Chapter 29: Waves and Wave Characteristics

Waves

Throughout the previous chapters, we have discussed things and effects, or rather objects and forces. While the first part of the text dealt with very visualizable phenomena, such as rotation, motion, and pushes or pulls, the second part dealt with fields and their applications. This third part of the text begins to deal with a natural phenomena that is very important and prevalent in the physical universe. It is also a sort of combination between the visualizable and the conceptual and the moving and the static. It is a wave. We must begin by defining exactly what a wave is. A wave is a traveling disturbance of energy. Take a moment to think about what each word in this definition means, because each one is important. Traveling implies that the wave is moving, thus changing values over time and over space. Traveling also implies a propagation. Propagation means that the wave can sort of "move itself", where one disturbance creates another disturbance in the area next to it, which creates another disturbance in the area next to that, and so on, and so on. In this fashion a wave, which is not a "thing" and thus has no inertia, can continue in its motion (think about this a bit, it is an interesting phenomena). Disturbance means that as the wave passes, things in the area will change. Energy means that what is traveling is the energy, not the material or the medium. For example, when a wave passes under a duck that is swimming in a lake, the duck is not swept along with the wave, it simply rises up and down, indicating that the water is not moving, the disturbance of the water is moving (However, it should be noted that ocean waves are a very complicated form of waves and their complexity often makes them poor examples of true, simple wave motion. In reality, some water is carried along with an ocean wave for short periods of time.) Energy also implies that a wave is not a "thing" like an atom or a giraffe. A wave is an increase in the energy in an area of space. For example, when a light wave passes by, the energy in the space the wave passes through increases and then decreases as the wave passes. Here, once again, ocean waves are somewhat illustrative of the principle. When the wave passes under the duck, the duck (and the water) goes up and then down. Obviously, they both gain and then lose gravitational potential energy.

There are four different types of waves that exist:

mechanical, electromagnetic, matter, and gravitational. Mechanical waves are waves that pass through matter, such as a sound wave or an earthquake wave. These waves are disturbance of the material (generally the density) and thus can only travel through materials. We say that these waves need a medium to propagate. For this reason, mechanical waves cannot travel through space (if you scream on the moon, you will produce no sound!). Mechanical waves and their behavior can be explained by Newton's Laws.

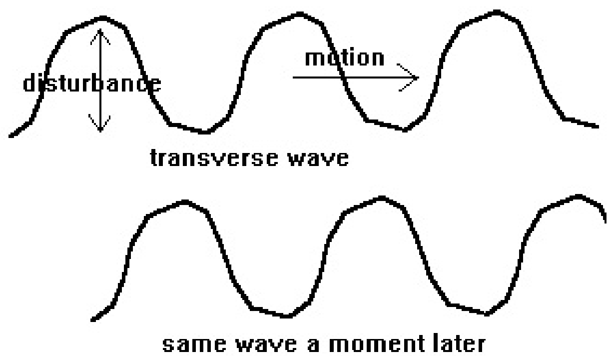
Electromagnetic waves are another story altogether. These waves are traveling disturbances of electric and magnetic fields that propagate through either a medium or "free space" (a physics term for a vacuum). Light waves come in many different varieties, such as visible light, radio waves, ultraviolet, and x-rays, just to name a few. When one of these waves pass by, the electric and magnetic energy in the area of the wave will change. We will devote an entire chapter to these waves in the future, but for now two individual comments should be made. First, unlike mechanical waves, these waves can travel through a vacuum, they do not need a material (good thing for us, since light must reach us through the vacuum of space from the sun). Secondly, Newton's Laws do not apply to electromagnetic waves, they are governed by Maxwell's Laws for electricity and magnetism.

The third type of wave, a matter wave, is even stranger still. Matter waves are representations of things (electrons, for example) as waves instead of particles. We are used to envisioning an electron as a little ball, such as represented to us in the Rutherford-Bohr model of the atom. However, modern physics tells us that electrons can be represented as waves in some instances. This thinking causes us to look at matter in a whole new light. Since this is realm of Quantum physics, we say that matter waves obey the laws of Quantum Mechanics. Since they are matter themselves, they do not need matter to propagate.

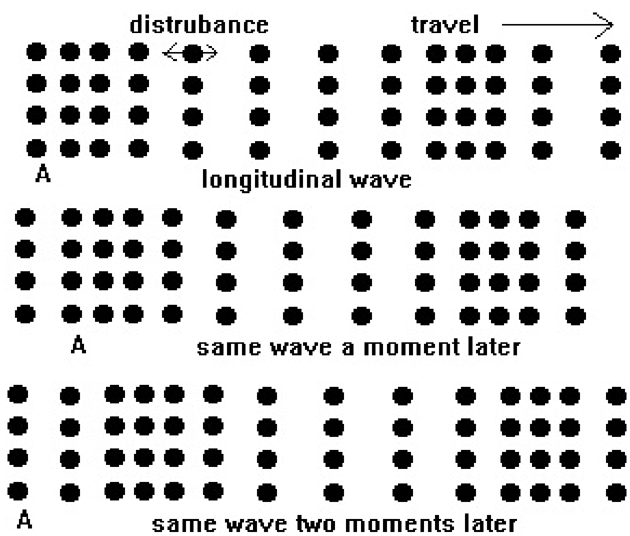
The fourth and final kind of wave is a gravitational wave. Gravitational waves are "ripples in the fabric of space and time" (sounds like it is right out of Star-Trek, eh?). And they are hard to explain. Einstein's general theory of relativity says that space and time are a sort of four dimensional surface which matter will distort, causing us to see the effects of "gravity". A gravitational wave, therefore, will be a ripple or disturbance in this fabric. As gravitational waves pass by, they actually shorten the distance between two points by warping the "space" in between them. These strange waves have never been seen directly, there existence has only been postulated. However, the theory states that these waves, if they do exist, will be very rare and very, very, very weak. It may be a long time before we can actually be sure they exist. Since they are a wrinkle in the universe itself, they obviously need no medium to pass through.

They are governed by Einstein's General Theory of Relativity.

Besides coming in four different types, waves also come in three different kinds. All waves are either transverse, longitudinal, or surface waves. Transverse waves are waves where the disturbance of the material is perpendicular to the direction of motion. Imagine taking a string, tieing one end to a wall and shaking the other end up and down. This would cause a transverse wave to travel through the string. The disturbance in the string is the shaking, which is up and down, and the wave itself is traveling sideways down the string.



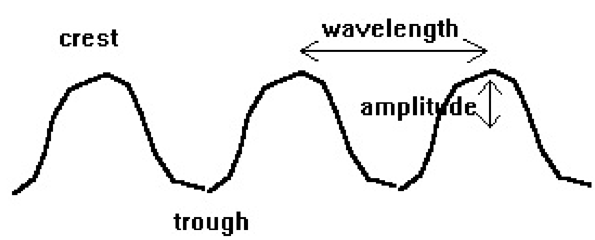
A longitudinal wave is a wave where the disturbance is parallel to the direction of travel for the wave (These are often also called "compressional waves"). This is a little harder to imagine. If you think about a long piece of metal, such as a solid bar, and you hit one end, a "shock wave" will travel through the bar, from on end to the other. This wave will travel because the original particle that was hit will "bump" into the next one, and so on down the line. This is an example of a longitudinal wave, since the disturbance of the particles is side to side and the wave moves to the side. This situation is harder to draw because of the motion of the particles, however, an attempt is made below.

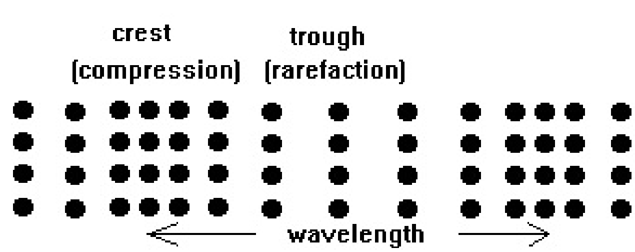


Here, you can see that the particles are moving side to side, along the same line that the wave moves. In each drawing, there is one particle labeled "A", and this is the same particle in each of the three diagrams. Particle A first moves right, then back to where it started from, then left, then back, etc. It is important to note that "A" is not going anywhere. It is simply vibrating back and forth, in concert with other particles, giving the illusion of motion. In fact, the energy is moving, not the particles. In some ways, it helps to think about this wave as if it were a transverse wave viewed from the top, when the particles are bunched together, it is a crest, when they are spread apart, it is a trough. However, don't take this allusion too far, since that is not really what these waves are. An example of a compressional wave is a sound wave.

The third kind of wave is a surface wave. These waves are a combination of compressional and transverse waves, resulting in the disturbance being in a sort of circular motion. Ocean waves and some waves that come from an earthquake are surface waves.

Whatever their fundamental differences, all waves have a number of things in common. These common characteristics are shown below.





Wave Characteristics

Crests and Troughs: The are the "high" and "low" areas of the waves. Notice how in the compressional wave, these are areas of high and low density.

Amplitude (A): This is the height of the disturbance above the equilibrium point (not from crest to trough). Notice how this is not labeled on the compressional wave. That is because compressional waves often have two amplitudes that can be discussed. We can talk about the distance each particle moves from its equilibrium or we can discuss the density change from equilibrium to crest where the particles are the densest. Another interesting thing about amplitude is the variety of units and measurements that can be used for amplitude. Normally, as in the case of ocean waves, amplitude can be measured in units of distance (meters), but in the case of other waves, it might be measured in different units. For example, in the case of a compressional (sound) wave, it might be measured in g/cm^3 (density), or in the case of light it might be measured in units of electric fields (N/C). Amplitude tells not how "high" the wave is, but how much the disturbance is from the equilibrium. It is abbreviated with the letter "A".

Wavelength (λ): The wavelength of a wave is the distance from one crest to the next (or any one point to the next corresponding point on the wave). Notice how on the compressional wave it is from one compression to the next. It is measured in units of distance (m) and abbreviated with the Greek letter lambda (λ).

Wave Number (K): Associated with wavelength is another characteristic, called the wave number of a wave. It is the number of wavelengths per unit length for a wave. So if a wave has a wavelength of 0.1 m, it would have 10 waves/meter or a wave number of 10 m^{-1}. This is abbreviated with the capital Greek letter kappa (K). You should be able to determine the formula for the wave number, common sense tells us that it is:

$$K = 1/\lambda$$

Angular Wave Number (κ): The angular wave number for a wave is the number of complete waves per unit length expressed in radians. You may remember that we did the same sort of thing back in the chapter on harmonic motion. Instead of saying one wave per meter, we say 2π radians per meter, etc. The abbreviation is a lower case kappa, and it is measured in radians/meter. The formula for the angular wave number is simply:

$$\kappa = 2\pi K = 2\pi/\lambda$$

Besides the number of characteristics that are shown on the diagram, there are quite a few that have to do with the time aspect of the waves. Since waves are traveling and changing, they are time dependent. The following topics are related to that aspect of the wave.

Period (T): The period of a wave has the same meaning as the period of a simple harmonic oscillator. It is the time for one complete cycle to pass. For example, if you were just sitting there, watching a wave pass by, it would be the time from when one wave crest passed you until the next came by. It is abbreviated with a capital "T" and obviously measured in seconds. There are two associated characteristics.

Frequency (ν): The frequency of a wave is the number of cycles per second that occur. In other words, if you were sitting there watching a wave go by, it would be the number of waves that went by in one second. It is abbreviated with the lower case Greek letter "nu" and is measured in 1/sec. As we have learned in the past, 1/sec or sec^{-1} is a special unit called Hertz. The formula for frequency is:

$$\nu = 1/T$$

Angular Frequency (ω): Once again, instead of waves, we can use radians to represent cycles. Thus the angular frequency of a wave is abbreviated with lower case "omega" and has units of radians/second. It is found by:

$$\omega = 2\pi\nu = 2\pi/T.$$

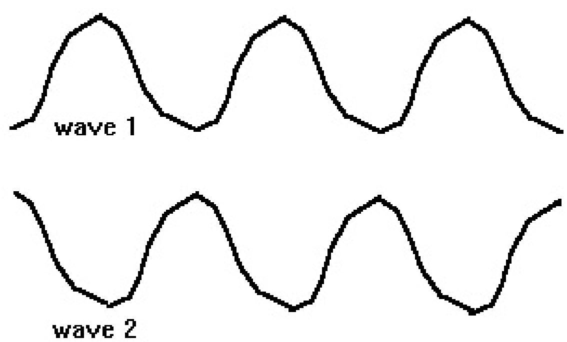
The two main characteristics above (wavelength and frequency) can be combined to form a third important measurement of waves. It is helpful for the student to determine the relationship themselves, since it will help them remember it. To derive the formula, consider a passing wave to be a passing train. If each boxcar is 15 m long (what would that correspond to in a wave?) and you see 3 boxcars pass you every second (which corresponds to...), how fast is the train traveling? Think about how you came up with that answer. Thus we see that we can also discuss:

Wave Speed or Velocity (v): The speed at which the energy of the wave propagates. Given by the formula:

$$v = \lambda\nu$$

Another helpful way to remember this formula is to look at the units involved. Speed must be in m/sec, which can be arrived at by multiplying m (from wavelength) times 1/sec (from frequency). I would <u>very</u> strongly encourage the student to remember this equation since it is probably the most important equation in the chapter. It is one of those little, simple equations that you must be able to recall at any given time and it is used in many of problems involving waves. It must become second nature to you. There is one last wave characteristic to discuss and that is a little harder to understand.

<u>Phase Angle(φ):</u> The phase angle of a wave is a sort of "fiddle factor" that we use to fit a mathematical equation to a wave. Consider the two waves as shown below:



If we take a good look at these two, we notice that they have the same wavelength, the same frequency (provided they have the same speed) and are essentially identical. The only difference between the two waves is that they start at different positions in their cycles. In physics, we must somehow be able to distinguish between these two waves. We do so by assigning each wave a different phase angle. For wave number 1, since it begins at zero amplitude, we assign it a phase angle of 0 radians (actually, the phase angle depends on the trigonometric function used to describe the wave, here we are assuming a sine function. This will be clearer later.). For wave number 2, since it begins half way through the wave cycle, we assign it a phase angle of π radians (half way through the full cycle of 2π).

<u>Factors that Affect Wave Characteristics</u>

Before we proceed to examine waves from a mathematical point of view, we need to take a few minutes to discuss what factors in the real world will affect the characteristics discussed above.

Amplitude: The amplitude of a wave is determined by the source creating the wave and in the most realistic cases of waves in more than one dimension, the distance from the source. Since the amplitude is a measure of the energy in the wave, it makes sense that the device creating the wave would set the amplitude. As a wave spreads out and gets larger (like the ripples from a stone dropped in a pond growing), the energy gets spread out over a greater area and thus the amplitude of the wave will decrease. Besides this, loss of energy to useless heat (remember the second law of Thermodynamics?) will decrease the energy of the wave over time. However, for our beginning treatment of waves, we will focus on one dimensional, ideal waves that do not lose energy as they travel. Thus our discussion will assume amplitude to be a constant. In a following chapter that deals specifically with sound, we will reincorporate the idea of waves in more than one dimension, since that is important in order to understand such concepts as decibel ratings, etc.

Wave Velocity: The velocity of a wave is determined by two factors: the type of wave and the material through which the wave is passing (the medium). Please note that these are the only two factors that affect the velocity. Any two of the same waves traveling through the same material must travel at the same speed. For example: all sounds travel the same speed through the same materials, as do all forms of light (however, light and sound do not travel at the same speed). If the material changes, the speed will change. As sounds pass through the air, sometimes the air will change (the temperature, the humidity, etc.) and this will change the speed of sound, but it will change it equally for all sounds. The world would be a very confusing place if sounds did not all travel at the same speed or all colors of light did not travel at the same speed (in fact, later we will learn that this statement is not entirely true).

Wavelength: We have learned that the wavelength of a wave is related to the frequency and the speed of the wave (according to $v = \lambda\nu$). Thus if the speed of the wave changes (due to a change in material), either the wavelength or the frequency must change. it turns out that the wavelength is the one that changes. For this reason, we say that the wavelength is dependent on the velocity and the frequency of the wave (and not the other way around).

Frequency: The frequency of a wave is determined by the source of the wave, and thus can only change if the source does. Every wave must be created by something that is vibrating or oscillating. Once it creates a wave, that wave will oscillate at that same frequency forever. If the source changes frequency, it will change the frequency of the wave produced after that time, but the wave that is already in motion will not change itself. In nature, there is only one way to actually change the frequency of the wave, and that will be discussed later (it is called the "Doppler Effect" and it involves changing the relative motion between source and receiver.).

Wave Equations

It is not time to quantitatively describe waves and wave motion so that we can solve problems and make numerical predictions involving waves. One possible wave equation is:

$$y(x,t) = A\sin(\kappa x-\omega t+\varphi)$$

As we said, this is one possible wave equation, there are many others. In fact, in advanced physics, this is not even called the wave equation. The wave equation is a term given to an equation that generates all possible wave equations as solutions. Before we begin to use this equation, it is important to understand some things about the wave equation.

First, we must emphasize exactly what this equation really is. This equation will tell you the height of the wave (y) at any given time and place. This is what we call an equation of two variables. In this case, the two variables are x and t. The student is probably used to working with equations of one variable, where you plug in one value and get out another, however, working in two variables is more complicated. If you only plug in one value, you do not get a number, you get an equation (which is often very useful). To get a numerical answer, you must plug in both variables.

Consider the equation above if you plugged in a value for x. What exactly would you have? You would have an equation for the height of a wave at some given location as a function of time. Now what would happen if you plugged in a value for t (this idea is very important). What would this (the wave equation) look like if you graphed it? How could you graph it?

One last thing that the student needs to be aware of regarding the wave equation we are using is that very often the goal of the problem will be to find the wave equation in some given situation. This is very akin to what was done in the chapters regarding fields, where finding the field equation was the important thing, it was assumed that if you found that, you could apply it in any manner necessary. Once again, it is assumed that if you found the wave equation in a given instance, you could use that equation to find out many other things. Finding the wave equation consists of filling in all the blanks except for the variables. Thus to write the wave equation, the student must find the amplitude, the angular wave number, the angular frequency, and the phase angle (A, κ, ω, φ). Of course x and t are left as variables.

One last warning: remember that these are all still one dimensional waves that are represented by these equations.

EX. AEIOU.) Waves are seen traveling through a medium with a speed of 250 m/sec and a wavelength of 8 m. If the disturbances are 0.25 m high, what is a.) κ? b.) ν? c.) ω? and d.) the equation for the waves (with $\varphi = 0$)?

Ex. HHJU: Imagine a wave with the following formula:

$$y(x,t) = (0.16\text{ m})\sin(23x-5t+4)$$

What is the height of the wave at $x = 12$ and $t = 2$ seconds? What is the height of the wave at 0 m and 0 sec? What is the equation of the amplitude, with respect to time at $x = 5$ m?

There are two other major points to discuss before we leave this section regarding wave equations. An astute student might have been wondering, "The one thing that we have not discussed is how to tell whether a wave is traveling to the right or to the left." In order to distinguish these two different waves, we would need to follow a crest (maximum height) over time and see where its position is going. Looking at our wave equation;

$$y(x,t) = A\sin(\kappa x-\omega t+\varphi)$$

as t gets larger, x must get larger to keep the argument (the stuff in the parenthesis) the same and keep a maximum. Thus the wave is going to the right. Thus the equation above is for a wave traveling to the right. However, if the ωt part of the argument were positive, increasing t would mean that x must decrease to keep the argument constant. Therefore, an equation for a wave traveling to the left would be;

$$y(x,t) = A\sin(\kappa x+\omega t+\varphi)$$

In short, the direction of motion of the wave is opposite to the sign of the ωt term in the wave equation.

The second thing to point out is that calculating the φ for a wave is often difficult. In order to write a wave equation with a φ, you must be given one extra piece of information, and that is the amplitude of the wave at some given time and place (x and t). Then you use the wave equation backwards (insert all values and solve for φ) to find the phase. You must then rewrite the wave equation, including the phase, but removing the x and t so that it is general. The example below illustrates this.

EX VFVFVV: Write the equation for a wave that has a wavelength of 5 m, travels at a speed of 2000 m/s, has an amplitude of 0.0002 m, is traveling to the left and is 0.000014 m high at x = 2 m and t = 1.5 seconds.

We should also point out that the same wave can be written in many different formats, as shown below.

EX HYTG: Rewrite the following wave using a cosine instead of a sine function.

$$y(x,t) = (5.3\ \text{m})\sin(1.7x-9t+3)$$

The problem above can be done one of two ways, either intelligently (in one quick step, taking no more than 2 seconds) or in a "clunker" fashion, taking lots of time and energy for a simple answer.

Assignment #31

1.) Visible light has a frequency of 6.5×10^{14} Hz and travels at a speed of 3×10^{8} m/sec. Write the equation of a light wave that has the above characteristics and an amplitude of 6 N/C. Write the equation in the form $y(x,t)=A\sin(kx-wt)$.
(W10)

2.) Given the following equation of a wave:

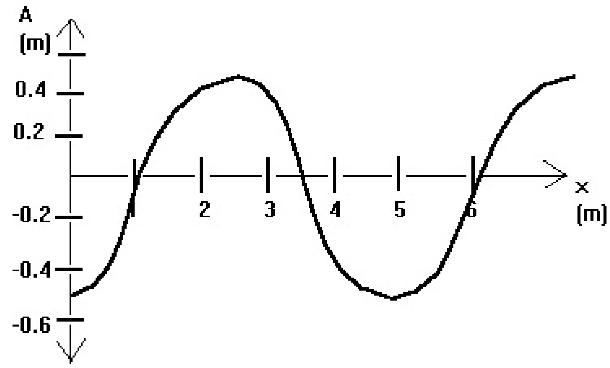
$$y(x,t) = (3.4 \text{ m})\sin((4 \text{ m}^{-1})x - (0.3 \text{ sec}^{-1})t + (2 \text{ rad}))$$

determine:
(a.) the amplitude
(b.) the phase angle
(c.) the angular wave number
(d.) the angular frequency
(e.) the wavelength
(f.) the period
(g.) the speed of the wave
(W7)

3.) While enjoying a relaxing evening around a campfire in Big Bend National Park, a student decides to work on a little Physics homework. Soon the tension builds and he lets out a primal scream of anguish (at 1500 Hz). He hears the echo return after bouncing off a cliff (distance = 670 m) 4 seconds later. Just to pass the time he decides to calculate the speed of the sound and the wavelength of his primal scream. What are they ? (W15)

4.) One day, without warning, Kristy engages in primal scream therapy during Physics class. She screams at 3000 Hz with an amplitude of 0.02 Pa and a phase angle of 12 rad. If the scream travels at 350 m/sec what is the equation of the wave produced ? What is its value at x = 2 m and t = 3 sec ? (W31)

5.) From the following diagram, write down the equation of the wave described. (consider the speed of the wave in the medium to be 3000 m/sec and consider the graph to be made at t = 1 sec) (W5*)



7.) As the tides rise and fall in a harbor, a ship rises and falls with it, going up once and returning to the same level (equilibrium) in 12 hours and 10 minutes. How long does it take the water to rise from its lowest level to one half of its maximum level above equilibrium? (Clarification: how long does it take to go from -Max Amp to +1/2(Max Amp)) (S10)

Lab #12 - Waves and Wave Characteristics

In this lab, you will create waves in a container and attempt to measure the relevant quantities (wavelength, amplitude, frequency, wave speed). You will also vary how the waves are created and determine how this affects (or does not affect) each of the quantities.

Note: This lab requires five students per lab group.

Materials: a large, plastic tub (the larger the better, a very large plastic ice chest would work), a permanent marker, two stopwatches, meter stick, water, marbles.

Procedure:

1.) Use a black, permanent marker to mark the side of a large tub in centimeters (vertically, to measure water level) and tape a meter stick a across the top of the tub. To improve the performance of the lab, foam rubber can be taped around the tub on three sides at the water level to reduce reflected waves that might interfere with the experiment.

2.) Fill the tub with water, leaving about six centimeters of markings between the water level and the top.

3.) This activity requires five people working at the same time. One will drop marbles at a steady pace into the water, creating waves. As those waves move across the tub, one of the people will measure their wavelength by looking down on the waves and reading the meter stick, another will measure the frequency by counting the number of waves that strike the side of the tub in a given number of seconds, another will measure the amplitude by observing how height the waves splash against the water level markings, the fourth will measure the speed by timing how long it takes a crest to travel from the source to the side.

4.) Write the equation for the waves produced, determining all of the relevant quantities.

5.) Redo the experiment, dropping the marbles from the same height, but at a slower pace.

6.) Redo the experiment, dropping the marbles at a faster pace.

7.) Redo the experiment, dropping the marbles at roughly the same pace, but from a higher height.

8.) Redo the experiment, dropping the marbles from the higher height, but at a slower pace.

Conclusions: From your observations, draw as many conclusions as possible, showing how the activity supported what we have learned about waves, wave characteristics and factors that affect them. Be very specific in showing how exactly how the equations either did or did not support what was explained in this chapter.