Chapter 30: Wave Behavior

Principle of Superposition

Now that we have a general grasp of what a wave is and how to describe it, we need to move on and discuss exactly how waves behave in certain situations. The first of these situations is to examine what happens when waves combine or pass through each other. When waves intersect each other, we call this "interfering". Interference is a term that is probably familiar to the student, when radio stations or cellular phone calls get "fuzzy", we say that we are getting some interference. In fact, that is exactly the case. The wave that is carrying either the phone call or the radio station wave is intersecting and being interfered with by some other wave, causing the static. When two waves pass through the same point at the same time, the outcome is determined by the Principle of Superposition. The Principle of Superposition is a fancy way of saying that when waves interfere, they algebraically add together and when they pass, they continue as if nothing ever happened. We will restate these comments in a moment, but first another point should be mentioned, and that is that only waves of the same kind can interfere (this is not completely true, but for the next few years of studying physics you can believe this and it will cause no problems). For example; two sound waves passing through each other can interfere, as can two light waves passing through each other. However, one sound and one light wave will not interfere, they will pass through each other without having any effect on each other.

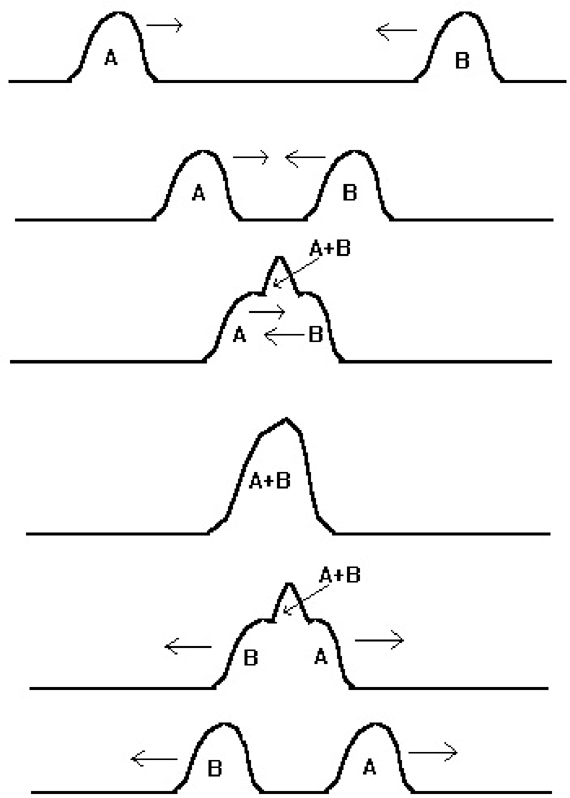
Once again, the Principle of Superposition has two parts:

1.) When waves (or wave pulses) interfere, they result in a new wave whose amplitude is the algebraic sum of the two original waves.

2.) When waves are done interfering, the two waves continue unaffected by the interference as if nothing had ever happened.

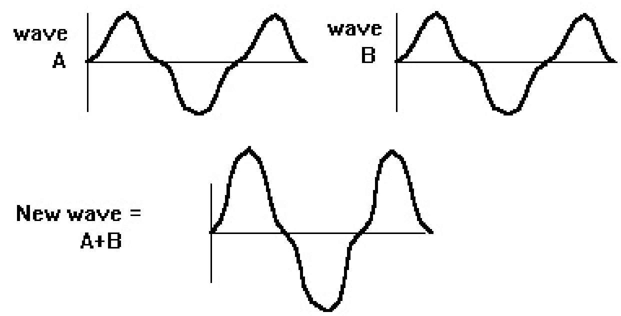
Notice how unique this phenomena actually is. The two waves intersect, create a brand new wave (the sum) and then go off as if nothing ever happened. Compare this to two cars "interfering" (in the strictest sense, they cannot, it would violate one of the oldest and most revered of the philosophical laws of the western world). When two cars interfere, they certainly do not just "continue on as if nothing ever happened". This illustrates one of the major differences between waves and particles. It also shows you a glimpse of how complicated things can get when you start dealing with things like electrons as waves (something that is done in modern or quantum physics).

The way we will approach understanding how to use the principle of superpostiion is to first deal with it on a conceptual level and then look at the mathematics behind the phenomena. The principle can be easily illustrated by first showing the simplest case. On the diagrams below, two wave pulses (single crests) are approaching each other on a string (although the student should remember that this scenario is completely general and the two pulses could be sound or light or ocean waves). Notice how they interfere.

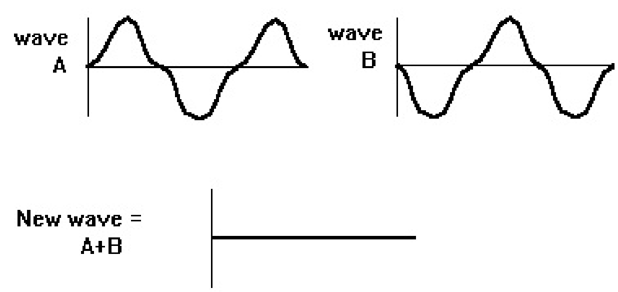


Notice how the two waves combine when they pass through each other and then return to normal after the interference. Likewise, although it is not obvious or apparent from the illustration, while they are combined, the new wave (A+B) is exactly as high as the values of each of the individual waves added together at each point. When they are entirely on top of each other as in the fourth diagram above, if the two waves are identical, the resulting bump should be double the height and yet the same width as either individual wave.

We can, of course, take this concept further and demonstrate how it works on full waves, and not just wave pulses. Imagine two waves, completely identical, that are traveling the same direction (along the same one dimensional line). A little common sense along with the Principle of Superposition, shows us that the new wave that results is exactly double the height as the original waves and yet keeps the same wavelength and frequency an important fact). The diagram below illustrates this.



Once again, to stress the results, notice that the two waves combine to form a new wave that is twice the height, but the same wavelength (if the above graph is an amplitude versus position, with time held constant). However, an astute student might suggest that the only way this will work is if the two waves are "in sync" or "in step" with one another. For example, if we take two waves that are identical, it is possible for them to cancel each other out, as shown below.



In such a case, every positive value of wave A is perfectly canceled by a negative value of wave B and visa versa. It is not hard to notice the difference between the two cases, in one case the two waves are "in phase" and in the second case they are perfectly "out of phase". In phase means that the phase angles of each wave equation are equal (in the first example, the phase angle of each wave was zero, however, a little thought should show the student that they could be anything, as long as they were equal) and out of phase means that the angles differ by π, or 180^0 (as in the second diagram where wave A has a φ = 0 rad and wave B has φ = π). From these observations, we can draw some conclusions. First, the phase difference (a very important term) determines the outcome of two waves interfering. Second, since a phase difference of zero gives double the amplitude and a phase difference of π give no amplitude, it is logical to suggest that phase difference between 0 and π should give values in between zero and double for the waves.

Before we progress on to discussing this mathematically, there are a number of important points that should be either stressed, defined of mentioned.

- The outcome of interference between two waves in dependent on the phase difference between the two waves at the point or points in question (notice the ending of this statement, we will clarify it later).
- When the phase difference between two waves is zero, the waves double. This is called <u>perfectly constructive interference</u>. When the waves do not have a phase difference of zero, yet add together to produce a wave greater than either of the two original waves, this is called <u>constructive interference</u>.
- When the phase difference between two waves is π, the waves cancel. This is called <u>perfectly destructive interference</u>. When the waves do not have a phase difference of π, yet add together to produce a wave of less amplitude than either of the two original waves, this is called <u>destructive interference</u>.
- There is a great difference in discussing the phase difference of two waves and the phase difference of two waves at one point (the student should ponder this for a while until it makes sense).
- Only two identical waves, traveling the same direction can have a phase difference between the waves that is constant. You cannot discuss the phase difference between two non-identical waves, since it is different at every different point in space and in time.
- Two identical waves traveling opposite directions do not have a constant phase difference, but each and every point in space has the same phase difference that changes over time. In short, two identical waves traveling opposite direction have a time dependent phase difference.
- Two identical waves, traveling random directions, will have phase differences that are constant at points of intersection, but different at different points.

The last four comments require a rather sophisticated mental exercise to understand, but I encourage the student to attempt it and not just to read them and say "yea, sure". Experiments with two strings laid on a table in the shape of waves might help the understanding along. Remember that you are looking for the phase difference between the two waves, which is equivalent to noticing where they are in each cycle at any given point and comparing them.

We should now turn our attention to a mathematical treatment of interference. I will remind the student before we begin that because of the difficulties outlined above, we will at first only deal with one dimensional waves having the same wavelength and traveling in the same direction (of course these waves must be of the same type and kind, therefore same wavelength implies same wave speed and frequency). In short, our goal is to mathematically add two waves that are identical except for a phase difference.

We begin with two waves as such:

$$W_1 = A\sin(kx-\omega t)$$
$$W_2 = A\sin(kx-\omega t+\varphi)$$

Notice that we have taken wave to have a phase of zero and the other to have a phase of φ. In fact, whatever two waves you are adding, you can assign one a phase of zero and put the phase difference into the other as its phase. Thus our proof is still general. Now, the new wave will be:

$$W_3 = W_1 + W_2$$

$$W_3 = A\sin(kx-\omega t) + A\sin(kx-\omega t+\varphi)$$

$$W_3 = A\{\sin(kx-\omega t) + \sin(kx-\omega t+\varphi)\}$$

Recalling our trig identity of $\sin\alpha + \sin\beta = 2\sin\{(1/2)(\alpha+\beta)\}\cos\{(1/2)(\alpha-\beta)\}$, we get the following mess:

$$W_3 = 2A\sin\{(1/2)(kx-\omega t+kx-\omega t+\varphi)\}\cos\{(1/2)(kx-\omega t-kx+\omega t-\varphi)\}$$

which reduces to:

$$W_3 = 2A\sin(kx-\omega t+\varphi/2)\cos(-\varphi/2)$$

rearranging and recalling that $\cos(-\alpha) = \cos(\alpha)$, we have:

$$W_3 = \underline{2A\cos(\varphi/2)}\sin(kx-\omega t+\varphi/2).$$

Now, let us take a look at this in depth. What we have is a new wave consisting of two parts. The first, the underlined part, is not dependent on either position or time, and is thus a constant. This entire thing is considered to be the amplitude of the new wave. The second part is a wave equation, since it is dependent on x and t. Notice that the amplitude of the new wave is dependent on the phase difference (as we reasoned earlier) and the new wave has the same k (and thus wavelength) and ω (and thus frequency) as the original waves. Also notice that the phase of the new wave is half of the phase difference between them.

Now let us do three example problems using the principle of superposition. The first uses straight mathematics and logic along with the principle, the second uses the result just found and the third once again uses logic.

EX. DFRE:) A duck sits on a lake 20 meters to the right of a boat and 60 to the left of another. As the boats pass, they send out wakes according to the equations below. How high from his or her resting position is the duck after 10 seconds?

Boat on the right sends out wake according to:

$$W_1 = (0.45\ m)\sin\{(2\ m^{-1})x - (3\ s^{-1})t\}$$

Boat on the left sends out wake according to:

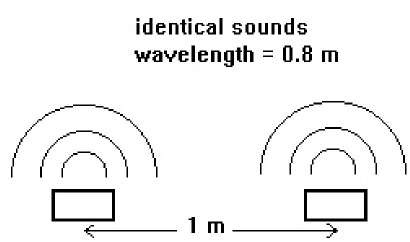
$$W_2 = (0.7\ m)\sin\{(1.3\ m^{-1})x + ((0.6\ s^{-1})t\}$$

EX. UYHGT:) Imagine two waves shown below interfere. Find the amplitude of the resulting wave and sketch all three waves at t = 0.

$$W_1 = (1.2\ m)\sin\{(2\pi/3\ m^{-1})x - (\pi/4\ s^{-1})t + \pi/6\}$$

$W_2 = (1.2\text{ m})\sin\{(2\pi/3\text{ m}^{-1})x - (\pi/4\text{ s}^{-1})t + \pi/3\}$

EX. BNM:) Imagine that two speakers, facing the same direction and positioned 1 m apart, emit a sound with a wavelength of 0.8 m in perfect semicircular waves. Find a position somewhere in front of the waves where they will interfere perfectly constructively, perfectly destructively and one where they will produce a wave with the identical amplitude of each interfering wave. Also discuss what these results mean in terms of the sound heard and explain what would happen if these were light bulbs instead of speakers. See diagram.

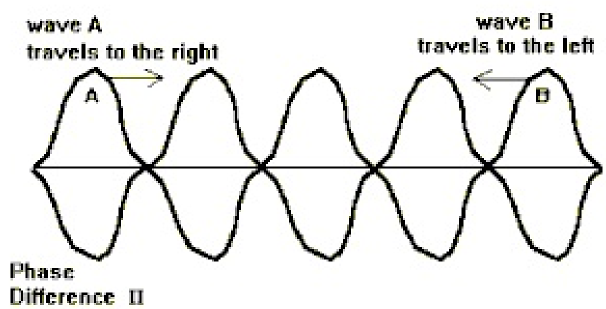


The last problem had an infinite number of correct solutions, the points simply had to satisfy a simple equation. The student should keep this example in mind, since it illustrates another principle that will be discussed in detail later.

Standing Waves

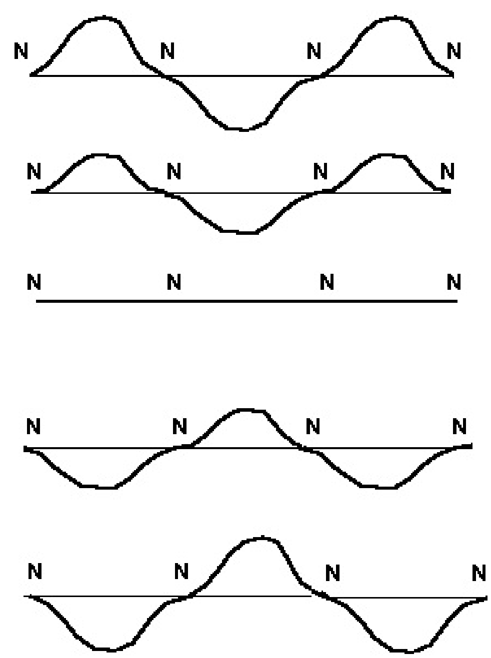
The next topic to discuss is another type of interference produced under a special set of circumstances. This interference is called "standing waves". Standing waves and standing wave production is one of the most important, if not the most important, type of interference to understand. In order to comprehend it, we will attempt to look at it logically first (which is actually very difficult to do) and then look at it mathematically. Once we understand it, we will look at cases where standing waves occur and how they are produced (which will require us to introduce another side topic) and then we will look at some of the consequences of standing waves, namely resonance.

Put quite simply, standing waves are produced whenever two identical waves traveling opposite directions interfere. As will be discussed later, this is a very common occurrence. If the student will recall it was discussed earlier that identical waves traveling opposite directions do not have a set, constant phase difference.



However, each individual point in space (along the string, or whatever medium through which the wave travels) has the same phase difference at any one time as any other point. This is hard to visualize, but the outcome is illustrated below. Imagine that along the line, two identical waves are interfering. The phase difference between the two waves is the same everywhere (see diagram). Now imagine that the two waves move. The phase difference has changed, but is still uniform.

If the waves each moved one quarter cycle in the diagram above, the new phase difference would be zero. If they once again moved one quarter cycle each, the phase difference would be π again, etcetera. In short, the phase difference cycles from zero to π to zero to π, and on and on. Consider what this means according to the principle of superposition. We have an ongoing cycle of constructive to destructive interference. The wave should grow and disappear and grow and disappear. That is in fact exactly what happens. However, there is one further important behavior, which is even harder to visualize. The points where the zero amplitudes of the waves meet in the diagram above are called nodes. Imagine what happens at these specific points as the waves move. At those points, the waves will <u>always</u> add up to zero (this is different that destructive interference). At those points, there is never any wave. The same logic can be applied to the other points. At any point on the line, there is a set amplitude which will never be broken. The points between the nodes are called anti-nodes and they are points of maximum amplitude. The diagram below attempts to show a standing wave in action (remember, it is the result of two other waves interfering that produces one resultant wave). The nodes are labeled "N".



Since in this scenario, notice how the wave "crests" always appear in the same spot. Thus wave seems to "grow" and shrink without moving, ergo the name: "standing wave".

Some further conclusions can be made from this conceptual understanding of these interference patterns.

- The new standing wave has the same wavelength as the original interfering waves.
- Nodes occur at points of zero amplitude, every half wavelength.
- Maximum amplitude occurs every half wavelength and has a value of twice the amplitude of the original waves.

Mathematically, the situation is thus. We begin with two identical waves traveling opposite directions:

$$W_1 = A\sin(kx-\omega t)$$

$$W_2 = A\sin(kx+\omega t)$$

Adding together gives:

$$W_3 = W_1 + W_2$$

$$W_3 = A\sin(kx-\omega t)+A\sin(kx+\omega t)$$

$$W_3 = A\{\sin(kx-\omega t)+\sin(kx+\omega t)\}$$

Using the same trig identity that was used in the previous proof gives:

$$W_3 = 2A\sin\{(1/2)(kx-\omega t+kx+\omega t)\}\cos\{(1/2)(kx-\omega t-kx-\omega t)\}$$

$$W_3 = 2A\sin(kx)\cos(\omega t).$$

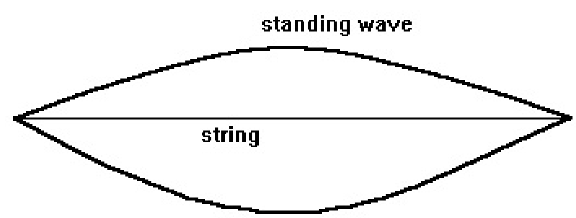
This simple equation gives the format for a standing wave. Notice how the sine part depends only on position. This is the amplitude that was discussed earlier. Each position has a maximum amplitude given by 2Asin(kx). Nodes are where this function equals zero, occurring once every half wavelength. Antinodes are where this function equals 2A. The second part of the equation, the cosine part, shows how the wave varies at that position over time.

Before we progress to the next topic, I would like to reenforce that standing waves and understanding them is very important. Standing waves explain how and why musical instruments work, and show themselves in an incredible number of other situations.

Standing Wave Production

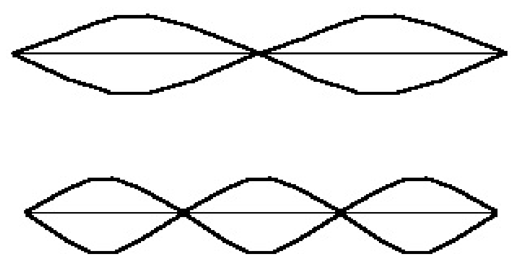
Now that we understand standing waves, we can turn our attention to how they are produced and look at situations where they might arise. It is in fact very rare that two identical waves would come at each other from opposite directions from different sources. Instead, most standing waves are formed when one wave bounces off a boundary and interferes with itself. Imagine tieing a string to a doorknob and shaking it once so that a pulse is sent along the string. When the pulse hits the knob, it does not disappear, it is reflected back. If you were to shake the string continuously, it would be reflected continuously and you would have two identical waves interfering. This sort of situation is very common. We will begin to explain it using the limited case of a wave on a string and then expand to more general cases.

If we has a string or wire, stretched between two posts, and we plucked it (as in the case of a guitar string), waves would travel along the string in both directions from the "pluck", hit the ends and bounce back, creating standing waves on the string. These waves would eventually die out (due to the second law of thermodynamics), but the waves would produce a sound. It is possible that the string might vibrate as shown below.



Notice the relationship between the string and the standing wave. If the string were 80 cm long, the wavelength of the standing wave produced would be 160 cm. (why double?) This brings us to an important point: When a standing wave is produced, it must be of the proper wavelength to have a node on each end (this is only for a string, we will discuss other cases later). Now, an astute student might ask, "But we just plucked the string, how did we know to pluck it at the proper frequency to produce standing waves?" The answer is, of course, we didn't. What happens in reality is that when the string is plucked, we sent out a myriad of different waves along the string. However, only the frequencies that produced standing waves remained long enough to be noticed. These special frequencies, called resonant frequencies, set up standing waves in the string. All the other frequencies, because of their different wavelength, worked to cancel each other out after bouncing back from the ends of the string. This process is a little difficult to visualize, but an attempt will be made later when we discuss boundary behavior.

Another question that might be asked is "Since there are obviously more than one frequency that can set up standing waves, why was only on illustrated?" In the same situation as above, the following wave, with λ = 80 cm, would also set up standing waves as would one with λ = 53.3 cm.



The answer to that question is that we didn't know for sure which one would show up, because it depends on many other conditions, such as how quickly the string was plucked, etcetera (it is also very possible that more than one of these waves could show up at the same time, more about that in the next chapter). The lesson here is that any number of standing waves could be produced and therefore, there are an infinite number of resonant frequencies for any given string. The only condition is that there must be a node at each end.

EX. HYGTF:) Determine the five longest wavelengths that would produce standing waves on a string 120 cm long. Attempt to find an equation to determine the proper wavelengths for any resonant frequency on any string. Next, assuming that the speed of the waves in the string is 760 m/s, determine the frequencies that go along with these waves. Sketch all five standing waves and write the equations for the first two.

This same sort of effect arises just about any time you strike an object and it makes a noise (especially a noise that lingers after you have stopped striking it). A perfect example of this is a drum. When you strike a drum, waves go out in all directions, any wave that is not of the proper frequency to produce standing waves is diminished and standing waves are set up across the top of the drum. When you speak, your vocal chords are vibrated by the air rushing across them and they form standing waves. By changing the tension with your muscles, the chord changes to accept different standing waves and different sounds are made. When you strike a glass with a spoon, the glass vibrates, causing standing waves which vibrate the air and produce a sound. When you blow air across the top of a soda bottle, the turbulence in the air produces standing waves in the column of air, thus producing the sound you hear. All of these, and many others, are examples of standing waves. However, not all of the standing waves are produced in the same fashion. Before we investigate that further, let us reiterate some facts about standing waves.

- Standing waves on a string must have a node on each end.
- Standing waves must be produced by waves with the proper frequency to position a node on each end.
- All the frequencies that produce standing waves are called resonant frequencies. These are also sometimes called <u>natural frequencies</u> since these are the frequencies produced

when the object is struck.

- Other frequencies are also produced, but their waves die off quickly, while standing waves often linger.

Boundary Behavior

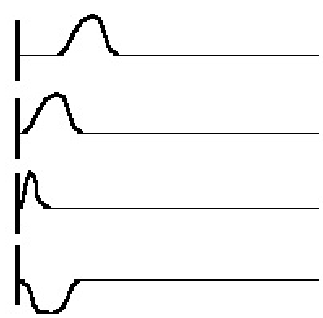
We have stated that standing waves on a string must have a node at each end. We have not stated why this is so. The obvious answer is that since the string is fixed, it cannot vibrate there. However, this would not explain the behavior of a column of air in a soda bottle, since it is free to vibrate however it wishes. The answer lies in the behavior of waves at boundaries.

When a wave encounters a boundary, a number of things happen, all dependent on the type of boundary it encounters. First and foremost: When a wave hits a boundary, a portion of the energy of the wave is transmitted into the new material and a portion is reflected back. A boundary is considered to be the interface between any two materials. For example, the string attached to the post is a boundary, thus every time the wave hits it, some energy actually goes into the post and some is reflected. Another example would be the boundary between the air in the soda bottle and the bottle itself, in that case it would be called an air-glass boundary. Some of the sound wave that hits that boundary would be reflected and some would be absorbed (or transmitted) into the glass. Yet another example would be the air glass boundary between the air in your room and the window. When the light in the room bounces off objects and hits the window, some is transmitted into the glass and in turn escapes outside, while some is reflected back into the room. At night, when very little light is coming into the room from the outside, you can see the reflected portion of the light clearly.

Not all boundaries reflect and absorb (or transmit) waves equally. The amount of reflection depends on a number of factors. The two most important factors are the angle at which the wave hits the boundary and the make up of the materials involved. The angle will be discussed in a later chapter, but it behooves us turn a little attention to the factor regarding the materials. The amount of reflection (and thus transmission) that occurs depends on the difference between media involved in the boundary. However, each different type of wave involves a different property of the material. For example, sound is a mechanical wave and therefore the properties that determine reflection would depend on such things as the elasticity, rigidity and density of the material. Boundaries that contain similar materials (with regards to these properties) would cause little reflection. For example, a sound wave traveling through brass that encountered a brass-steel interface, would suffer little reflection. However, going from air into glass would cause a great reflection or the sound. In the case of light, since light is an electromagnetic phenomena, the properties of the materials that matter are electrical (and magnetic) in nature. In fact, in most cases (both light and sound) the single predictor of reflection that is most reliable is the speed of the wave in the material. If the speed in both media on either side of the boundary is similar, there should be little reflection.

One conceptual method of dealing with boundaries is to classify them as either "hard" or "soft" depending on the difference in the material. Naturally, no boundary is completely "hard" or completely "soft", but instead there is a continuum from hard to soft. Hard boundaries tend to reflect a large portion of the wave and soft boundaries very little. There is yet another boundary behavior that should be discussed, and that is what is called "phase shift by reflection". To understand this, consider the following two illustrations of hard and soft boundaries. A hard boundary can be seen as a string tied to a pole at one point. For a soft boundary, consider a string hanging vertically, not tied to anything. The soft boundary is the free end of the string, where string meets air.

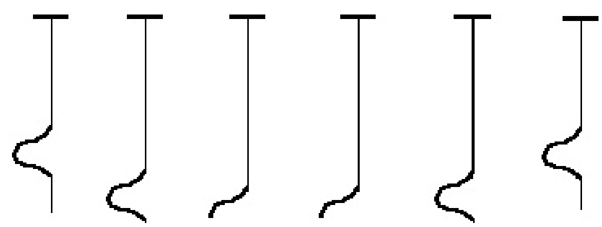
If we shake the first string (the hard bounded one) and send a single pulse down the string, imagine its behavior as it hits the boundary and returns. It might look something like the diagram below.



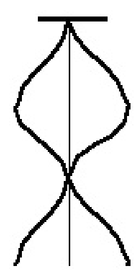
This is called a full phase shift by reflection, since when the wave returns, it is inverted (or phase shifted automatically by $\pi/2$). Hard boundaries invert waves when they reflect them. Naturally, this is the extreme case, but depending on the hardness, the phase shift can be any value up to $\pi/2$.

This thinking explains why standing waves on strings have nodes on the end. Since these are obviously hard boundaries, the incoming and reflecting wave are completely out of phase. Since they combine to produce the standing wave, at the boundary they will automatically produce destructive interference.

Now consider a soft boundary. In this case, logic will tell you that there is no phase shift on reflection and the two waves add constructively, producing not a node, but an antinode. The situation with a free hanging string will show this.



This leads us to our second important conclusion: <u>when waves encounter soft boundaries, the standing waves produced must have an anti-node at that end</u>. Thus if you were to set up standing waves in the vertically hanging string, you would have a node at the end you are holding (hard boundary) and an anti-node at the free end. The wave would look like this:

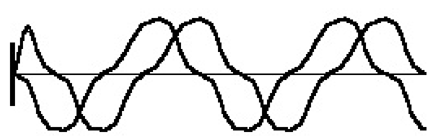


Notice that this is only one of many possible standing waves (resonant frequencies), and not even the longest. I leave it to the student to draw the others as an exercise. This type of standing wave occurs in the case of blowing across the top of a soda bottle, or tapping a cup that is sitting on the counter. It also occurs in situations involving open-ended pipes. If a pipe is open on one end, the standing wave will form an anti-node at that boundary. "Wait a minute," says the astute student, "why is an open end of a pipe considered a boundary? The wave doesn't change media." True, it does not, but the change in conditions (restricted air to unrestricted air) is in fact considered a boundary. One factor that we are ignoring is that open ended pipes do not actually form a perfect anti-node. There is a slight correction needed to deal with this situation, but we shall ignore this in our discussion. Logically, the student should be able to reason what type of standing waves form in a pipe with two open ends (a perfect example of this phenomena that is easily visualized is to grab a flexible meter stick somewhere about one quarter of the way from one end and shake it. If you shake at the proper frequency, you will form standing waves in the stick.).

EX. OPINU:) By blowing on an empty soda bottle (35 cm deep), a sound is heard. Supposing that the sound heard is longest standing wave possible, determine the frequency of the sound given the speed of sound in air is 340 m/s. If the bottle is filled with 10 cm of water and a second sound is heard, determine the frequency of the second sound.

EX. NMJB:) Draw the first five standing waves that can be formed in a.) a closed pipe and b.) an open pipe of length 1.5 m. Determine the wavelength of each and determine the frequency of the longest wave in each case using the speed of sound given above.

Before we move onto the next topic in this section, I wish to revisit a comment made earlier regarding standing waves. It was said that many waves are formed when an object is struck, but only the standing waves "survive". Let us reexamine that statement. If a wave of an improper frequency to set up standing waves encounters a boundary, we might have a situation as such:



Notice that because the waves do not match up on the return, at some points the wave will actually be working against itself, cancelling some of its energy. In this fashion the wave will die out and only the proper (resonant) frequencies will remain on the string. This is one of the reasons that often, if you try to make music on something other than a tuned instrument, when you first make the note you might hear a bunch of gibberish, but as time goes on, only a clear note remains. One excellent example of this is to take a solid, metal bar (cylindrical, preferably) and hold one end at any random point. Now strike the top with a hammer and you will hear some (jarring) gibberish and then a clear note. It is also interesting to hold the bar in other random place and hear the different notes that are produced. As an exercise, the student might attempt to determine and draw the different standing waves. One hint: at the point where the pipe is being held, you are"forcing" a node at that position. This same technique is used when a guitar is played and the string is held down on a fret to achieve a different note.

Resonance

At this point we should understand what exactly standing waves are and how they are produced. It has been mentioned earlier that standing waves occur at what is called the resonant (or natural) frequency and that just every object has at least one resonant frequency. Thus it should be possible to set up standing waves in anything; a book, a table, a string, a bridge, a building. Indeed it is possible to do just this. Setting up continuous standing waves in an object is done by a process called resonance (or driven oscillations). Amazing and powerful things can happen using the concept of resonance. In this section we will begin by explaining what is occurring and then discuss some situations where this phenomena can be observed.

Resonance occurs when an object is struck or "driven" at its resonant frequency. When this occurs, standing waves are formed in the object. But since the object is continually struck, the standing waves combine constructively and the disturbance in the object grows. Driving an object at its natural frequency causes what is (erroneously) called a "perfect energy transfer" from the driver to the object. <u>Perfect energy transfer occurs only at the resonant frequency</u>, no other frequencies will work. Because this transfer is highly efficient, the energy leaves the driver and enters the object where it continually builds. Thus the energy of the object increases to incredible levels. To further understand what is occurring, let us use an illustrative example. In all of these situations, there is a driver (some device causing a wave to form or causing the object to be struck) and some object that is resonating.

In this first example, let us consider a child on a swing to be the object and ourselves to be the driver. Imagine pushing a child on a swing set. If we just closed our eyes and randomly pushed our arms forward, sometimes we would push the child and cause them to go higher. However, sometimes we would actually be working against the swing by pushing them at the wrong time (as they were still moving backwards) and sometimes we would be pushing into thin air. That is the normal situation that occurs when an object is driven at some frequency other than its resonant frequency. Occasionally we are working to cause it to vibrate, but occasionally we are not. In such an instance, the net result of our pushing is zero (although it is possible to have some positive effect).

However, now imagine that we are pushing at just the proper frequency to help the child go higher and higher. This illustrates resonance, because every push that we give goes into helping the child swing (vibrate). This is what is meant by a perfect energy transfer from the driver (us) to the swing (the object). Every push helps and no pushes hinder. Notice how the second law of thermodynamics is not suspended here. Every transfer must be accompanied by some loss, but at least every push is a gain in energy for the object.

If we follow this logic to its conclusion, we see that when a resonant frequency is matched, there is only two possible outcomes. Either the system can begin to lose all the energy gained in one cycle during the cycle or the system can go out of control. In the case of the child on the swing, either the child must lose all the energy gained from one push in one swing (in the form of air resistance, etcetera) or the child will go higher and higher until the chain breaks or they go all the way around the top bar (experience tells us which of the two occurs). In other systems, the outcome might be the other of the possible choices. Let us now examine a number of examples of resonance.

The first is a common example that is often shown in physics classes. Two tuning forks of the same frequency are mounted on wooden boxes with one end open. The two open ends are set facing each other and one of the forks is struck. After a second, that fork is silenced (by grasping it) and it is noticed that the second fork is vibrating. What has occurred here is that the first fork sent out sound waves (air vibrations) at the resonant frequency of the second fork. These air vibrations hit a resonance with the second fork and caused it to vibrate. The fork began losing energy as quickly as it was gaining it.

Another example is the shattering of a wine glass by singing. There are singers that can actually sing loud enough to shatter a wine glass, yet not hurt your ears if you are in the room. This occurs because the singer is singing exactly at the resonant frequency of the wine glass (a difficult task) but not at the resonant frequency of your ears. This gives a Perfect energy transfer from their sound waves into the wine glass. If they are singing loud enough, the glass cannot get rid of the energy faster than it is coming in. The glass vibrates more and more until it finally shatters.

Resonance can also be seen on larger scales. It is possible to set up standing waves in buildings. Imagine a building that is near the site of an earthquake. Even if the earth quake is not very powerful (just enough to rattle plates and dishes) it might possibly hit the resonant frequency of a large building. In such a situation, if the building cannot get rid of the energy fast enough, it could collapse. One of the ways that they make buildings safer in earthquake prone areas is to check models for resonances near earthquake frequencies before they are built and to install devices that will allow for release of energy quickly. On such building is the Citicorp Building in New York City. After construction, it was found that the building was resonant with a certain cross wind that could occur during a large storm (the building used some new construction techniques that were untested for this situation). Fortunately, the problem was corrected before such a storm occurred.

Another such famous incident occurred at the Tacoma Narrows Bridge in the 1950s. It turns out that the wind over the bridge created small vortexes which caused the wind to very slightly "push" the bridge at certain frequencies (this actual process is too difficult to bother to explain here, suffice it to say that wind can cause pushes. The same situation occurred in the Citicorp Building). These pushes were at the resonant frequencies of the bridge, causing it to shake and ripple tremendously. For a while, the bridge was able to disperse the energy as quickly as it was being inputted and people actually drove across the bridge for kicks. However, one day the wind was slightly stronger than usual and the bridge began to twist and shake out of control. Quickly, it collapsed. Today, bridges must pass wind tunnel tests before they are constructed. Interestingly enough, the military was already aware of the fact that resonance can destroy bridges. When a troop of soldiers march across a bridge, the "break step" and walk randomly, instead of marching in their precise military fashion. They do this on the off chance that their march might be a resonant frequency for the bridge. In such a case, the bridge might collapse. In the past, this has actually occurred (the last time being in the 1800s to a squadron of British troops) and the militaries around the world have leaned from such mistakes.

One other example that is worthwhile mentioning is the Bohr model of the atom. You may recall from chemistry that electron can only orbit on certain energy levels. It turns out that if we view electrons as waves, they will only orbit on resonant levels. In other words, electrons only exist on levels where their waves create standing waves.

Many other examples of resonance occur in everyday life, such as the fact that not any vibration of the lips will produce a sound out of a trumpet, the fact that occasionally washing machines will jump around violently and the strange pendulum that was witnessed in a previous lab. I will leave the explanation of these and other event to the logic and creativity of the student's own mind.

There is one other interesting aspect to resonance that deserves mention. Consider that you had a very small wave that you were attempting to detect. One way you could do it is to set up a device that had the same resonant frequency as the wave. Therefore, if the wave was present, its small effect would be magnified and detected. Resonance can be used in such a manner to detect small disturbance and magnify them. In fact, it is a type of electrical resonance that is used in a radio to detect the very faint radio signals sent out from the radio station. When you tune the radio by changing the setting, you are actually changing the resonance of the radio to match the wave of the station you want to listen to.

To close this section, let me reiterate the main points regarding resonance.

- When a resonant frequency is struck, standing waves are produced in the object.
- At resonant frequencies, there is a "perfect transfer of energy" from the driver to the object.
- Objects being driven at resonant frequencies must either lose energy as fast as it is being obtained or go out of control.
- Resonance can be used as way to amplify or detect very minute waves or disturbances.

Assignment #30

1.) Suppose the following two waves interfere:

$$y1(x,t) = (6\ m)\sin(9x-3t)$$
$$y2(x,t) = (9\ m)\sin(6x+2t+6.28)$$

What is the value of the resulting wave produced at t = 3 sec and x = 9 m ?
(W11)

2.) Imagine adding two waves that are identical except for a phase difference. Is it possible to create a wave that is identical to the two waves that you started out with ? If so, how ? (W13)

3.) Two waves that are identical except for a phase difference are combined together to form a wave of the equation:

$$W_3 = (2\ cm)\sin\{(24\ m^{-1})x - (55\ s^{-1})t + 2.5\ rad\}$$

Write the original equations for the two waves.

4.) A 180 cm string is stretched between two fixed supports. (a.) What are the four longest possible wavelengths for the standing waves that could be set into motion in the string ? Sketch these waves. (b.) What is the ratio of the frequencies of the longest to the second longest ? the first to the third ? the first to the fourth ? the second to the third ?

5.) What are the four longest possible wavelengths that would produce standing waves in an open ended pipe? Sketch these and determine the ratios between the frequency pairs as described above.

6.) In a certain situation, the fundamental frequency that produces standing waves is found to be a wave described by the following equation:

$$y(x,t) = (6\ m)\sin\{[2\pi/9\ m)((x)-(343\ m/sec)t)]$$

where this wave equation is in the following form:

$$y(x,t) = A\sin\{(2\pi/\lambda)(x - vt)\}$$

Write the equation for another wave that will produce standing waves.
(W12)

7.) If waves in a string travel at 35 m/sec and the string is 6 m long, find two frequencies that will set up standing waves in the string. (W32)

8.) Given the following wave equation:

$$y(x,t) = (3\ m)\sin((4\ m^{-1})x - (2\ sec^{-1})t + (1.2\ rad))$$

write the equation for a wave that when it interferes with the wave above will:

(a.) produce standing waves
(b.) give constructive interference
(c.) give destructive interference
(d.) give a resulting wave with a new amplitude of 2.5 m
(W9)

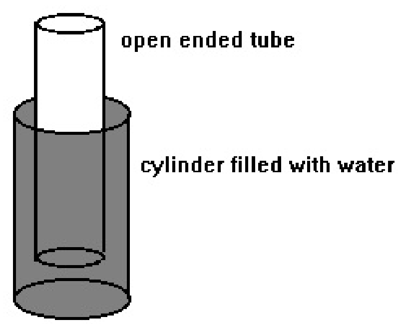
Lab #13 - Standing Waves

In this lab you will investigate standing wave production and resonance in a tube with one open and one closed end. You will use this information to determine the speed of sound in air (at the present pressure and temperature). The way you will do this is to use a tuning fork of a set frequency and an adjustable pipe. The pipe will be adjusted until a resonance occurs, telling you that standing waves are set up in the pipe. The pipe will be measured and then the length of the standing waves determined. By using the wavelength just found and the set frequency of the fork, the speed of sound will be determined.

Materials: Deep, open graduated cylinder (demonstration type) filled with water, long open ended pipe (that easily fits inside cylinder), two tuning forks, meter stick, good ears.

Procedure:

1.) Place the tube in the graduated cylinder filled with water as shown. Leave only a few centimeters of tube above waver level.



2.) Strike the tuning fork above the tube and slowly raise the tube until you strike a resonance in the pipe. By raising the tube, you are changing the length between the open end and the closed end (where the water is). You will know that you have standing waves in the tube because the sound will get suddenly loud.

3.) At the moment, you know standing waves are being produced, but you do not know their wavelength (why?). To remedy this, measure and record the length of the tube and then keep rasing the tube until standing waves are produced once again. Record this length and use this information to determine the wavelength of the waves.

4.) With this information, determine the speed of sound in air.

5.) Repeat the above procedure with another tuning fork and compare the two speeds in your conclusions.

Notes Regarding Conclusions: To determine the wavelength, why did we not just determine one length? How did measuring two lengths convince us of the proper wavelength (to answer, be sure you include drawings of the waves being produced)? How did the two speeds compare? How should they have compared? How do you think temperature, pressure and humidity affect the speed of sound? A motivated student might want to look up the equation for the speed of sound with temperature and humidity taken into account in order to find a percent error.

Special Note: As was mentioned earlier, there is a special correction that needs to be used to determine the standing waves that are formed in an open pipe. The student might research this and redo their calculations to find a more accurate result.