Chapter 27: Circuit Analysis

Kirkoff's Loop Rule

Now that we have a basic conceptual understanding of electrical circuits, it is time to begin analyzing them quantitatively. Our goal here is to be able to use our knowledge, along with some laws to be introduced shortly, to calculate the voltage across devices and the currents in different branches of more complicated circuits. There is one important note to make before we begin. It should be stated that we will only consider equilibrium situations in electronics. All of our laws and theories depend on the circuit having reached equilibrium. When an electrical circuit is first connected (switched on) the current will surge through all the branches in a manner that is difficult to predict (consider turning on an empty hose). Very quickly (in a matter of less than milliseconds), the surges will disappear and the circuit will settle into equilibrium. Although that first period is very interesting (recall that most light bulbs burn out when the switch is flicked on, they rarely burn out while they are on), it is too difficult to handle mathematically. Our interest is only in that period following this, when equilibrium has been reached.

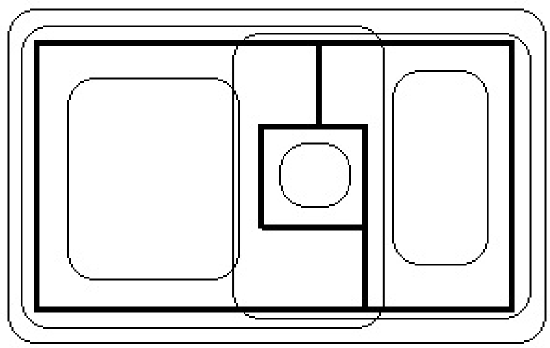
It is important to view circuit problems in one of two ways, depending on the complexity of the circuit. The first method, using your basic knowledge along with V=iR, can quickly get you the answer in simple situations. However, when the circuit is more complex, it is helpful to approach the problem from a purely mathematical standpoint. In other words, locate your unknowns and then attempt to write down the correct number of equations (for example, if you are asked to find two different currents in a circuit, you would need two different equations that involve those currents). While the first method is actually preferred, since it demonstrates that you understand what is going on, our presentation will basically neglect it, even for simple problems. This will make some problems more complicated than necessary, but it will help us build up from the simple to the complex in an organized manner.

As mentioned, it is very helpful to view these problems as a situation with a number of unknowns for which you must construct a number of equations. These equations are constructed using two rules: the Loop Rule and the Junction Rule (both are usually preceded by the name of their "creator", Kirkoff).

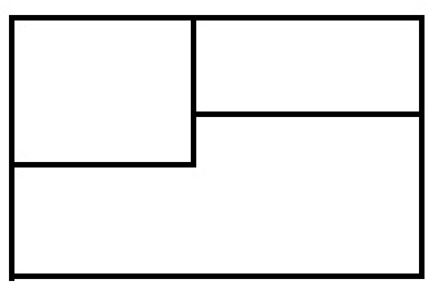
Kirkoff's Loop rule is actually a manifestation of the conservation of energy applied to electrical circuits. It states that in any closed loop in a circuit, the sum of all the voltages must be zero. Applying this rule to any loop in the circuit will yield an equation. There are a few notes that are worth mentioning before we go on to the examples:

Notes Regarding Kirkoff's Loop Rule

1.) This loop rule will apply to any loop, regardless of how meandering, convoluted or simple the loop is. Picking the proper loop is the key to using this law. In the diagram below, a complicated circuit is shown in thick lines (the batteries, resistors and such have been left off for simplicity) and different loops are shown in thin lines to illustrate the many choices you have.



2.) Before
you begin an electrical problem, you should always go through the circuit diagram and label all the different currents that are present, assigning each one a direction (don't worry if the direction is wrong, the mathematics will tell you so by giving you a negative answer). Remember what we have learned about currents, that they change at a branch and are unaffected by obstacles. For practice, label all the different currents in the circuit below:



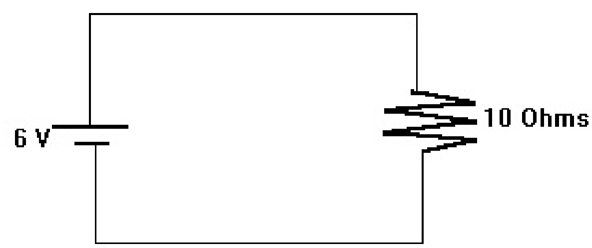
3.) If you pass through a battery from negative to positive, the voltage difference is a positive (a rise in voltage). Passing through a battery from positive to negative is a drop in voltage (a negative).

4.) Passing through a resistor (or other device) <u>in the direction of the current</u> is a negative voltage (a voltage drop). Passing through against the current is a voltage rise (a positive). This is the solution to the problem hinted at in an earlier section.

5.) Do not forget that V=iR still applies and that you must go around the loop in a particular direction.

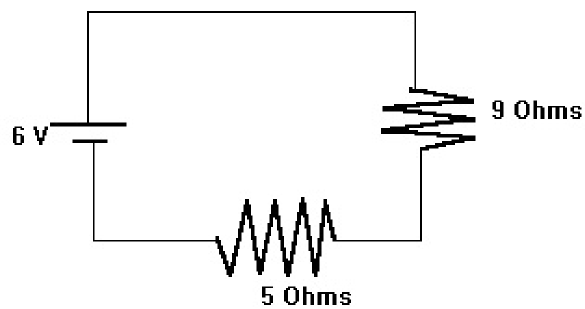
Let us begin to do some of the examples, from simplest to more complex.

EX. R.) Find the current passing through the resistor below.

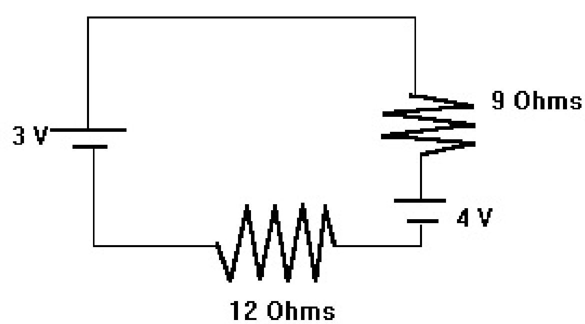


This example was very simple and could have been done using your basic knowledge.

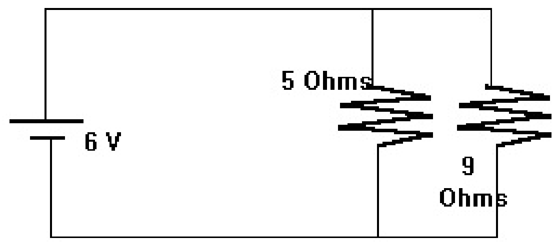
EX. J.) Find the current passing through the second resistor below and the voltage across it.



EX. V.) Find the current in the circuit below.



EX C.) Find the currents through each of the resistors below.



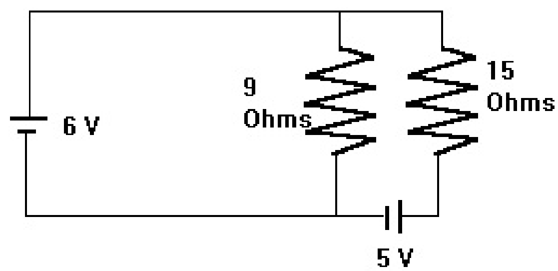
Kirkoff's Junction Rule

If we look back at the last example, we might ask: "What is the current coming out of the battery?" Looking at the diagram, there is no way to answer that question using the loop rule. There is, however, another rule that can be used, the Junction rule. Simply stated, the Junction Rule says that the algebraic sum of the currents into a junction of wires must equal the sum of the currents leaving the junction (this rule is actually an application of the conservation of mass or charge). This very simple rule allows us another means for writing an equation for the circuit.

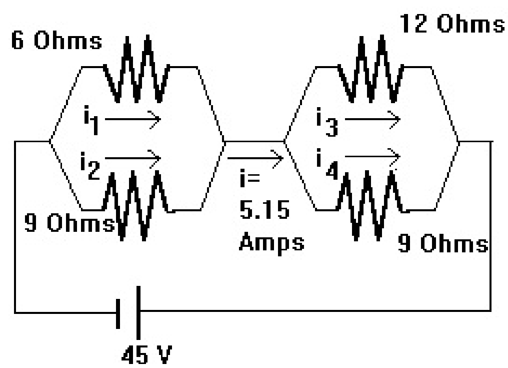
Ex. O.) Go back to the previous example and calculate the current coming out of the battery using the junction rule.

With this second law understood, it is time to move onto more complicated examples.

Ex F.) Find all three currents in the circuit below.

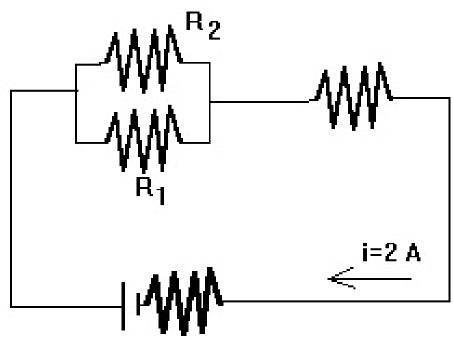


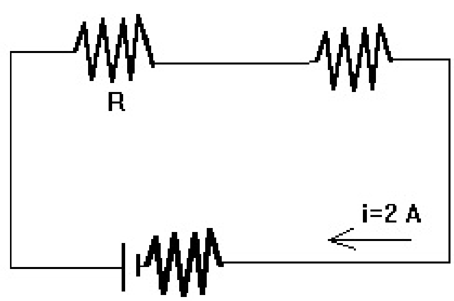
Ex. I.) Find the currents labeled i_1, i_2, i_3, and i_4.



Equivalent Resistance

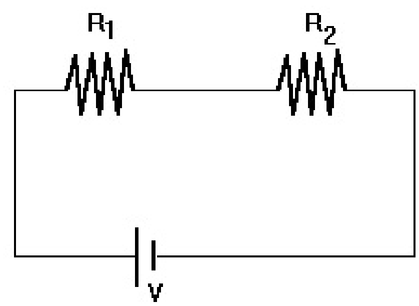
There is one more tool available to us when analyzing circuits, the concept of equivalent resistance. The equivalent resistance of two or more resistors is the value of the one single resistor that could be used to replace the others and leave the external currents the same. In other words, imagine taking the combination of resistors out of a circuit and replacing them with one resistor. The value of that single resistor would be the equivalent resistance of the combination. In the circuit below, notice how R_1 and R_2 have been replaced by R, leaving the rest of the circuit unchanged.





Being able to calculate the equivalent resistance of a set of resistors enables us to simplify many circuit problems. We calculate this using two principles; resistors in series and resistors in parallel.

Resistors in series are the simplest case. Consider the two resistors in the circuit below:



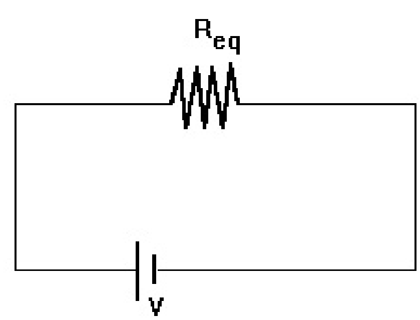
Using the loop rule on this circuit gives:

$V-iR_1-iR_2=0$

or

$V=i(R_1+R_2)$

What we want is to find R_{eq} so that the circuit looks like this:



Here, the loop rule gives:

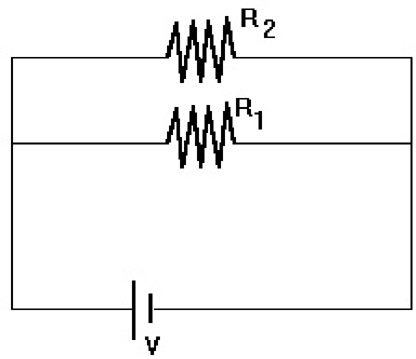
$V=iR_{eq}$.

Comparing, we see that for resistors in series:

$R_{eq}=R_1+R_2$.

Resistors in series simply add together to produce an equivalent resistance. Does this result make sense? Consider what resistance means. If two object impede the flow of electrons, then it makes sense that for the electrons to pass through both of them, they must overcome their combined resistance.

EX. M.) Find an equation for the equivalent resistance of resistors in parallel.



Thus we get the result for resistors in parallel:

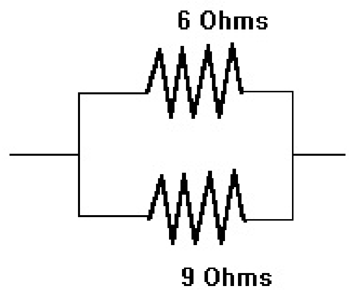
$1/R_{eq}=1/R_1+1/R_2$

or

$R_{eq}=R_1R_2/(R_1+R_2)$

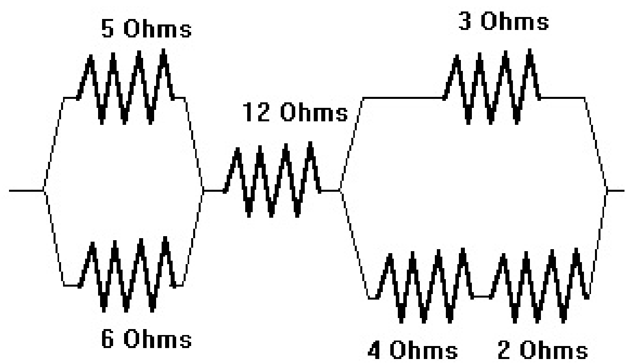
Although the first equation is easily expandable to include three or more resistors in parallel, the second equation is only valid for two resistors and cannot be immediately expanded.

Ex. Q.) Find the equivalent resistance of the following two resistors:



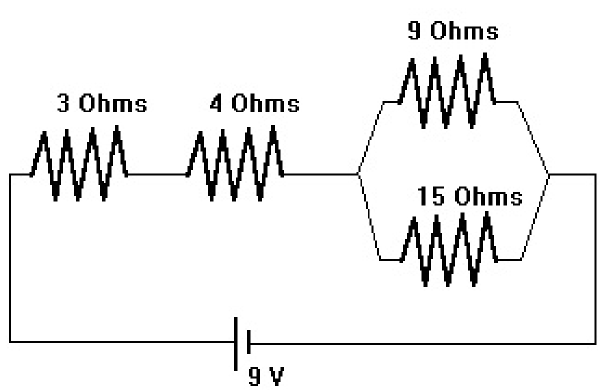
Notice how the equivalent resistance of resistors in parallel is less than the resistance of either resistor. This is always true and it makes sense. If there are two paths, regardless of the difficulty of either one, the net result is that electrons will flow more easily through the combination than through either one alone (think of water pipes and water flow). This same principle is applicable in the case of dual exhausts in cars, two pipes, regardless of the size of the second one, will always allow the exhaust to escape the engine more easily than just one. This also explains why you should not overload wall sockets. Since they are wired in parallel, adding another appliance will <u>reduce</u> the overall resistance on the circuit and increase the current. This could lead to overheating in the wires and possibly fires.

Ex. H.) Find the equivalent resistance of the following set up.



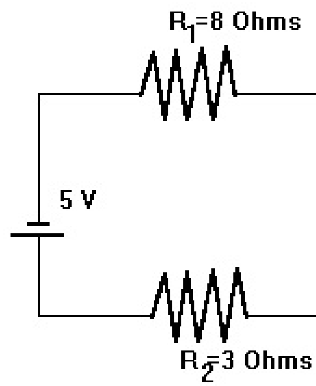
As was mentioned, this concept can be used to help analyze electrical circuits. When looking for a current, especially a main current, it is often easier to solve the problem by simplifying the circuit using equivalent resistance. Consider the next example:

Ex. X.) Find all the currents and voltage drops in the following circuit.

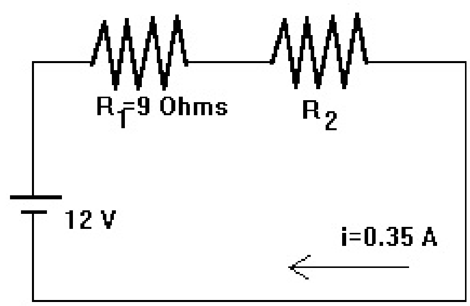


Assignment #27

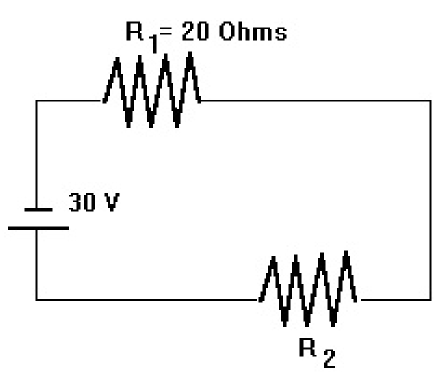
1.) Determine the voltage drop across each resistor in the circuit below.



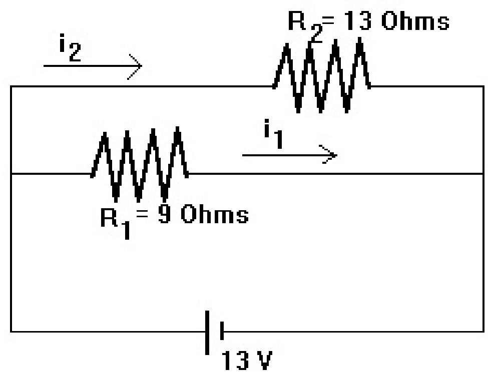
2.) Determine the value of R_2.



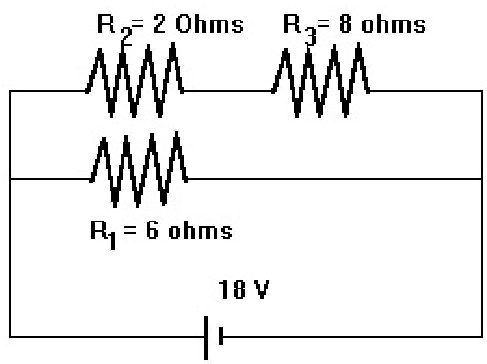
3.) Determine the value of R_2 if the voltage across R_1=18 V.



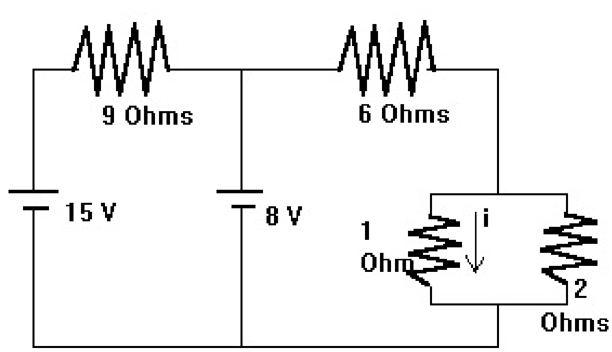
4.) Determine i_1 and i_2.



5.) Find the current through and the potential across each resistor.

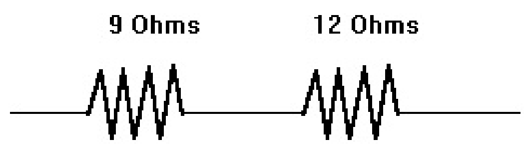


6.) Find i.

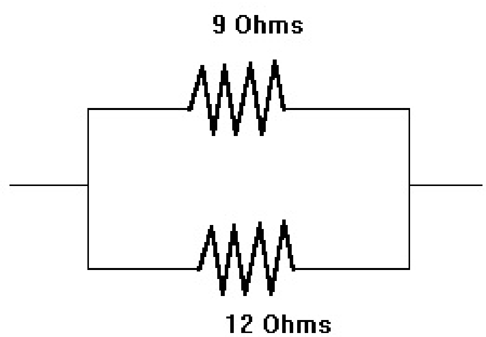


7.) Find the equivalent resistance of the following setups.

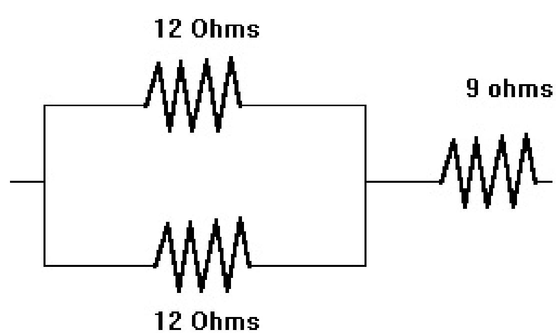
a.)



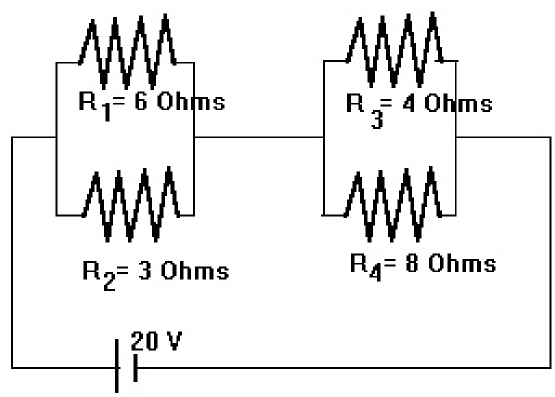
b.)



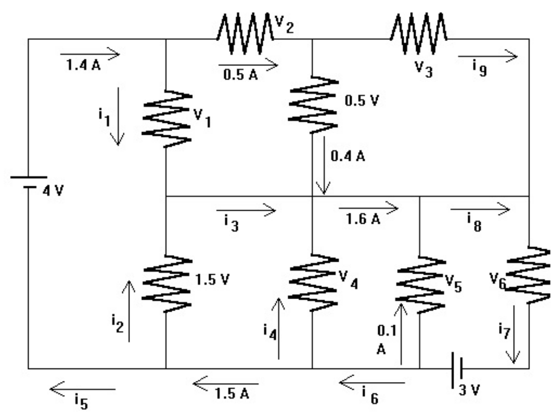
c.)



8.) Find the current through each resistor below.



9.) Using Kirkoff's two laws, find the currents labeled 1 through 9 and the voltages labeled 1 through 6 (the voltages are the drops across each resistor).



10.) Consider the equation for equivalent resistance of the form $R_{eq} = (R_1R_2)/(R_1+R_2)$ for resistors in parallel. If we try to expand this to the equivalent resistance for three resistors we would think it would give: $R_{eq} = (R_1R_2R_3)/(R_1+R_2+R_3)$. However, this equation is not correct. Show why. Also, determine the correct equation by manipulating the equation: $1/R_{eq} = 1/R_1 + 1/R_2 + 1/R_3$, which is appropriate.

11.) Decipher: "Abstention from any aleatory undertakings precludes a potent escalation of a lucrative nature." (DNCTHWG)

Lab# 10 - Kirkoff's Junction Law

In this lab you will attempt to experimentally verify Kirkoff's Junction Rule, otherwise known as Kirkoff's Current Law. The law states that the algebraic sum of the currents entering a junction is the same as the algebraic sum of the currents leaving that junction. Simply stated, what comes in must go out. Two more complicated names for this rule are "the law of continuity" and "the conservation of mass or charge". In this lab you will not measure the currents directly, but instead you will measure the voltage drop across a known resistor and use the Pseudo-Ohms Law to determine the current.

Notes:

1.) For the best results, do not use identical resistors but try to find resistors that are close in their ratings (i.e. do not use a 5 Ohm and a 50 Ohm).

2.) Be sure to record the value of each resistor. A complete lab write up should have schematics with the resistors and voltage drops labeled clearly.

3.) Put a switch in all circuits and only turn it on when you are about to take readings. If left on too long the resistors can get very hot.

Procedure:

1.) Construct circuit 1 (next page). As in the last lab, the value of the resistors is not important, as long as they are not too far apart.

2.) The junction you wish to investigate is labeled "A". The resistor prior to A is the current that comes in and the resistor after A is the current that comes out.

3.) Record the resistance of the first resistor.

4.) Measure the voltage drop across the first resistor using a voltmeter and record.

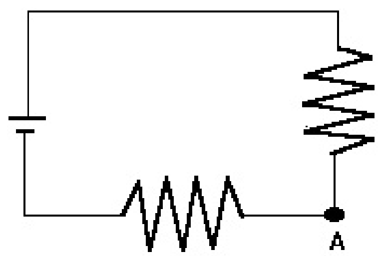
5.) Repeat steps 3 and 4 for any other resistors prior to A.

6.) Repeat steps 3 and 4 for all the resistors after A.

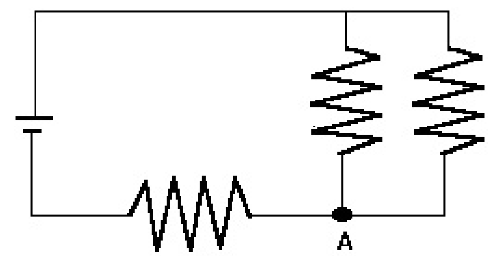
7.) With the data you have gathered, use the Pseudo-Ohms Law to calculate the current through each resistor.

8.) Compare the sum of the currents entering A to the sum of the currents leaving A.

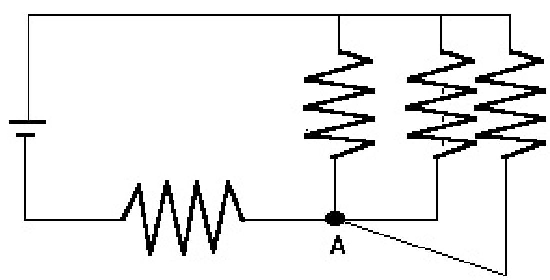
9.) Repeat the procedure for circuits 2, 3, and 4.



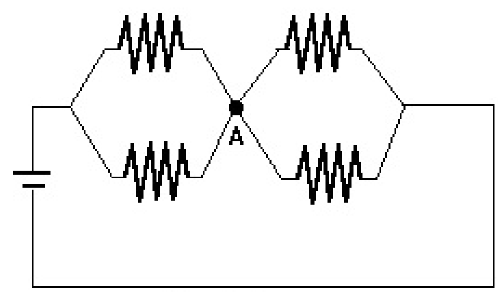
Circuit 3



Circuit 2



Circuit 3



Circuit 4

<u>Conclusions:</u> As usual, comment on any discrepancies from the expected values. Percent error ? Have you seen any evidence for Kirkoff's other law in this lab ?