly, since each instrument is unique in its sound production. A key board might be able to replicate most of the harmonics on a piano, but there will still be some subtle ones that are left out.

Before we move onto the next section, let us take a minute to restate some notes about musical quality.

Notes Regarding Musical Quality

- Quality is the number, combination and intensity of the harmonics played.
- The quality of an instrument is what gives its note the distinctive sound that differentiates it from other instruments playing the same note.
- The fundamental frequency is the actual note being played and almost always is louder (more intense) than the

harmonics that accompany it.

- Some books call the fundamental frequency the first harmonic or the first overtone and then label the others from there. In this text, the first harmonic refers to the second longest wave that produces standing waves on the instrument.
- The above discussion and diagrams focused only on strings. It is important to note that the principles stated apply to all instruments, but the details vary from stings to woodwinds to pipes.

EX. ZSW:) Draw the fundamental and the first four harmonics that are formed on a.) a string, b.) an open pipe and c.) a pipe closed on one end. Determine an equation of the form f(n) where {n = 1,2,3,4...} for the frequencies of all the harmonics.

Octaves, Scales and Chords

Most students that have had some introduction to music know that music is played not by frequency numbers, but rather by notes. The question is then how do notes relate to our ideas about frequency and wave characteristics? Before we begin, we should realize that although music theory had its beginning in mathematics (with the ancient Greek Pythagoras discovering the law of strings) much of music has progressed through the ages qualitatively "by ear". Musicians have been more concerned about the notes "sounding right" than they have been about their frequencies fitting a certain, rigid pattern (and correctly so, I might add). Thus as we examine the notes and the scales, we shall see that they do not always fit nice, easy patterns.

Before we begin to understand musical notes and their relations to frequencies of sounds, a warning must be issued. The first step, understanding octave, is relatively simple, but understanding scales and chords is very difficult. Do not be discouraged if you do not follow the logic completely on your first read through. This is a subject that very well might require a number of slow and careful readings to grasp. Of special note is that there are two terms used in the section that are similar in spelling but mean very different things. They are frequency difference (or simply difference) and sound difference. Frequency difference is the absolute value of the subtraction of the two frequencies. For example, the difference between 500 Hz and 507 Hz is 7 Hz. The sound difference is a qualitative difference between how those two sounds are heard. The two sounds might not sound very different to your ear, thus we say they do not have a great sound difference.

To understand notes, one point must be made clear. In order for the notes to sound correct, it is the ratio between their frequencies that is important, not their differences. This point should not be understated. For example, suppose you heard two sounds, one at 200 Hz and the other at 400 Hz. You could easily tell they were two different sounds. Now imagine that you hear a sound of 5000 Hz. If you attempted to find another sound that was as different from 5000 Hz as 400 Hz was different from 200 Hz, what sound would you find. The answer is not 5200 Hz. The difference is not what mattered, the ratio is. Thus the second pair of sounds that sounded as different as the first pair would be 5000 Hz and 10000 Hz. As you go up in the frequency range, you must get farther apart (difference wise) in order to hear a difference. You would be hard pressed (unless you were a musician with a good ear) to tell the difference between 5000 Hz and 5200 Hz. However, the same difference is striking lower on the scale. Because of this, what matters in music is the ratio of two notes. For example, suppose you started out with a note of middle "C", 264 Hz. If you played a note of 528 Hz you would hear a difference between the two. Now suppose you wanted to find a note that sounded as different from 528 Hz as 528 Hz did from 264 Hz. In that case, you would find that 528/264 = 2 and you would have to play a note of 1056 Hz. This span, with double the ratio, is called and octave. An octave is a span of notes bounded by two note whose frequencies are in a ratio of 2:1. In the example above, all the frequencies listed correspond to notes that are given the name of "C". The next "C" would be 2112 Hz. Notice how the differences in frequencies are growing larger, as they should be. In order to maintain the same sound difference, as you go higher in the frequency numbers, the difference must get greater.

EX. URRCV:) How many octaves does the range of human hearing span?

Within an octave, there are six "notes" between the boundary notes. How then are we to divide an octave? The first temptation might be to divide it into equal segments, putting a note on each dividing line. For example, we could put notes like such:

Note	Frequency
C	264 Hz
D	306
E	344
F	382
G	420
A	458
B	496
C'	528

In this scale, each note is 38 Hz above the one before it (with the exception of the last two notes which did not fit due to rounding off). However, the astute student should notice that this would not work. Remember that it is not the difference that matters, it is the ratio. The way the above scale is laid out, the sound difference between A and B would be much less than the sound difference between D and E. It is the ratio that matters. Historically, the way the scale was divided up was to issue set ratios between the notes. This resulted in the Just Major Scale:

Note	Frequency
C	264
D	297
E	330
F	352
G	396
A	440
B	495
C'	528

Notice how in most instance, the differences between notes increase as you go up the scale. There are a few places, however, where the differences actually decrease. The will be explained in a moment. The ratios that these notes produce are:

Notes	Ratio of Frequencies
D/C	297/264 = 9/8
E/D	330/297 = 10/9
F/E	352/330 = 16/15
G/F	396/352 = 9/8
A/G	440/396 = 10/9
B/A	495/440 = 9/8
C'/B	528/495 = 16/15

Notice that not all the ratios are equal, but with the exception of the two ratios of 16/15, they are roughly the same. The ratios of 16/15 are actually called half steps and the other ratios are called full steps. Musically, they have introduced notes in the middle of the full steps. These are called sharps and flats. The notes C# (sharp) and D♭ (flat) are the same thing and this note exists one half step up from C or one half step down from D. At the places in the scale where only half steps exist, there are no sharps of flats. Thus there is no such note as E# of F♭ and no such note as B# of C♭. The sharps and flats on the piano are the black keys spaced in between the white keys. If you have ever noticed, there are spaces where there are no black keys on the keyboard. These spaces are the half steps.

There are, however, some problems with the just major scale. The ratios that you have seen above are not exactly equal. This becomes a problem if you are to set your instruments to a key other than middle C. For example, the key of D produces a very different scale, as seen below:

To rectify this problem, a different scale was created, the Equal

Tempered Scale. This scale (a more recent invention) sets an equal ratio between all notes. In this case a half step is $\sqrt[12]{2}$ and a full step is twice that: $2\sqrt[12]{2}$. This gives a scale that looks like this;

Note	Frequency
C	262
D	294
E	330
F	349
G	392
A	440
B	494
C'	524

and it gives the ratios as:

Notes	Ratio of Frequencies
D/C	294/262 = $\sqrt[6]{2}$
E/D	330/294 = $\sqrt[6]{2}$
F/E	349/330 = $\sqrt[12]{2}$
G/F	392/349 = $\sqrt[6]{2}$
A/G	440/392 = $\sqrt[6]{2}$
B/A	494/440 = $\sqrt[6]{2}$
C'/B	524/494 = $\sqrt[12]{2}$

In comparing the two scales, there are only two notes that have the same frequency in both, the E and A. The Equal Tempered Scale makes more logical sense than the Just Major and is easier to work with when designing electronic instruments. However, it is important to notice that the notes are different, thus a piece of music that was written years ago using the Just Major Scale will sound slightly different when played on an instrument tuned to the Equal Tempered Scale.

When playing musical instruments, often chords are played, not just single notes. A Chord is a special set of three or four notes played together that sound pleasing to the ear. Besides sounding pleasing, they also must fit a ratio pattern. Chords are made up of triads which are sets of two notes whose ratios fit a pattern. There are two different triads:

major triad (M^3) = ratio of 4/5
minor triad (m^3) = ratio of 5/6

These ratios are used in combinations to produce the four different chords:

Chord	Triad Combination	Ratios

Major	$M^3 + m^3$	4/5/6/8
Minor	$m^3 + M^3$	4/4.8/6/8
Diminished	$m^3 + m^3$	4/4.8/5.76/8
Augmented	$M^3 + M^3$	4/5/6.25/8

Thus a chord always spans one octave (notice the 4/8 ratio from beginning to ending note, although often the final note is not mentioned since it will be an overtone of the first) and includes two other, intermediate notes. An example of a major chord is the notes C,E,G,C' played simultaneously and an example of a minor chord is C,E♭,G,C'.

EX. JKHY:) Verify, by using ratios that the above chords fit the frequency ratios given and then determine a diminished and augmented chord beginning with middle C in the equal tempered scale.

The Laws of the Strings

It is interesting to note that much of what we have been studying was developed over time by ear and not directly by numbers. However, music had its first organized origins with Phytagoras who determined such things as octave and chords by experimenting with different lengths of strings. He noticed that.........XXXXXXXX include section on Pythagoras here...........

His conclusions about the notes produced by strings are actually related to Newton's Laws and an analysis of how strings vibrate. It makes sense that four factors affect how a string will vibrate: length, diameter, tension and density. By evaluating each of these separately, we can find relations that can predict what will happen when one factor is changed.

We can understand that a longer string will produce lower pitched notes, since the notes that form are standing waves formed on the string. It turns out that this is a direct relation to the wavelength produced, thus an inverse relation to the frequency. In other words, by lengthening a string, we change the frequency according to:

$$\nu_{old}/\nu_{new} = L_{new}/L_{old}$$

where ν is the frequency produced and L is the length of the string in each instance. It is important to note that to make this kind of comparison, all other factors must remain constant.

If the diameter of a string increases, it makes sense that the pitch will decrease, since there is more string to vibrate. Just like the case above, it results in an inverse relation.

$$\nu_{old}/\nu_{new} = d_{new}/d_{old}$$

Once again, only if all other factors remain equal.

Tightening a string, or increasing its tension, will cause the note produced to rise in frequency. This is a direct relation:

$$\nu_{new}/\nu_{old} = \sqrt{T_{new}}/\sqrt{T_{old}}$$

If the density of a string is changed (or is being compared to another string), we find that an inverse square root relationship exists.

$$\nu_{new}/\nu_{old} = \sqrt{\rho_{old}}/\sqrt{\rho_{new}}$$

Once again, only if AOFACE.

EX. JUHY:) If two strings are identical except for their radius, what is the frequency of a string with radius = 0.2 cm if a string with a radius of 0.12 cm has a frequency of 400 Hz?

Ex. HGTF:) A violin string is 35 cm long and stretched with a tension of 27 N, so that it vibrates with a frequency of 256 Hz. What is the new frequency when the length is changed to 30 cm and the tension is increased to 32 N?

<u>Assignment #34</u>

1.) Using either a graphing calculator or the Principle of Superposition, draw the resulting composite waves that result from an instrument that has the following musical quality.

a.) fundamental amplitude = 10
first harmonic = 5
second harmonic = 3

b.) fundamental amplitude = 10
third harmonic = 6
fifth harmonic = 4

2.) Discuss the relationship between harmonics and octaves.

3.) What would be heard if you played a note of B on two identical instruments (simultaneously), one tuned to the Just Major Scale and the other to the equal tempered? Be precise in your answer.

4.) Determine the frequencies of all the sharps and flats on the equal tempered scale.

5.) Determine a major, minor, augmented and diminished chord beginning with the note of F in the equal tempered scale. Represent the notes both with frequencies and musical names.

6.) Using a graphing calculator (or by hand, using the Principle of Superposition), draw the wave forms for the major and minor chords beginning with middle C.

7.) While tuning a piano string that should vibrate at 550 Hz, you heat 6 beats per second with a tuning fork of that frequency. If the string is under 40 N of tension, by how much should you change the tension to get the desired note ? (two answer, either one is correct) (SO4)

8.) A tension of 233 N on a guitar string gives a fundamental frequency of 414 Hz. If the guitarist wants to tune it to a frequency of 408 Hz, what should they do to the tension? (SO6)

9.) One string on an instrument (string #1) has a density twice that of the string next to it (string #2). If all other factors are equal, and the frequency of the string #1 is 256 Hz, what is the frequency of the string #2? (SO7)

10.) A violin string has a resonant frequency of 512 Hz. If the tension in the string is halved and the length is doubled, what will the new frequency be ? (SO2)

11.) Imagine that a stringed instrument contains just four strings, all the same diameter, under the same tension and of the same density. If the four strings are set to play a major chord, and the longest string is 60 cm, what are the lengths of the other strings? What (exactly) would have to be changed to be able to play a minor chord on the same instrument.

Activity # 14

The goal of this activity is to become accustomed to relating notes to frequency and to hear first hand the differences in musical quality between instruments. This will be accomplished by first listening to individual notes on a tuning fork, xylophone, and piano and then progressing to chords and finally to a song on each.

Materials: tuning forks, xylophone, piano, ears, striking mallet

Note: Do not hit the tuning forks with anything other than a striking mallet, doing so might chip the forks and change their frequency of vibration.

Procedure:

1.) Find a tuning fork that corresponds to a note in the Just Major scale. Strike the fork and strike the corresponding note on the xylophone. Listen for beats. If you do (or do not) hear them, what does that mean? It could mean one of two things. Determine a way to find out which of those two things is implied. Do the same with the piano.

2.) Once you have found a fork that matches both the piano and the xylophone (if that is possible...), strike each one at a time and listen for the difference in quality.

3.) Now gather a set of forks that makes a complete chord. Strike all four and then do the same for the xylophone and piano.

4.) Below are the notes for "Mary Had a Little Lamb". Play the song on the piano and the xylophone and then determine what frequency each note stands for and find the correct tuning forks to play the song. Listen carefully to each to determine the quality.

Short Write Up: Comment on each of the three instances (notes, chord, song) and the quality you heard in each. Was any instrument out of tune? Which scale was the piano and/or the xylophone tuned to? How did you determine which of the two possible reasons for hearing beats in step one was the correct answer?

Lab #23 - The Laws of the Strings

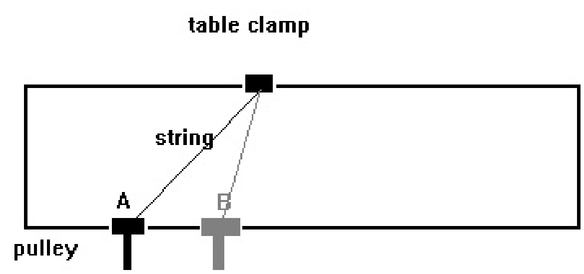
Purpose: In this lab you will experimentally verify two of the four "Laws of the Strings". You will do so by tuning a string to certain note (using tuning forks) and then change the length of the string to tune it to four others. By graphing the data, you will verify the law relating string length to frequency and then repeat the process by varying the tension in the string to verify the tension law.

Materials: guitar string, table clamp, mounted pulley clamp, five tuning forks, meter stick. This lab can also be accomplished with an oscilloscope to determine the frequency, however, less will be learned and some of the fun will be removed.

Procedure:

1.) Mount the table clamp to one on the long sides of the lab table. Attach the guitar string securely to the clamp.

2.) On the other end of the string, attach a weight hanger. Attach the mounted pulley on the other long end of the lab table, but not directly across from the clamp. Lay the string over the pulley so the weight hanger drapes down and is suspended.



3.) Using the lowest frequency tuning fork, adjust the tension in the string by adding or removing weights from the hanger until string is playing the same note as the fork when plucked.

4.) Record the length of the string (from clamp to pulley) and the frequency of the tuning fork.

5.) Keeping the tension the same (not removing or adding weight), move the pulley along the table to another location (B) and using the next lowest tuning fork, tune the string to match it. You may have to move the pulley back and/or forth along the edge of the table to accomplish this. Be sure the string remains fixed firmly and that no other factors except length are changed.

6.) Record the length and the frequency, the repeat with the remaining forks.

7.) Graph the data to verify the law. You may to directly to the appropriate graph that the law predicts.

8.) Moving the pulley back to the original position, retune the string to match the original fork. Record the frequency and the weight (tension).

9.) Now add weights to the holder until the string is tuned to the next fork. Record this information and repeat the procedure with the remaining forks. Be sure the string is attached firmly and that no other factors are being changed.

10.) Graph this data and verify the law.

<u>Notes Regarding Conclusions:</u> Did the laws match the experimental data? How far off from a perfect line were the points? What were the slopes of each line and what meaning did they have? Did the graphs go through (0,0)? Should they have? What would be the physical meaning of this point? What factors might have affected the outcome of this lab? Where was there room for error?