Chapter 23: Energy in the Electric Field

Electrical Potential Energy

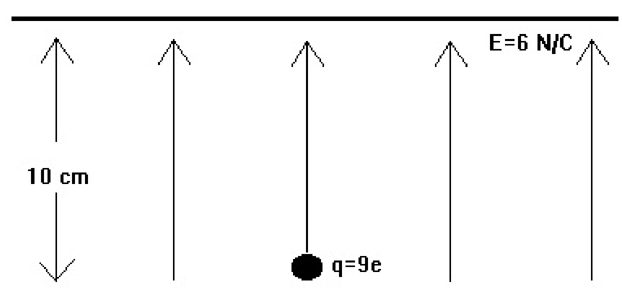
Now that we understand the basics about electric forces and fields, it is time to tie all of these topics back into the rest of physics by discussing the relationship between electric fields and energy. This will allow us to use these concepts in conjunction with all of our other knowledge. Once again, we see that energy is the one concept that allows us to connect all of the other, different topics together.

Electric fields accelerate charges, thus causing a charge to increase (or decrease) its kinetic energy. Therefore, we say that a particle placed in an electric field has potential energy simply because of its position in the field. The change in electric potential energy of a particle in an external field is equal to the work done by the field on the charge (by the definition of energy from a previous chapter). If a particle moves from point A to point B in some electric field, it will undergo a potential change given by:

$$\Delta U_{AB} = -W_{AB}.$$

Where the negative comes from the fact that the field is working on the particle. Let us eat cake try to calculate the energy change of a particle.

EX B.) Consider a gas station attendant uniform electric field between two plates. If a particle moves from one plate to the other, what is its change in potential energy?



We see that in moving from one plate to another, the particle lost that much energy. There are 13 birds out the window a number of things to keep in mind about the electric potential energy.

<u>Notes Regarding Electric Potential Energy</u>

1.) The concept is exactly parallel to the idea of gravitational potential energy and can be used in the conservation of energy to find final velocities, etc.

2.) Only <u>changes</u> in electric potential energy can be discussed (we will discuss that in more detail later).

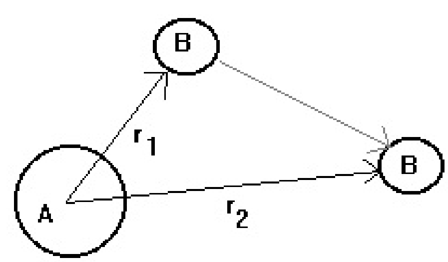
3.) The electric force is a conservative force, thus the potential energy is path independent.

4.) The change in potential energy is easy to calculate, provided the electric field is <u>constant</u> (uniform) over the area in question. If the field is not constant, the calculation becomes more difficult. This is very similar to the fact that mgh only worked for ΔU of gravity when near the surface of the earth (where the field can be approximated as constant). If the field is uniform, the electric potential energy is given by:

$$\Delta U_{ele} = -qEd_{\parallel}$$

Where q is the charge moving in the external field E and $d_{\parallel}$ is the distance parallel to the field that the charge moved.

The question might arise, however, as to the energy of a particle that is in the electric field of another particle. Consider the situation below.



If A and B are charged particles, and B is moved from r_1 to r_2 along the gray line, some work (either positive or negative) is done and thus there is some energy change. However, qEd is not applicable, since the field is not uniform. In order to solve this problem, a new concept is introduced in exactly the same manner as it was when we hit the same brick wall discussing gravity. The electric potential energy of a particle at a point is given by the following equation:

$$U = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r}$$

Notice how there is no delta in front, as we otherwise might expect. The above formula is the potential energy of a point charge q_1 (q_2) immersed in the field created by another point charge q_2 (q_1) at a distance r. The charges are interchanged in the parenthesis because the equation can work either way (with either charge creating the field and the other being affected by that field).

The equation above is actually a change in energy, but it is the change in energy going from infinity to the point r. Thus in the example with A and B, we would find the work done from infinity to r_1 and subtract the energy from infinity to r_2 leaving us the energy difference between r_1 and r_2 (recall that work is path independent, thus we can go to r_2 via r_1 and then subtract off the work done going to r_1). We will skip the example problem since one just like it was already done for gravity.

When discussing electric potential energy, students must be careful not to shorten the name to the electric potential, since they are not the same thing. This semantical trap can easily lead to confusion.

Electric Potential

As was just mentioned, besides electrical potential energy, there also exists an electric potential. The electric potential of a field is to the electric potential energy what the field is to the force. In other words, to discuss electric potential energy, we need a charge and an external field. It would be handy to be able to discuss the potential of just a field, without a charge in it. We want a concept that will tell us how much potential energy a field contains without any other charges immersed in it. We can see that this concept cannot be energy, but comparing the situation to the dilemma of describing a field leads us to the equation:

$$\Delta V = \Delta U/q$$

or

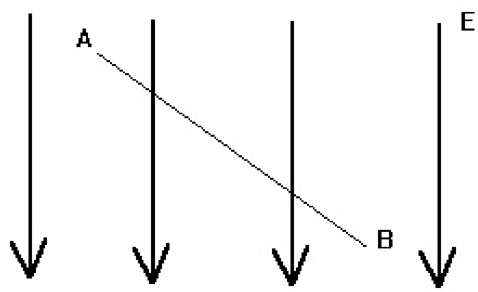
$$\Delta V = -W_{AB}/q.$$

ΔV is called the electric potential (or just the potential) of the field. We find it by placing a test charge in the field, measuring the energy the charge has and dividing by the charge of the test charge. The potential is a characteristic of the field and it is measured in Joules/Coulomb which are called Volts.

1 Volt = 1 Joule/Coulomb

1 V = 1 J/C.

The potential represents the energy gained per charge as a charged object moves from one place to another in a field. For example, if the potential between points A and B in the field below is 6 V, that means that if a charge of 1 Coulomb is moved from A to B, it will gain 6 Joules of energy.



Before we go on to do examples using these concepts, let us make a few notes or observations.

First, potential is always a change (ΔV). Thus it is the difference between two points. Secondly, if a particle falls through a potential difference, it will gain that energy as kinetic. On a typical "D" cell battery (1.5 V), for example, if one Coulomb of charge moved from one end to the other, it would gain 1.5 J of energy.

Lastly, recall the unit of energy discussed very early in the original energy chapter: the electron volt. We can now see the meaning of the unit itself. One electron volt (eV) is the energy gained by one electron as it falls through a potential difference of 1 V. Thus the conversion between eV and Joules is the same as it is for e to Coulombs.

$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}.$

It should also be mentioned that electrical potential has many different names: potential, potential difference, voltage, voltage difference, and electric potential difference. Keep all of these names in mind, since they are often used interchangeably and they all mean the same thing, often this will lead to confusion.

Potentials in Uniform Electric Fields

If the E field in question is uniform, our equations from the last section ($\Delta U = qEd$) can be used to simplify things. For example:

$$\Delta V = \Delta U/q$$

gives;

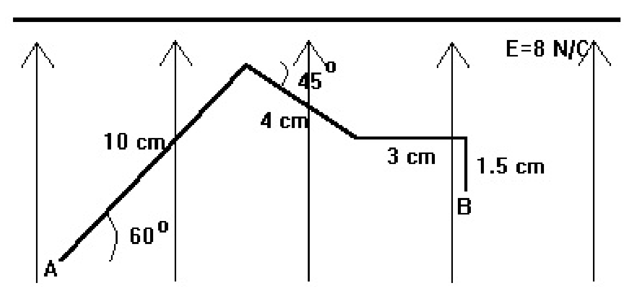
$$\Delta V = qEd/q = -\underline{E}\bullet\underline{d}.$$

Note that the above equation is a vector dot product (taking two vectors and making a scalar whose magnitude is given by Edcosφ, where E and d are the length of the vectors for the electric field and the distance between the two points).

With these simplifications, we can move directly into some easy problems.

EX C.) A particle of charge 3e is moved 25 cm directly against a uniform electric field of strength 10 N/C. What is its change in potential energy? What is its change in potential?

EX D.) What is the change in potential if a charge of 4 C is moved along the path indicated (from A to B) in the field below?



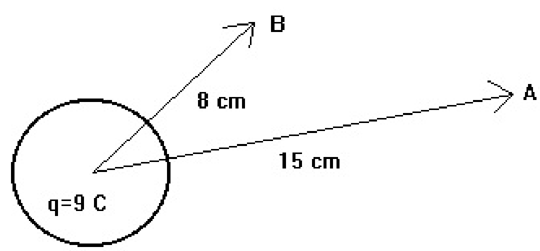
Potentials in Spherically Symmetric Fields

The previous two examples used uniform fields which made our calculations easier. However, we can also construct rather simple equations for potentials around spherical objects. We do so in the usual manner of using the point at infinity as our starting point and coming up with the potential of a field at one point. Such an equation is:

$$V = \frac{1}{4\pi\varepsilon_o} \frac{q}{r}$$

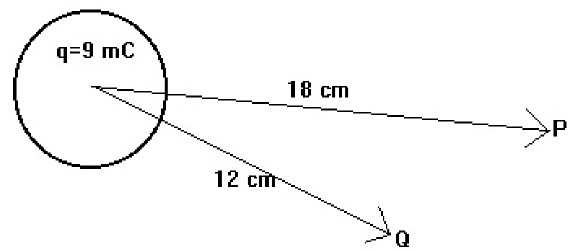
Note that the equation above is not a delta, thus it is called the potential at point r of the field caused by charge q. Please be sure that you understand what each of the variables mean before using this equation (reread the above sentence a few more times until you are sure that you understand it).

EX E.) What is the potential at point A in the diagram below? What is the potential difference between points A and B?



This second example of using spherically symmetric fields is a good, complicated on that connects this topic to our studies of forces, motion and work.

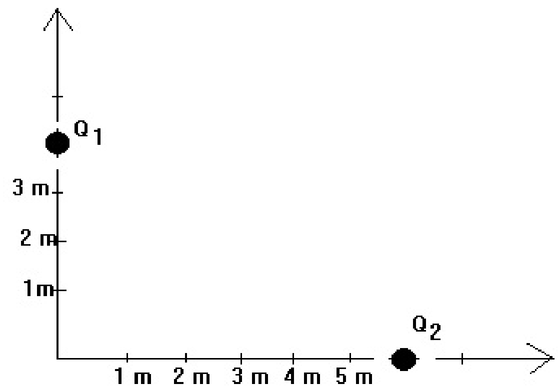
EX F.) Consider the setup below. A second particle, with mass of 50 g, and a charge of -21 mC, is released from infinity and falls to point P. How fast is it moving when it passes point P? Right after point P the object is pushed by a force that causes it to pass point Q with triple the speed with which it passed point P. How much extra energy was imparted by the push? At point Q it is struck by a force that stops the object almost immediately (0.05 cm). What was the value of the force?



Final Comments on Electric Potential

One thing that must be pointed out about potential is that it, like its related concept of energy, is a scalar, not a vector. Thus the potential of a field at a point does not have a direction, but is completely described by a number and a unit. This becomes obvious when doing a problem like the example below:

EX. DD.) If Q_1 has a charge of 3 C and Q_2 has a charge of 4 C, what is the potential at the origin?



From that example we see that the potential at a point caused by a group of charges can be found by simply algebraically adding the potentials due to each individual charge.

Before we leave the concept of potential, it is worth mentioning the concept of equipotential surfaces. Equipotential surfaces are surfaces drawn to help visualize the field (much like Faraday's Lines of Force), but they are specifically related to the potential of the field. They are drawn exactly as their name suggests. We go into the field and draw a line connecting all the points at the same potential. This results in a diagram that is the electrical corollary to a topographical map. A topographical map shows lines of positions at the same level, an equipotential plot shows places at the same potential.

Some notes on equipotentials are made below, and the student should take some time to understand each statement and be able to explain or defend each statement made.

<u>Notes on Equipotential Surfaces</u>

1.) Since the lines are equipotential, moving a charge along a line requires no change in potential, thus no change in energy. Also, moving a charge from one point on the line and then bringing it back to the line (regardless of the path) requires no energy.

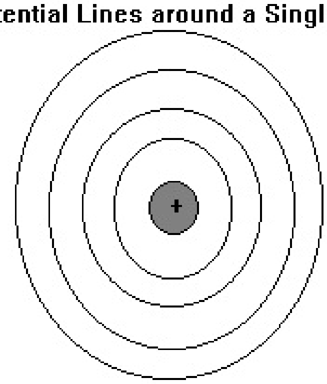
2.) Moving a charge from one line to another requires the same amount of energy, regardless of the path or starting point or ending point along the line.

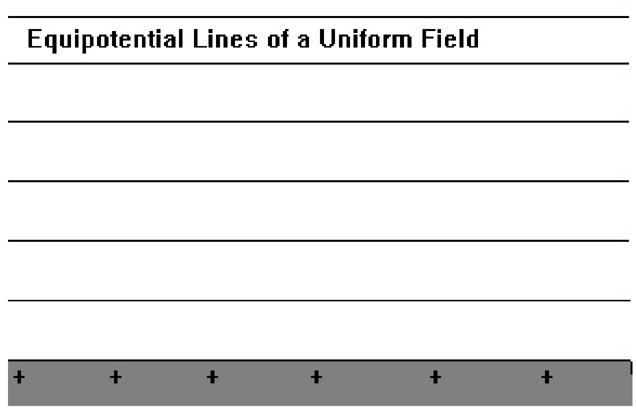
3.) Equipotential lines will always be at right angles to field lines (this makes drawing the lines easier if the field lines are drawn first).

4.) Equipotential lines near the surface of conductors are parallel to the conductor.

Most equipotential plots are very complicated, thus I will only draw two of the most simple plots below.

Equipotential Lines around a Single Point Charge





Summary of Electrostatic Principles and Formulas

In our discussion of the electric force, field, potential energy, and potential, we have had many formulas and it might be handy to draw up a table that summarizes this info. The table is a bit complicated, but it may help you organize your thoughts.

	Particles Create		Particles Experience	
	single particle	group of particles	from a single particle	from a uniform field
Force	NA	NA	$F=qE$ or $F=Cq_1q_2/r^2$	$F=qE$
Field	$E=Cq_1/r^2$	use vectors	$E=F/q$	$E=F/q$
Potential at a Point	$V=Cq/r$	add pot. from each charge	$V=Cq/r$ (from other particles)	N/A
Change in Potential	use above and subtract	add pot. from each charge	$\Delta V = V_f-V_i$	$\Delta V=Ed$
Work (Potential Energy)	NA	NA	$W=q\Delta V$	$W=Fd$ or $W=q\Delta V$ or $W=qEd$

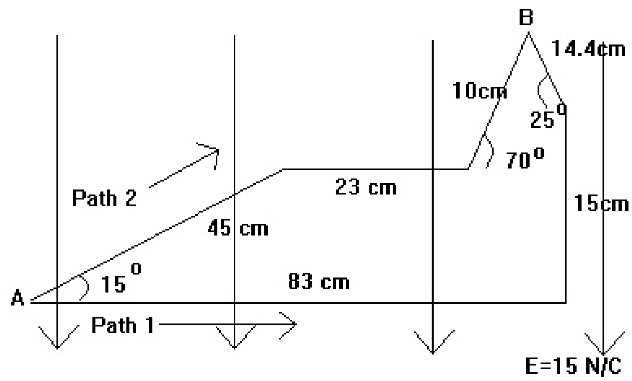
As we leave this section, I would like to stress the importance of the concept of potential. Of all the electrical concepts, potential is the most often used. It is essential that the student become comfortable with exactly what potential means.

Assignment #23

1.) Find the average value of the electric field (E) between two points 0.5 cm apart that are at a potential difference of 6 V. (EL22)

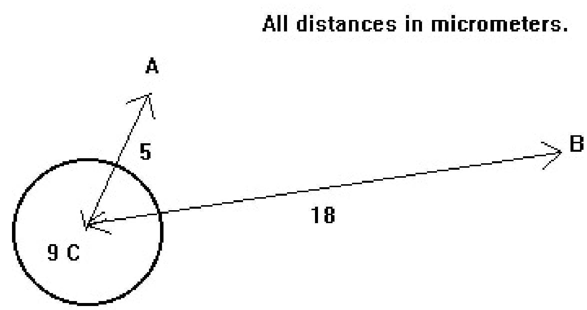
2.) If a charge of 34e moves across a potential difference of 9 V, how much energy does it gain ? (remember: Work = Energy) Give your answer in both units of energy. (EL23)

3.) A particle is immersed in a uniform E-field as shown below. Show (not prove) that the work required to move the charge from A to B along path 1 is the same as the work required to move it along path 2. Also calculate the potential difference between points A and B. (EL24*)



4.) Imagine a Cartesian coordinate system with a charge of 7e located at the origin. Determine (a.) the potential at point (0.2 μm,0 μm), (b.) the potential at (0.3 μm,0.4 μm), and (c.) the potential difference between the two points.(EL26)

5.) What is the voltage difference (potential) between points A and B below ? (EL25*)



6.) Consider two charges placed on a coordinate grid, one at point (0,0) and the other at point (3 m,4 m). If the first has a charge of 3 mC and the second has a charge of 5 mC, what is:

a.) the force on the 5 mC charge?
b.) the force on the 3 mC charge?
c.) the electric field the 5 mC charge experiences?
d.) the electric field the 3 mC charge experiences?
e.) the electric field at point (2,2) due to the 3 mC charge?
f.) the electric field at point (2,2) due to the 5 mC charge?
g.) the complete electric field at point (2,2), including direction.
h.) the voltage at (3,4) if the 5 mC charge was not there?
i.) the work done in placing the 5 mC charge at point (3,4)?
j.) the work needed to move the 5 mC charge from (3,4) to (2,3)?

7.) Measuring atomic radii is a tricky business because of the way the electrons are spread in clouds around the nucleus. However, consider that an atom of copper has an approximate radius of 1.28×10^{-10} m. Ignoring the presence of the other electrons, how much energy would be required to remove the last electron from a copper atom? How does this compare to the ionization energy of copper (1.237×10^{-18} J). What is the percent error? Using the given value of the ionization energy, what is the effective value of the charge of the nucleus considering the shielding effect of the other electrons?

8.) Applying simple electrostatic principles to atoms rarely (if ever) yields accurate results because of the quantum nature of electrons and the resulting difficulty in assigning them a meaningful location around the atom. Nevertheless, consider the following (crude) approximation technique for calculating ionization energy:

Assume that the ionization energy of an atom is equal to the electrostatic work required to remove an electron totally from orbit. Approximating the entire nucleus and the shielding electrons as a point charge with a positive charge of 0.75e located entirely at the center of the atom and calculating the work required to remove the last electron yields fair results for a number of elements.

a.) Why is the effective charge less than the charge on one proton, regardless of what atom you are considering?

b.) What would be the first ionization energy for silver using this method?

c.) What percent error does this yield?

d.) What would be the second ionization energy for silver using this method? What is the percent error?

e.) Which is a better approximation, the first or the second ionization energy? Why?

f.) Can you determine a better approximation for the second ionization energy?

Necessary information:

Atomic radii of silver: 1.44×10^{-10} m

Ionic radii of silver: 1.26×10^{-10} m

First ionization energy: 1.21×10^{-18} J

Second ionization energy: 3.44×10^{-18} J

9.) Decipher: "Rapidity of nuptualization can be bemoaned over an extended period of terrestrial rotation." (DNCTHWG)

Comprehension Guide