Chapter 31: Sound

The Nature of Sound Waves

The term "sound" defines a broad variety of different waves that occur in nature. Technically, a sound wave is a wave that is classified as a mechanical, longitudinal wave that travels through a media. In fact, any compressional wave that travels through a material can be called a sound wave, even if it never reaches our ears. A perfect example of such a wave is an earthquake wave, which is a tremor that travels through the ground (although earthquake waves can be transverse as well as longitudinal). When we think of sound waves, we usually think about the sounds we hear. A sound wave, being a compressional wave, is a series of compressions and rarefaction of the air around us. The easiest illustration of this comes from an explanation of how a speaker works. A speaker (such as in a stereo) is simply a cone that vibrates in and out according to the sound being produced. Thus, when the speaker wishes to make a sound corresponding to 300 Hz, it moves in and out 300 times a second. When the cone pushes out, it compresses the air in front of it, when it moves back, it decreases the air in front of it. These disturbances, once created, move away from the speaker, producing an alternating pattern of high and low pressure air. This is a sound wave. If you had a barometer that could measure air pressure quickly enough, it would rise and fall according to the frequency of the sound being produced. Your vocal chords produce sound in much the same manner, by vibrating according to the appropriate frequency. When these disturbances reach your ear, they cause your ear drum to vibrate in the same manner as the sound patterns and your brain interprets these vibrations as meaningful sounds.

A question might arise as to why the disturbances move away from the source after they are created. The answer is that in a medium there is a "restoring force" that acts on the particles. Consider a piece of steel for example. If we push a number of particles together, they will naturally spring back apart, since every particle is attracted to every other particle by the intermolecular (or interatomic) forces that hold them in place in the lattice. Now, when the particles spring back together, they will not return exactly to the original position, but will in fact over shoot it because of their inertia (remember, simple harmonic motion is natures response to disturbing a stable system). When they over shoot, they will disturb other particles, creating another compression, which will send the other particles into SHM. Thus we have a chain reaction. Notice how the disturbance is moving, not the particles (as was stressed earlier).

Since sounds must travel through a media, it makes sense that the media must affect the speed at which the waves travel. In any given media, the speed of sound in that material is determined by the properties of the material itself. The three main factors that affect the speed of sound are: density, the restoring force of the particles (the intermolecular forces) and the movement that those forces allow the particles to undergo. To see the variety of speeds in different materials, a few are listed below.

Speed of Sound in Materials

Air (0^0 C)	331 m/s
Air (20^0 C)	343 m/s
Helium	956 m/s
Water (20^0 C)	1482 m/s
Seawater	1522 m/s
Lead	1227 m/s
Steel	5941 m/s
Aluminum	6420 m/s

When we attempt to write out the equation for a sound wave, we are immediately struck with a quandary. What do we use for the amplitude of a sound wave? If we think about it a little, we will see that sound waves have two different characteristics that could be used as an amplitude. They are pressure and distance. Recalling that a sound wave is a disturbance of air pressure, it makes sense that our equation could have pressure as an amplitude. The areas of highest compression are the areas of highest pressure and visa versa. Thus the amplitude could be half of the pressure difference between the compressions and rarefaction (notice how only the pressure difference, not the absolute pressure is of importance). In this case, the units used for the amplitude would be units of pressure: Pascals (N/m), atmospheres, mm of Hg. Thus our wave equation might look like this:

$$W = (2.3 \times 10^3 \text{ P})\sin(kx-\omega t+\varphi)$$

However, there is another type of amplitude that could be considered. Since the particles in the media are actually undergoing simple harmonic motion, we could discuss the amplitude of the wave in terms of their displacement from the equilibrium point (just as we did for oscillators). In this case, the amplitude would be measured in units of length: meters, centimeters, inches, etcetera). In such a case, the equation might look like:

$$W = (6.7 \times 10^{-4}\ m)\cos(kx - \omega t + \varphi).$$

Notice how one equation is written with the sine function and the other with the cosine function. This is done for a very specific reason. The pressure wave and the displacement wave are $\pi/2$ rad out of sync. We could have written the second equation the same as the first and added $\pi/2$ to φ, but the same result is accomplished by using the other trig function. The student should take a moment to convince themselves that these two waves (while describing the exact same thing) are out of step by $\pi/2$. If you need a hint, think about the displacement position when the pressure is at its maximum.

The pressure amplitude is usually the amplitude that is discussed when sound waves are considered, but the displacement amplitude is often interesting. It turns out, that by using Newton's laws, we can relate the two amplitudes according to:

$$\Delta p = (v\rho\omega)\Delta x$$

where:

Δp = pressure amplitude
v = speed of wave
ρ = density of material
ω = angular frequency
Δx = displacement amplitude

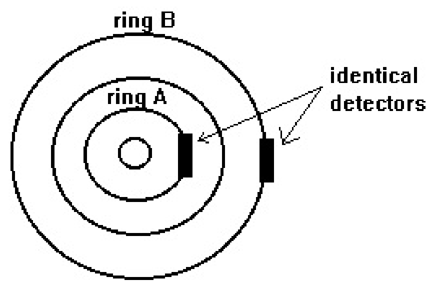
The actual derivation of this relation is relatively complicated, and only the sharpest of students should attempt it without some help. The following example shows some interesting results.

EX. OICDX:) The maximum pressure change that human ears can withstand is approximately 28 P and the minimum that an ear can detect is about 3×10^{-5} P. Consider a sound of 264 Hz. What is the displacement amplitude for the maximum sound of this frequency that the ear can withstand and the displacement amplitude for the minimum sound that can be detected. Assume the sound is traveling in air (density 1.29 kg/m^3) and comment on these distances.

EX. WSZJ:) Assuming that sounds keep the same displacement amplitude as they pass from one material to another, how is the pressure amplitude affected as sound goes from air into water.

Wave Power and Intensity

While we have been discussing the different aspects of waves and sound, one aspect has not been fully investigated, and that is the energy of a wave. Because waves are continuous and time dependent, it is not often common to discuss the energy of a wave. Instead we will consider the power of a given wave. Recall that Power = Energy/time and that power is measured in units of Watts, abbreviated W (1 W = 1 J/s). If we, for the moment, ignore the loss of energy as a wave travels, then it would make sense that the energy or power of a wave is constant. In short, energy given to a wave would never disappear. However, this appears to contradict common sense because we know that a sound cannot be heard from millions of miles away and that a flashlight cannot shine on the clouds over and be noticeable. The reason there is no contradiction is obvious after a little consideration. Consider a wave being produced as shown below. In this case a wave (imagining it to be a sound wave might help the illustration) is produced and spreads out in all directions equally, which is what all waves do unless otherwise obstructed or directed.



In this situation (which is actually three dimensional, not just two) we see that obviously the wave front grows larger as the wave travels out. The energy given to each front remains the same, however, farther out the energy is spread over a greater area (consider the ring marked A as having 10 J of energy spread over it and compare it to the ring marked B having the same 10 J spread over it). When dealing with waves, it is not usually the energy or even the power we are concerned about, it is the amount of power that reaches some detector (our ears or eyes for example) that is of concern. In the diagram you see that if identical detectors are placed at rings A and B, a greater energy will reach the detector at ring A. This idea is called the intensity.

Intensity is defined as the amount of power per unit area at a given distance from the source. Another definition is the rate at which energy is transmitted to a given are at a given location. From the definition we see that:

Intensity = Power/Area

$I = P/A$

and the units come out to be W/m^2. If we imagine our above diagram to be three dimensional, we see that the area in question is the surface area of a sphere, giving us:

$I = P/4\pi r^2$

Where r is the distance from the source. Notice how intensity is related to $1/r^2$. Here we once again have an inverse square law, just as we had for gravity and electromagnetism, complete with all its ramifications. The primary one being that the intensity drops off quickly as r increases. Before we continue, there are a number of comments that should be either made or repeated.

- The above equation only applies to spherical waves from a single point source in the center.
- The diagram above, while instructional, should be three dimensional, not just two.
- Intensity is position dependent, thus it makes no sense to talk about the intensity of a wave, only the intensity of a wave at some distance.
- The intensity of a wave at a point is an inverse square relation.
- The intensity of a wave is directly related to its loudness (in the case of sound) or its brightness (in the case of light).
- The intensity of a wave is related to its power, not its energy. To find the energy deposited on an area, you would need to know the time over which the wave acted.
- In all our discussion so far (in the remainder of this chapter) we have assumed that energy remains in the wave and none is lost as it travels.

EX. KBGVF:) A trumpet gives out a power of about 3.15×10^{-3} W. What is its intensity at a distance of 5 meters? Assume the trumpet to be a point source sending waves in all directions.

EX. NMHB:) The average intensity of the suns light arriving at the surface of the earth on a sunny day is 75 W/m^2. How much energy hits the roof of an average house 25 m by 20 m during 10 hours of such sunlight? Note: these numbers are simply approximations, the actual power of the sun reaching the surface of the earth would depend on latitude, time of the year, etcetera.)

As far as sound waves and human hearing is concerned, the ear can detect intensities as low as 10^{-12} W/m^2 and as high as 1 W/m^2 (without sustaining damage). Because this range is so broad, a new concept was introduced to make these numbers more meaningful. This concept is called the sound level, or volume and it represents the intensity of a sound in units called decibels. The unit decibels is named after Alexander Graham Bell, who did much work with sound and hearing besides inventing the telephone. Since the last name appears in the unit, it is abbreviated as dB. The decibel scale assigns a value of 0 to the lowest intensity that can be detected by the average ear and a value of 120 dB to the intensity which usually begins to cause pain to the listener. Intensities and sound levels are related in the following fashion:

$$\beta = (10\ \text{dB})\log(I/I_0)$$

where

β = sound level in decibels
I = intensity of the sound at the point in question
I_0 = constant, intensity of lower limit of average human hearing, 10^{-12} W/m^2

Notice that decibels are not true units, since they are the result of a logarithmic function and since it is a log scale, a jump of 10 decibels means an increase in ten times the intensity. For example, a sound of 60 dB is 100 times as intense as a sound of 40 dB and a sound of 100 dB is 10,000 as intense as a sound of 60 dB. This is not a linear relationship! There is another famous scale that is similar, in that it uses a log function. That is the Richter scale to measure earthquake strength. In that case, a jump of one unit on the Richter scale is a jump of ten time the strength of the previous unit. For example, an earthquake of magnitude 7 on the Richter scale is 100 times as powerful as an earthquake of 5. That is why there is a big difference between two earthquakes, even though one may be 7.1 and the other only 7.3. Below are listed some typical sound levels:

Sound Levels

Threshold of hearing	0 dB
Rustle of leaves	10 dB
Whisper	20 dB
Normal conversation	60 dB
Jack hammer (1 m)	90 dB
Concert	110 dB
Threshold of pain	120 dB
Airplane engine (50 m)	130 dB
Saturn Rocket (50 m)	200 dB

It should be noted that both the threshold of hearing and of pain are set for average ears and are frequency dependent. Thus some people might be able to hear sounds of -5 dB, and others might not be able to detect a sound of 5 dB. Besides this, the frequency dependency means that a sound of 1000 Hz might be heard at 0 dB, but a sound of 2000 Hz might not be heard until it reached 5 dB. This is more obvious in the threshold of pain, where some sounds cause pain even at low levels, while others might not.

EX. MJNB.) The table states that the intensity of a Saturn rocket at 50 m is 200 dB. What is its intensity? If it were to be considered a point source, what is its power? How far away should you be to hear the rocket at a comfortable level (60 dB)?

The above example shows you the effect that the second law of thermodynamics has on waves. Obviously, much of the energy of the wave is lost as it travels.

EX. OINH.) If a 60 W light bulb emanated its energy in the form of sound instead of light, how loud would it be at a distance of 4 m?

EX. WMOX:) How much more intense is a jack hammer than normal conversation?

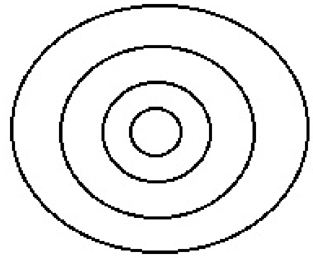
EX. THFU:) Derive the following two equations:

a.) An equation that directly relates the ratio of the intensities of two sounds to their decibel levels.
b.) An equation that directly relates the amplitude of two waves to their decibel levels.

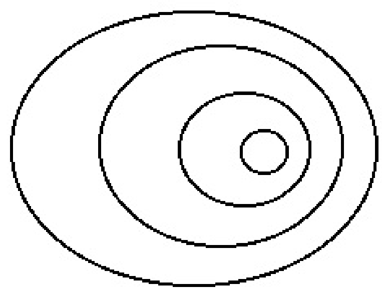
The Doppler Effect

Sound, as a wave, demonstrates all the properties and behavior of waves that were discussed in previous chapters (i.e. superposition, constructive and destructive interference, standing waves) as well as many others. In the remainder of this chapter, we will look at a number of other behaviors that arise in sound waves, and in fact, in waves in general. The student should remember, that although these behaviors are most obvious in the case of sound waves, they arise in all types of waves including light.

The first of these phenomena is called the Doppler Effect. When we first discussed waves, we said that the frequency of a wave was set by the source and would not change unless the source changed. The Doppler effect is the only situation where the frequency of a wave will change, and it comes about by the relative motion of the source and the receiver. This is best illustrated by using an imaginary example. Imagine that you, the source, were creating waves in a lake by dropping pebbles in the water at a rate of one pebble per second. Now imagine that a friend was on the far bank, counting how often the waves reached the shore and measuring their wavelength. The pattern you created on the lake might look something like this:



(Imagine that all the wavelengths were equal and let us not worry about how you can be suspended above the surface of the lake without being in the water!) A little thought will tell you that your friend will measure the same wavelength as you and that the frequency measured will be 1 Hz (one wave per second). Now imagine that you begin moving toward your friend, dropping stones in the same manner. Will the readings be the same? The answer is no. As you move you will create a pattern like the one below:



(Once again, the drawing is not perfect, the waves are still circles, but they are not centered on the same spot any longer.) The wave pattern will look like this provided your speed approaching your friend is not very close, or greater than, the speed of the waves in the material. Your friend will now measure a shorter wavelength and a higher frequency that they did in the past. If your friend were situated on the opposite side of the lake, they would measure a lower frequency and a higher wavelength than previously. Also consider what would happen if they were off to the side or at some angle. A little further imagination will tell you that the result would be the same if you stood still and the receiver approached or moved away from you. This is called the Doppler effect, and the change that occurs in the wavelength or frequency is called the Doppler Shift. It shows that the frequency and wavelength of a wave will change if the source or the receiver is in motion. In other words, measurements made either while moving or of waves from a moving object will not match the measurements made at the source. Before we try to understand this quantatively, it behooves us to restate a number of important comments.

Notes Regarding the Doppler Effect

- The effect is seen whenever the receiver and the source are in relative motion. Anytime one moves with respect to the other, the effect is seen. It is not seen, however, if they both move together (not relative motion).
- If the source approaches the receiver (or receiver approaches the source) the wavelength is reduced and the frequency increases.
- If the source moves away from the receiver (or receiver moves away from the source) the wavelength is increased and the frequency decreased.
- When discussing Doppler shifts in sound waves, the air (or medium through which the waves travel) is generally considered to be still, i.e. no wind. This would throw yet another complication into the situation.
- When discussing the Doppler effect, it is usual only to discuss objects in straight line motion away from each other or towards each other. If the object is off to the side of the motion, the situation becomes more difficult to consider.
- The Doppler shift is velocity dependent, not distance dependent. This is one of the most common mistakes made by students first learning about the effect. In other words, if a source approaches you, you will measure a higher frequency than the source is producing. However, that higher frequency will remain constant, it will not get higher as the source gets closer.

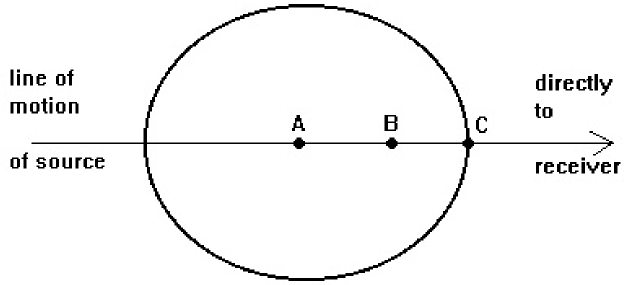
There are a number of example of the Doppler Shift that will help to clarify the situation. The most common example, and one that you have probably already noticed is that police and ambulance sirens sound different depending on whether they are approaching or moving away from you. Imagine that you are standing on a straight road and an ambulance is approaching. You will hear a higher pitched (higher frequency, shorter wavelength) siren than the ambulance driver hears. Once the ambulance passes, the siren becomes lower pitched. This example can be used to emphasize the last two notes made above. If the ambulance maintains a constant speed, you will hear a higher pitched sound that gets louder as it gets closer. The frequency of the sound will not get higher as it gets closer. You will hear the same sound the entire time.

Do not confuse volume (intensity) with pitch (frequency). The fifth note is also worth rementioning. For the easiest case of the Doppler Effect to apply, the ambulance must be directly approaching you and directly moving away from you. If you are off to the side of the line of motion, it is more complicated. I encourage the student to ponder this example and see if they can determine how the phenomena is affected by being off to the side. They might also consider the case where the speed of the ambulance is not constant.

Two other examples of the Doppler Effect are worthy of note. If you have ever watched car races on TV (or seen them live), you now that as the cars approach they sound different than they do as they pass and drive away. That "VROOOOOOOOOMMMMMM" sound that kids often imitate is an example of the Doppler Effect. A third example has to with light from distant stars. Astronomers know what kind of light different stars give off (by recognizing the spectral lines of different elements) and they have noticed that practically all the stars in the universe give off light that is a little bit too red to be normal. Since red light is longer wavelength, it shows us that the stars are actually all moving away from us. This is the primary evidence that the Universe is expanding. In fact, many police "radar guns" that determine the speed of an oncoming car are based on the Doppler Effect (they bounce waves off the oncoming car, making it appear to be a "source" of waves. When these waves return to the gun, the Doppler Shift is analyzed and the speed of the source determined).

The Mathematics of the Doppler Shift

As was mentioned, dealing with the Doppler effect for sound requires that the medium remain still (with respect to the earth) in order not to overcomplicate matters. Let us consider such a situation where the air is still, the receiver is still, and the source is approaching the receiver. Referring to the diagram below, the source begins at point A at time 0 seconds. After time t, the source has moved to point B and is ready to produce another wave crest. During the time interval that has passed (t seconds), the wave that was first created at point A has grown to the circular crest shown below, intersecting the line of motion at point C.



The old wavelength (that measured by the source) is the distance from A to C (abbreviated $\underline{AC}$), the new wavelength (measured by the receiver) is the distance from B to C ($\underline{BC}$) and we should define the speed of the wave in the material as V and the speed of the source as V_s. It is obvious that:

$$\underline{BC} = \underline{AC} - \underline{AB}$$

or

$$\lambda_{receiver} = \lambda_{source} - \Delta\lambda.$$

To change this equation to a more useful one that involves frequency, recall that $V = \nu\lambda$ or $\lambda = v/\nu$. Thus we get:

$$v/\nu_{receiver} = v/\nu_{source} - \Delta\lambda$$

Now consider $\Delta\lambda$. It is the distance that the source moved at velocity v_s in one period of the wave. Since distance equals rate times time: $\Delta\lambda = v_sT$. But $\nu = 1/T$ and thus:

$$\Delta\lambda = v_{source}/\nu_{source}$$

Combining this with the other equation gives us:

$$v/\nu_r = v/\nu_s - v_s/\nu_s$$

Rearranging:

$$v/\nu_r = (v-v_s)/\nu_s$$

Rearranging again:

$$\nu_r/v = \nu_s/(v-v_s)$$

And finally some more rearrangement:

$\nu_r = \nu_s\{v/(v-v_s)\}$ or $\nu_r = \nu_s\{1/(1-v_s/v)\}$.

Which is the Doppler Equation for sound with a source approaching a receiver.

EX. UVXCR:) If the speed of sound in air is 340 m/s, and an ambulance approaches you at a speed of 25 m/s, blaring a siren of 2000 Hz, what frequency do you hear?

Naturally, things change if the source is moving away from you. Consider for a moment our derivation. What would change if the source were moving away? After some thought, you would determine that the equation in that situation would be:

$\nu_r = \nu_s\{1/(1 + v_s/v)\}$

Furthermore, for the cases of the receiver moving and the source stationary (relative to the medium) we would get the following equations (the derivation is very similar to the one above, and is left to the student as a practice exercise):

$\nu_r = \nu_s(1 + v_r/v)$

for the case when the receiver approaches the source and

$\nu_r = \nu_s(1 - v_r/v)$

for the case when the receiver moves away from the source. Often these are combined into one great equation that looks like this:

$$\nu_r = \nu_s \frac{(1 \pm v_r/v)}{(1 \mp v_s/v)}$$

where on the top of the equation we use:

+ if receiver approaches source
- if receiver moves away from source

and on the bottom:

- if source moves towards receiver
+ if source moves away from receiver

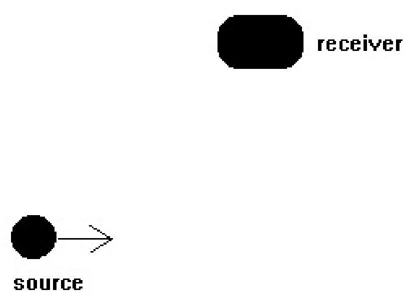
Although these conventions seem complicated, the key to remembering which is which is the understanding of what happens to the frequency. If you follow that, you should be able to determine which sign to use in each case (can you?). Notice how this reduces to all of our old equations in the cases of $v_r = 0$ and $v_s = 0$. In the case of light, it does not matter which is moving, the source or the receiver and in that case we simply set $v_s = 0$ and use the <u>relative</u> velocity between the two as v_r.

EX. LOPHY:) Imagine a 550 Hz sound is a.) moving away from you, b.) moving towards you and c.) you are moving towards the source and d.) you are moving away from it. In each case take the speed to be 25 m/s and use the speed of sound to be 340 m/s. For each part, determine the Doppler <u>shift</u> and compare them.

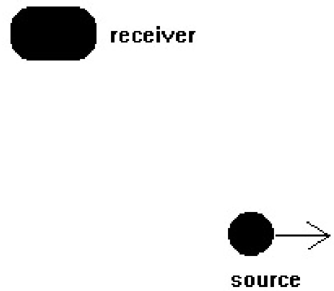
EX. OIEW:) If 750 nm light from a distant galaxy is shifted to 760 nm, how fast is the galaxy receding from us?

Doppler Shift at an Angle

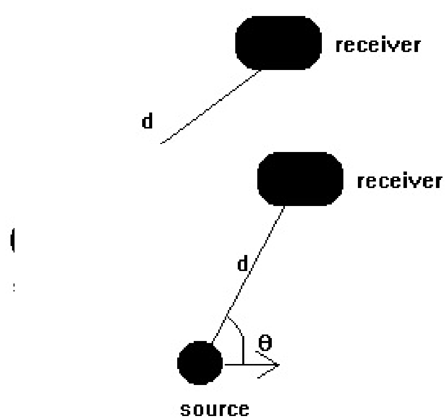
Over and over again I have cautioned the student that when discussing Doppler shift the source and receiver must be in a straight line and the motion must be along that line also. But now we should take a look at what happens when the motion is not viewed straight on. For example, imagine the situation pictured below:



Obviously, since the source is approaching, the frequency will be higher than normal. If we think about this for a few minutes, we will see that at some time later the object will have moved and we will have this situation:



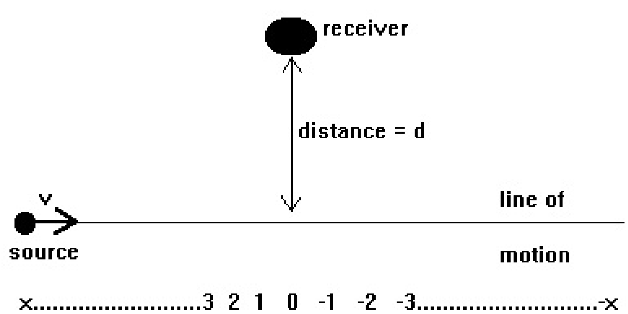
Now, the object is moving away, thus we expect the frequency to be lower. But how did the frequency change from high to low? Did it jump suddenly (as it does in the straight line case)? Anyone who has observed this listening to an ambulance will tell you no. The frequency starts off high, gets lower and lower until it reaches normal and then continues to get lower and lower. In short, the decrease is "continuous" (in the mathematical sense of the word, not the conversational sense). To explain how to handle this case, let us look at one instant in the objects motion. Certain lines and angles are labeled for later convience.



If we imagine the object at this instant, we could break its velocity into components; one that lies along the line d and one that is perpendicular to it. Consider these two velocities for a moment. The one that lies along line d represents that part of that motion which is straight "at us". The perpendicular motion is motion that is neither towards us or away from us. In that case, we would have a Doppler shift due to the motion along "d" but not due to the motion perpendicular to line d. In short, we would only use a component of the velocity v ($v\cos\theta$) in our Doppler equation for the velocity of the source. However, the complication arises a few moments later, when the object has moved. We now see:

And we notice that θ has changed dramatically. Since theta has changed, so has the velocity "towards" us (it has decreased as θ has increased and $\cos\theta$ decreased). Thus our Doppler shift decreases. Notice what happens when $\theta = 90^0$. No Doppler shift, as was predicted earlier. As it passes 90^0, it begins to increase, but negatively. This shows us the complications involved in a situation like this.

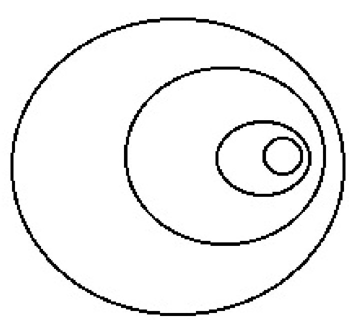
EX. KIJU:) For the situation below, write out the Doppler equation as a function of time. One hint is given, use the number line shown below.



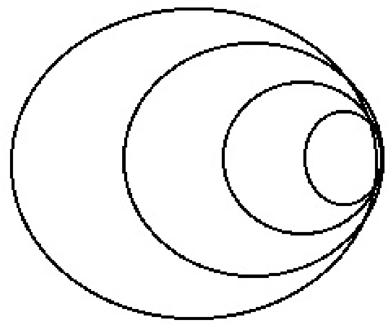
Congratulations if you somehow managed to pull that one off! What is more important is that you understand the concept being presented for these types of problems.

Bow Waves, Shock Waves and Sonic Booms

Bow waves, shock waves and sonic booms are all related to the Doppler effect. To understand how, let us take another "flight of fancy" and use our imagination. We are once again on a lake in some kind of boat that does not disturb the water (!). We begin dropping our pebbles again and moving in the direction of the waves, as before, and once again the Doppler effect occurs. Imagine that this time, however, we are very close to the speed of the waves. The pattern formed might look like this:



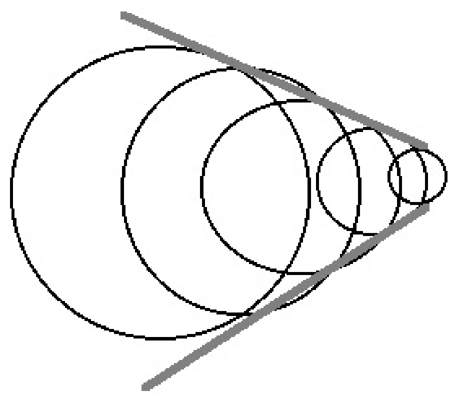
Now, imagine that you are moving at exactly the same speed at the waves travel. In this case, the pattern would look like this:



But what does this pattern mean? Think about the waves you are creating by dropping the pebbles. Every time you dropped one, the crest would remain directly in front of you and join all the other crests from the previous waves. Right in front of your motion a tidal wave would form, little by little. This is called a shock wave. In the case of sound, this would form a "wall of air pressure". This wall of air pressure is called the sound barrier. Our example, of dropping pebbles in not very realistic, since waves in water travel very slowly. The most common example is that of sound being produced by a plane, but the water helps us understand it better. If we continued in our boat and then suddenly decided to try and go faster, we would have a problem, namely a tidal wave in front of us that we would have to pass through or over. The same applies to sound. In that case, in front of the airplane we would have an enormously high region of air pressure. Since air resistance is greatly dependent on pressure, the force necessary to break through this tidal wave would be enormous. This explains why breaking the sound barrier was such a difficult feat. It might take double the amount of thrust from the engine just to break through the barrier and gain one mile per hour on the airplanes speed. However, once through, there is no more barrier to cross and there is simple air resistance to overcome (It should be noted that in the case of the airplane it is not the sound of the engines, although they add to it, that causes the barrier. It is actually the motion of the plane striking the air in front of it that produces the pressure waves that build up).

If we continued to gain speed, we would be outrunning the waves that we produce. In a plane that breaks the sound barrier, the noise produced by the engines would never reach the ears of the pilot. However, it should be mentioned that this does not mean that the pilot never hears the engines, since some of the sound of the engines is transmitted through the air inside the plane which is not in motion relative to the pilot. Also some sound is transmitted through the actual body of the aircraft. Nevertheless, a supersonic plane (flying faster than the speed of sound) is much quieter inside than a subsonic (slower than the speed of sound) plane. If we apply this same principle to boats, this is the reason that a boat is never disturbed by its own wake. The wake produced by a boat is, in a sense, a wave pattern produced by the propeller. Since the boat is going faster than the speed of the waves in the water, those waves never effect the boat until is slows to a stop.

The wake example is once again another good conceptual example used to understand what happens when a source is moving faster than the waves it creates. However, boat propellers do not produce perfect, circular waves, so we once again fall back to our example of dropping pebbles from our imaginary boat. If we were going faster than the waves travel, we would produce a pattern that looked like this:



The gray lines have been added to facilitate further discussion. What is occurring here is that by going faster than the wave speed, you have created a little triangle that spreads out behind you. In fact this is exactly the case of the wake behind a boat. It makes sense that the faster you go, the thinner the triangle and the slower you go, the wider. However, there is an interesting phenomena that occurs here along the gray lines and in the middle of the triangle. Along the edges, all the wave crests line up perfectly, and just inside of them there is an almost perfect line of troughs. In the middle, there is no pattern whatsoever. Water skiers are familiar with this. In the center of the wake the water is relatively turbulent, but with no real pattern. Off to each side, on the edge of the wake, there is a large, stable wave. The same thing happens to aircraft going above the speed of sound. Imagine a plane traveling at

Mach 1.5 (one and a half times the speed of sound - an objects Mach number is its speed divided by the speed of sound). Such a plane would produce a triangle behind it consisting of a huge, organized high pressure area (the crest) followed by a very organized and very strong low pressure area (the trough). In fact, since these patters must of course be three dimensional, not just two, it actually forms a cone of extremely high pressure that trails behind it. This cone forms what is called the shock wave. Naturally, since the cone follows the plane and expands behind it, it will eventually contact the ground. This is what is called a sonic boom. If you were on the ground and a supersonic plane passed by overhead, it would drag its cone across the ground some distance behind it. When this cone passed over you, your ear drums would react to the pressure as if to a sound, a very loud sound (it is usually helpful to think about a sonic boom as the "wake" of an airplane - when the edge wave hits you, you can hear it).

There are a number of interesting mistakes and false conclusions made about sonic booms. First, sonic booms do not only occur at the instant that the plane breaks the speed of sound. Sonic booms are formed and travel behind the plane for the entire time that the planes speed is above the speed of sound. You only hear it once because you only detect it when the cones intersection with the ground passes over your position. However, the boom remains intact the entire time. This is why supersonic aircraft are not flown over populated areas. When the military wishes to fly above the speed of sound, it does so in the desert or in the mountains, where the shock wave will not rattle peoples picture frames off the wall. The only supersonic passenger jet that is used is the famous "Concord" that flies between London and New York. However, it is not allowed to go above the sound barrier until it is over the ocean. It must slow down as it approaches land, so as not to disturb the people over which it flies. Another note that should be made is that since the cone grows, the intensity diminishes with distance. Thus a plane high up in the air will not make as great a boom as one close to the ground. The further from the ground, the more the cone has expanded and the less intense (likewise the more time that has passed, allowing the second law of thermodynamics to do its work). Lastly, although the sonic boom is actually a three dimensional cone shape, it should not be confused with the drawing presented earlier. That drawing is a two dimensional representation of a shock wave. It is by pure coincidence that it gives the illusion of a cone. The cone we are discussing would be formed by rotating that drawing around its axis.

Beats

In this last section we will address yet another common wave behavior that exists for all waves, but really is only detectable and reasonable to discuss for sound waves and that is the phenomena of beat. A beat is a form of wave interference that occurs when two waves, very close in frequency, but not identical interact. To understand what occurs, we will return to our mathematical treatment of waves. Consider two wave that fit the above description as described by the equations below:

$$W_1 = A\sin(k_1x - \omega_1t)$$
$$W_2 = A\sin(k_2x - \omega_2t)$$

Combining them we get:

$$W = A\{\sin(k_1x - \omega_1t) + \sin(k_2x - \omega_2t)\}$$

Now using the identity:

$$\cos\alpha + \cos\beta = 2\cos\{(1/2)(\alpha - \beta)\}\cos\{(1/2)(\alpha + \beta)\}$$

We arrive at (after a little rearranging):

$$W = 2A\cos\{(1/2)([k_1-k_2]x+[\omega_1-\omega_2]t)\}\cos\{(1/2)([k_1+k_2]x-[\omega_1+\omega_2]t)\}$$

Which is a horrible mess. Let us attempt to understand what is going on by doing two things. First, we will look at this wave at a particular location (a constant x), thus reducing the need for all the x terms (since they are constants we will ignore them, as if they were a phase angle adjustment). Secondly, we will make the following substitutions:

$$\omega_+ = (\omega_1 + \omega_2)/2$$
$$\omega_- = (\omega_1 - \omega_2)/2$$

Then we have (since $\cos(\alpha) = \cos(-\alpha)$):

$$W = 2A\cos(\omega_-t)\cos(\omega_+t)$$

Much like we did when we discussed standing waves, let us call the first part of this wave, the $2A\cos(\omega_-t)$ part, the amplitude. Thus we have a wave that has an amplitude that itself is also a changing wave. In analyzing this, we should notice that what we get is a new wave of new frequency ω_+ that has an oscillating amplitude with frequency ω_-. Putting this into terms of sound waves, the situation is this: If two sounds are plays, whose frequency is close, a third sound will be heard and its volume (amplitude) will oscillate between loud and soft. A closer look at ω_+ tells us more. This quantity is the two frequencies added together and then divided by two. In other words the average. The frequency of the amplitude is ω_-, which is the difference divided by two. However, because of the way in which the wave gets loud and soft, it will do so at twice this frequency (just as the wavelength of a standing wave is 2 "bumps", not just one).

This is worth emphasizing and repeating: <u>If two sounds close in frequency interfere, the sound that is heard is the average of the two frequencies and it gets loud and soft at a frequency that is the difference of the two waves</u>. Although this is hard to illustrate, this wave pattern can be seen easily with a graphing calculator, by inputting the above function, or even better, by inputting two sine functions and having the calculator add them together. It is suggested that the student try this, varying the two arguments to see the different results. A number of conclusions can be drawn about beats:

- The new sound produced is the average of the two frequencies.
- The beat frequency is the absolute value of the difference between the two frequencies.
- The two waves must be similar in frequency but not identical for beats to be noticeable.
- If the two waves are too far apart (say 200 Hz and 110 Hz) then the beat frequency is too great (in this case, 90 Hz) and it is getting loud and soft so fast that the oscillations cannot be detected.
- The closer the two waves are, the more noticeable the beats are. This is because as they get closer together in frequency, the beat frequency goes down, meaning the beats last longer.

Beats are used in a number of different ways, and like resonance, they can be used to detect small changes since the beats get longer as the sounds get more similar. A perfect example of this is that beats are used to tune instruments. If you wished to tune a piano, you would use a tuning fork set to the correct note. By striking the tuning fork and the piano string at the same time, you could listen for beats. If you did hear beats, you would know the string is out of tune. If it was, you could tighten the string and see if the beats get longer or shorter. If they get shorter, you know the string needs to be loosened. If they get longer, you keep tightening until they disappear. This same method is used for other instruments as well.

Another interesting example of beats is that beats were used to prove the Doppler Effect. When it was first theorized, scientists had to determine a method of measuring and proving the effect did exist. What they did is this: they took two trombone players and made sure they were in tune with each other (by checking for beats). Then they put one on a train that would move away from the listeners. When they both played, beats were heard, proving that the sound from one of the trombones was slightly "shifted" because of the motion.

EX. EVUH:) Explain what is heard if a note of 350 Hz is played simultaneously with the note of 358 Hz.

EX. HUYG:) While tuning a note on a piano, you hear 4 beats per second while using a tuning fork of 440 Hz. What note is the string playing?

Assignment #33

1.) Spherical waves are emitted from a 1.0 W source in a non-absorbing medium. What is the wave intensity (a.) 1.0 meter from the source and (b.) 2.5 meters from the source. (W4)

2.) Two sounds are emitted, one at 35 dB and another at 90 dB, how many times greater is the intensity of the 90 dB sound than the 35 dB sound ? (W16)

3.) While at a concert, a student notices that the main acts sound level of 105 dB is considerably higher than the warm up bands level of 87 dB. (a.) How many times greater is the main acts intensity? (b.) Later in the show, she runs into a friend who claims that the sound is much better nearer to the speakers. She claims it is about 135 dB at 15 m from the speakers. What is the power output of the speakers ? (W19)

4.) People often brag about their car stereos having outputs of 200 W. (a.) What would be the intensity of a stereo speaker (considered a point source) at 0.75 m if its output were 200 W ? (b.) What would its intensity be in decibels ($I_o = 10^{-12}$ W/m^2) ? (W24)

5.) While driving to school one day, a reckless student misjudges an amber traffic light and gets a ticket. In court he tries to defend himself by saying that the color of the light was "Doppler Shifted" from red to green. Thus to him the light was actually a green light. The judge does a little calculation and dismisses the ticket for running a red light and replaces it with a ticket for speeding. How fast was the student going ? (frequency of red light = 4.6×10^{14} Hz, frequency of green light = 5.7×10^{14} Hz, speed of light = 3×10^{8})(W20)

6.) Imagine there is a stationary source of 650 Hz sound. Is there any speed where the change in frequency when you approach the sound is the same (absolute value wise) as when you recede from the sound ? (W28)

7.) Imagine that you hear a car horn blast when it is 200 m away from you, it is however, not moving directly towards you. At that point the frequency is 445 Hz. A short while later, the car passes directly in front of you 60 m away and sounds it horn again, this time the horn is 440 Hz. What is (a.) the actual frequency of the horn, (b.) the speed of the car, and (c.) the time that has elapsed between the blasts of the horn? (assume the car has maintained a constant speed)

8.) A supersonic aircraft will a conical shock wave behind itself as it travels. Derive a simple equation relating the speed of the craft and the angle of the cone to the speed of the waves in the air. Explain how this would relate to boat wakes.

9.) If you struck two tuning forks, one with a frequency of 532 Hz and another with a frequency of 528 Hz, what would you hear ? (two parts to this answer) (W23)

10.) While playing with things in the room (which he was not supposed to be doing), a student uses two tuning forks to set up waves of the form:

$$y1(x,t) = 6\sin(5.0x - 1728t)$$
$$y2(x,t) = 6\sin(7.67x - 1765.5t)$$

What exactly does he hear? Explain conceptually (with numbers) and write out the equation (note: when you hear something, you are standing at ONE location).(W29)

11.) If a tuning fork with a frequency of 660 Hz produces a sound of 656 Hz that gets loud and soft 8 times per second when it is used with a second fork, what is the frequency of the second fork? (W33)