Chapter 22: The Electric Field

The Electric Field

Just as we did when discussing gravity, we can also construct a field equation for the electric force. The reasons are the same: we want to be able to discuss the electricity caused by one single object instead of always dealing with the force between two charged objects. The procedure and concept is essentially the same as it was when dealing with gravity. To remind you, the field was measured by placing a test object at the location, measuring the force on that object and dividing by the factor on the test object that determined the force. For electricity, that factor would be the charge on the test object. Mathematically,

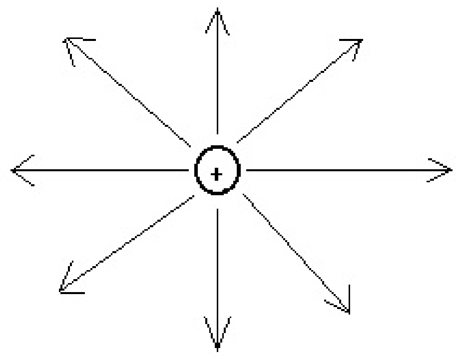
$$\underline{E}_1 = \underline{F}_{12}/q_2$$

Where E_1 is the field caused by object 1, F_{12} is the force between objects and one and two, and q_2 is the charge on the test object. Once again, we encounter the concept of the test object. In this case the test object must have a small enough charge so as not to disturb the field that is being measured. It is also important to notice that the test particle is defined as positive, therefore the field will always be given in terms of its effect on a positive charge. The above equation shows us the units of an electric field,

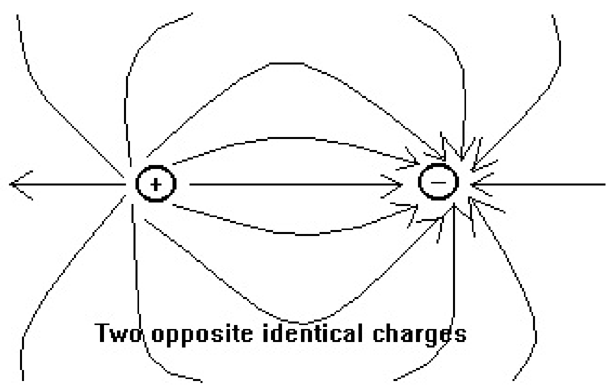
$$E = F/q = \text{Newtons/Coulombs} = \text{N/C}.$$

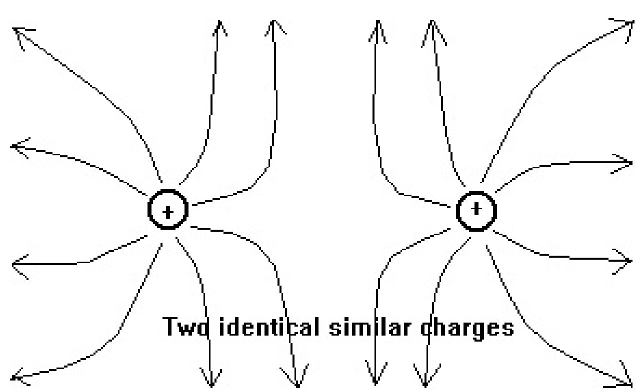
Notice how now the field is not in units of acceleration, as was the case with gravity.

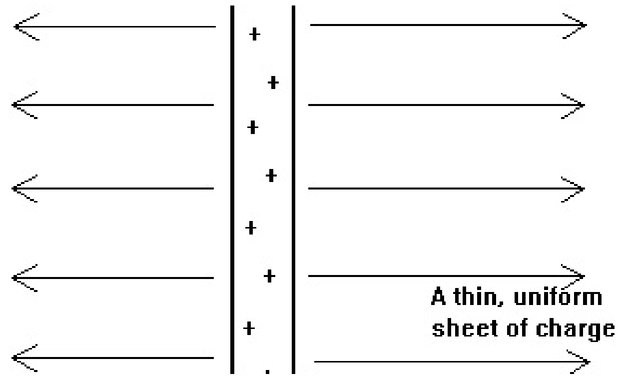
Because the concept of a field is often hard to understand, Michael Faraday came up with a conceptual method for visualizing a field. Faraday was a phenomenal physicist and chemist in the 1800's who was not only entirely self taught, but hated mathematics. He accomplished all his work by experimentation and conceptual explanations. Even without (or because of) his lack of mathematics he made some of the most important discoveries in physics. It was his lack of mathematical knowledge that propelled him to invent the concept of the lines of force. Faraday's line of force are lines drawn in the following manner. Imagine that we wanted to draw the lines of force around a single, positive charge. We would place a test charge at a location around the positive charge and draw an arrow in the direction the force acts on the test charge. We would then move the test charge to the head of that arrow and then we would draw another arrow in the direction of the force at that point. After doing this many times around the positive charge, we would end up with a pattern of lines that represent the field direction around the object, as shown below.



Notice how the lines point away from the positive charge, since the test charge is defined as positive. In the case of the field around a negative charge, they would point inward. The above diagram is very simplistic and probably doesn't shed too much light on the concept of forces. Faraday's lines of force are actually more useful when dealing with complicated fields. Consider the following diagrams which represent field lines for different situations.







Hopefully, this concept will help you visualize what exactly a field is. There are a couple of extra notes on Faraday's Lines of force that are listed below.

1.) Do not forget that although we draw these lines on two dimensional paper, a field is actually three dimensional.

2.) The line tangent to a line of force gives the direction of the field at that point.

3.) If the field lines are drawn correctly, the number of lines in a given area is proportional to the field strength in that area (when lines are close together the field is strong).

4.) The lines are always drawn using a positive test charge, a negative charge would feel the opposite force.

5.) Lines must either begin (or end) at a charge and start (or end) at infinity. Lines may not end or begin in the middle of a field.

6.) Lines cannot cross (why?).

7.) Lines cannot be drawn through an equilibrium point (why?).

8.) These are lines of force direction, not lines showing the motion of a particle. A particle released at the beginning of a line of force will not necessarily follow that line.

9.) Dipoles align themselves along lines of force (we will discuss dipoles later).

10.) Lines emanating from a charge on the surface of a conductor will always leave at 90^0 to the surface of the conductor.

One of the very important concepts of fields is the concept of a uniform field. A uniform field is a field that has the same value at all points. Recall our treatment of gravity near the surface of the earth. We stated that the field had a value of 9.8 m/s^2 at all points, thus we took the field to be uniform. As we discuss fields, it is important to keep this definition in mind. Uniform fields, either electric or gravitational, are very easy to work with (since they stay constant). If a field is uniform, we can see that the lines of force must be parallel (why?). Consider the last drawn example of Faraday's lines and you can conclude that that drawing represents a uniform field.

To help keep the student straight in understanding fields, we can classify fields into one of three categories:

Field Categories

1.) Uniform fields: These fields have the same value anywhere, the lines of force are all parallel and they are easy to work with.

2.) Fields around a spherical object or a point object: These fields are also easy to work with, but their value changes at different distances from the center. The field lines are not parallel, but spherically symmetric.

3.) Complex Fields: These are fields that do not fit either of the above descriptions. They are not easy to work with, as a rule, but can often by found by taking the situation to be a combination of different cases from above (example: a complex field might be the field due to two spherical objects). It is rare that we will do anything complicated with these situations.

Uniform Fields

We now turn our attention to mathematically handling fields and field equations. The most important fact is the simplest fact: a charged particle in an external field will experience a force. Thus we can always find the force on a particle if we know the field the particle is in. Things get more complicated if we wish to further than that, however. For example, if the field is uniform, F = ma can be used to find the acceleration. If the force is constant, the acceleration is constant, and our one dimensional motions equations apply. However, in a spherical or complex field, F = ma will give us the force, but since F is not constant, as soon as the particle moves slightly, the forces will change and thus the acceleration. Deriving the actual path and determining the velocity of an object in such a field is often a very complex procedure. Let us attempt to apply this knowledge to a few examples. The first type of field, the uniform field, is very easy to deal with. We already know which equations apply:

$$E = F/q$$

and

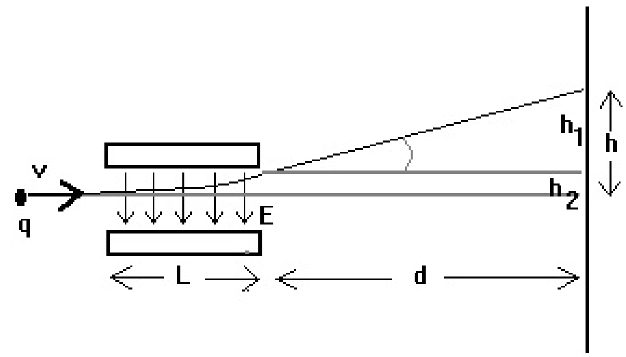
$$F = qE.$$

EX JUHY.) A 3 mC charge is placed in an external, uniform field and experiences a force of 4.5 N. What is the field strength?

EX. B.) A charge of 6.4 μC is placed in a uniform electric field of strength 2 N/C. If the charge has a mass of 65 mg, how fast is the charge moving after 3 sec?

As we can see, the equations couldn't be much easier. Before we move onto spherical fields, let us take a moment now and try to use this concept in a problem that involves many of our past discussions.

EX C.) In the diagram below, a charged particle q is launched into the electric field E as shown, deflected and sent to a screen. Determine the height above center for the impact of the particle in terms of the other givens (E, v, L, d, m, and q).



The above example shows us exactly how a cathode ray tube works (a tube like the kind used in hospitals to monitor heart rates, or a TV screen or a computer monitor). The "screen" is coated with phosphorous which glows when an electron hits it. A beam of electrons is shot out of a cathode and passed through two electric fields (one vertical and one horizontal). By controlling the electric field, the glowing dot can be made to move across the screen. Since the phosphorous will glow for a while after the electrons cease hitting it, it will leave a line across the screen. If the electric field is hooked up to the pumping of a heart, the dot will dance according to the beat.

The same principle is applied to ink jet (or sometimes called bubble jet) printing from computers. Each letter is fashioned by up to 100 dots of ink (a 10x10 matrix). Each little tiny dot is shot out of the printer at the paper with exactly the correct charge to hit the appropriate spot on the paper. Dots in the matrix that are not needed are shot out uncharged and gather in a gutter located directly across from the gun.

Spherically Symmetric Fields

Now let us turn our attention to spherically symmetric fields. Let us begin by deriving a simple law for spherically symmetric fields from Coulomb's Law. Remembering that:

$$E_1 = F_{12}/q_2$$

we can substitute in Coulomb's Law for the force:

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2 q_2}$$

then,

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r^2}$$

or

$$E_1 = \frac{Cq_1}{r^2}$$

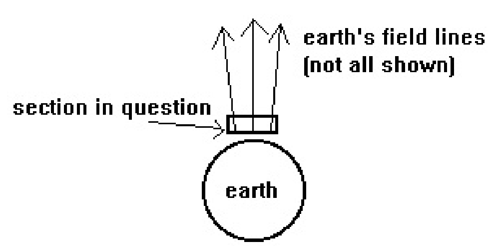
This formula above will give the value of the electric field at some distance r from a single point charge or spherically symmetric charge distribution. We should also remember that this is a vector equation, thus fields can be added or subtracted as vectors. Let us do some sample problems involving field calculations.

EX RTEF.) Find the field value at a distance of 2 m from a charge of 5 C. Where would the field value be one half of this value?

This next example shows how to use a spherical field as a rough approximation technique for a natural phenomena.

EX D.) An atom of uranium has 92 protons in its nucleus with a radius of 6.8 fm (6.8×10^{-15} m). As a rough approximation, consider the fact that each proton is repulsed by every other proton in the nucleus at a distance that averages out to be approximately r/2. (a.) What is the field that the one proton experiences? (b.) What force does this proton feel?

At this point, the astute student might be asking something like: "The earth produces a spherical field, yet we used a uniform field as an approximation. Because the two are so very different, why did our approximation work?" The answer is as follows. A uniform field can be used to approximate a spherical one <u>as long as the area under question is small compared to the overall field</u>. There are two ways for justifying this. First, if we look at just the first mile above the earth's surface, the field changes very little, thus it appears uniform. A second way of thinking about this is to imagine the field of the earth as a spherical field. If we focus in on one small section, as shown in the diagram below:

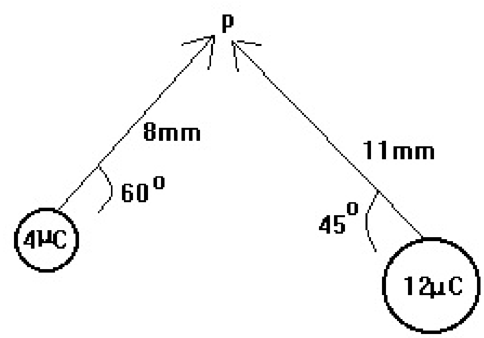


we can see that inside the rectangle, the field lines appear to be roughly parallel. As stated previously, parallel field lines indicate a uniform field.

<u>Complex Fields</u>

When dealing with complex fields, there is not much we can do with our limited mathematical ability. We can, however, do very simple problems like calculating the value of the field due to a set of charges by combining them as vectors.

EX. E.) Determine the magnitude and direction of the electric field caused by the two charges below at point p.



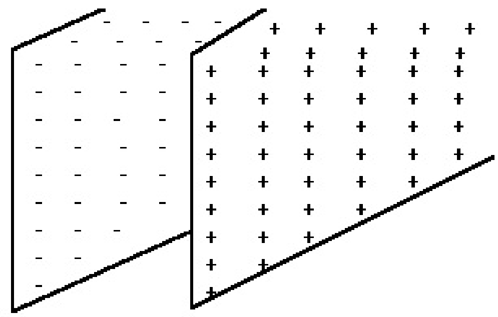
Notice, however, that this is as far as we can go with this. If the charge moves, even slightly, the field will change drastically. We cannot use our equations or motion since the force, and thus the acceleration, is not constant.

One other thing we can do, however, is to draw the fields around certain symmetrical objects. We draw these fields as an exercise and in hopes that by at least seeing them drawn we can draw some conclusions about them. This section also serves as a warm up for the next optional chapter concerning Gauss's Law. Gauss's Law is one of many methods that Physicist's use to find field equations for complicated fields. However, in the case of Gauss's Law, being able to use it requires that you are able to draw the field first.

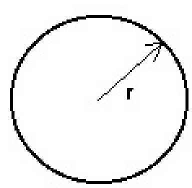
Drawing fields from symmetry is an exercise in logic. We simply look at a field situation and ask ourselves: "what is the most logical way for the field to look? We also ask: "Is there any reason it would not look like this?" We also need to realize what the different regions in an area would look like. In some cases the field will change abruptly as it passes through a region where situations change.

The easiest way to learn this concept is by example. Let us try to draw, describe and graph the following situations:

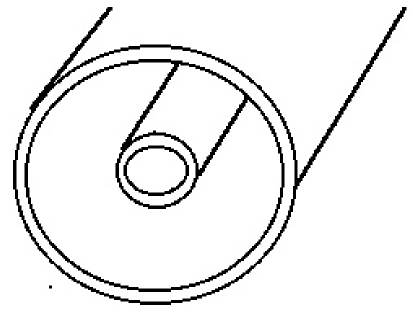
EX IUYT.) Draw, describe and graph the field between and on either side of two equal, opposite and infinite sheets of charge, a distance d apart.



EX. UIYH.) Imagine a solid, uniform ball of charge of radius r. Draw, describe and graph the field created by this charge from r = 0 to infinity.



EX. TRFD.) Draw, describe and graph the field shown below. It consists of a metal pipe with charges on it inside another, larger charged pipe.



This last example shows us an interesting thing that will be our next topic of discussion.

Electric Fields and Conductors

We will take a minute at this point to discuss electric fields around and inside conductors. Conductors, you will recall, are materials that allow electrons to flow through them easily. If a conductor is charged, two interesting things occur in relation to the field.

1.) The electric field will always radiate out from the conductor at right angles to the surface. No matter how irregular the object is, the field near the surface will always be at right angles to the conductor. Thinking about this a little will lead an astute student to the conclusion that very near the surface of the conductor the field is...

2.) The electric field inside a charged conductor is always zero. This interesting fact will lead to a number of conclusions, which are worth discussing in depth.

Recall that we discussed that electric charges will always reside on the outer surface of a conductor. We said that they do so because they are pushing each other apart and want to get as far from each other as possible. Another way of saying the same thing is to say that charges on a conductor will arrange themselves around the surface so that the electric field inside is always zero. If the electric field were not zero, then a charge placed inside would accelerate and charges would tend to move in one direction. We know, from experiments, that charges in a conductor are free to move in any random direction. The student should be reminded that this phenomena applies to any conductor, regardless of shape or solidity. It works inside a hollow sphere as well as a solid sphere. It would work on a metal box or a thin, solid wire.

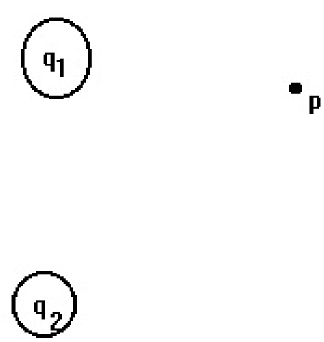
The result that shows that the electric field inside a conductor is always zero is the reason that you are safe from lightning inside of a car. Often people think that you are safe because you are insulated from the ground by your tires, but that is not the case. Because a car (or airplane) is basically surrounded by a sheet of conducting material (metal), when lightning strikes it, the electrons move around such that the field inside the object is zero. Humans feel no effect. It is of interest to note that as new, lighter plastics are developed and used for car body parts and parts of airplanes, this shielding effect will no longer protect the inhabitants of the vehicles.

This effect is also used in some "high tech" situations where equipment must be protected. In these cases, scientists use what is called a Faraday's Cage. It is basically a box made of metal. Instruments can be placed in the box, with the wires coming out, and then the instruments are protected from electrical interference. A situation where such protection might be necessary is in an electrical lab where large sparks might be common. Computers, for example, are instruments that can be destroyed by even small, stray electrical sparks contacting their inner chips. In such a lab, the equipment (and personnel) are usually located in a special room that is lined with copper. In such a Faraday's cage, no matter how big the spark is on the outside, noting inside will be affected. We can see this happen on a smaller level if you place a radio inside a metal box (this affect is somewhat evident inside buildings). Since radio waves are electrical phenomena, they cannot reach the antenna and you will get no reception.

Before we leave this discussion, two points should be made to remind the student of the limitations of this effect. First, it only occurs in conductors. It is impossible to set up an electric field inside a solid (or hollow) conductor, but it is possible to do so inside an insulating material. Secondly, this situation only applies in equilibrium, after the charges have settled moving around. If you dump a huge amount of charge on a conductor in a short amount of time, there can and will be an instantaneous field set up, but this field will vary and change quickly until the charges have settled and the field goes to zero.

Electric Dipoles

We have said that many electrical fields are too complex to handle mathematically at the moment (after a few years of calculus, you'll be able to take them on), but there is one situation that although deceptively simple looking and mathematically grueling, is of great physical importance. That is the configuration of an electrical dipole. This is a perfect example to use to show you how Physicist's use approximations for special cases to get a glimpse of the physics behind a complicated situation. What we are going to do is take a complicated situation, pick one special case, use approximative techniques and gain some physical insight on the situation. We will begin by examining the dipole to see just why it is mathematically difficult. Consider solving for the field at some random point p around two charges, q_1 and q_2. Try to develop one simple equation that will give the field at any location, regardless of where p is located.



If we attempt to examine the field at point p, we will find that the equation is not simple or reducible. In fact, it is a vector equation. The way we work around this is to use an approximation formula that utilizes the Binomial Theorem. Let us take a minute and take a side trip to review how to use the Binomial Theorem.

The Binomial Theorem is a method of expanding a set of variables or numbers raised to a power. The theorem states that:

$$(1\pm x)^n = 1 \pm \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} \pm \ldots$$

or $$(1\pm x)^{-n} = 1 \mp \frac{nx}{1!} + \frac{n(n+1)x^2}{2!} \mp \ldots$$

These are used when expanding a polynomial and can be used to approximate answers to otherwise unsolvable problems. Let us do one simple example.

EX F.) Approximate the value of $(1+0.25)^3$ to the first, second and third order.

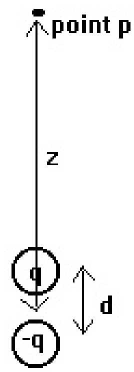
The above example shows not only the definition of "to the first order", but also how the binomial theorem can be used for an expansion. The problem itself is useless, since it could have been easily plugged into a calculator in its original form, but the next example shows how to use this to solve an otherwise unsolvable problem.

EX G.) Solve $(14+y)^6 + (14+y)^3 = 16781312$

We were able to get a reasonable answer just by doing a first order approximation. However, one thing should be mentioned. We can only discuss first order approximations if the higher order terms are negligible. In other words, each consecutive term must be significantly less than the term before it. This will only occur if the variable in the parenthesis (the x in the $(1+x)^n$ term) is much less than one. <u>Approximations using the binomial theorem are only valid if x is less than one.</u>

Although the above is interesting, it does not explain how this relates to physics. We can use the binomial theorem to approximate the field mentioned above to first order to gain some physical insight into the nature of the field. The student is urged to recall that what follows is only an approximation for a special case, and the approximation is only valid if the second term in the parenthesis is much, much less than one.

EX H.) What is the field along the axis of the dipole below at large distances?

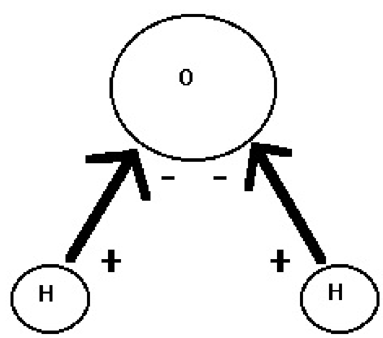


In the above equation, a substitution can be made where $\underline{p} = q\underline{d}$. Then we have the following:

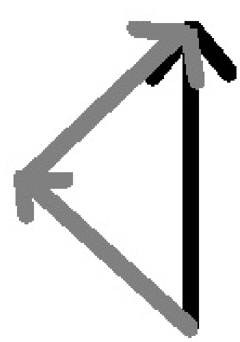
$$E = \frac{p}{2\pi\varepsilon_o z^3}$$

Which is an approximation for the field due to a dipole at distances along its axis much greater than the distance separating the charges. p is an interesting quantity called the dipole moment of the charge distribution. The dipole moment is an inherent property of the set-up, much like charge or mass. In other words, in many objects we would describe their electrical properties in terms of dipole moments, just as we would describe their mass or charge. A dipole is defined as any charge set up that has properties where one side is primarily positive and the other is primarily negative. Interestingly, at large distances from a dipole, it is impossible to measure the charge of the set up, one can only measure the dipole moment. Notice how the moment will remain the same if the charge is halved and the distances between them doubled. Also notice that the field goes according to $1/z^3$, instead of the usual $1/z^2$ that we would expect from an electric field. Dipole fields drop off more quickly because the negative and positive charges effectively cancel each other out, leaving only a component of the field.

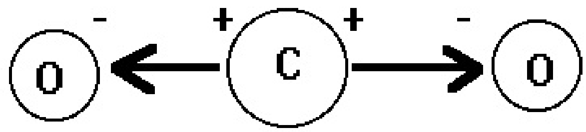
Dipoles play an important role in physics and in chemistry. Consider a molecule of water. In such a molecule the electrons in the covalent bonds spend more time around the hydrogen atoms than they do around the oxygen atom. Because of this, each hydrogen-oxygen bond forms a dipole moment as shown in the diagram below.



Because of the bent shape of the molecule, the two moments add as vectors into one total dipole moment for the water particle (as shown below).

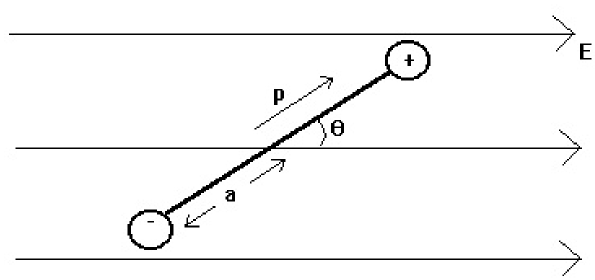


Thus each and every water particle acts like a tiny dipole and although it is electrically neutral, it does have a field and does react to external fields. This is what is commonly meant by "polar" molecules. An example of a non-polar particle would be CO_2, where the dipole moments cancel each other out because they point in opposite directions (due to its straight line form).



A few random notes should be made before we move on. First, notice how in the example of the water particle, it had a dipole moment without having a net charge. This shows that the dipole moment of an object is simply another inherent property, like (but separate from) mass and charge. Also, in the water diagram, the lone pairs of electrons were left off for clarity.

EX I.) Describe qualitatively, but using quantitative arguments, what would happen to a dipole placed in the uniform E field below.

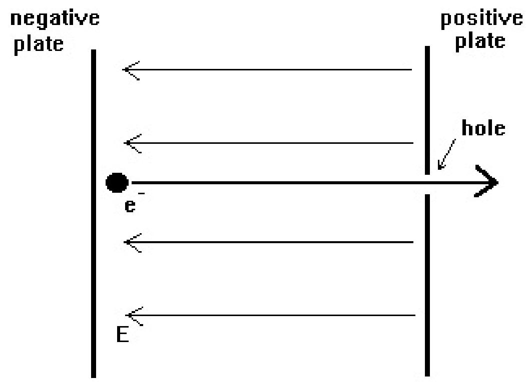


This example shows you an important property of a dipole, it will always align itself along the field lines of an external field. This explains compasses and the little magnetic fillings that are often sprinkled over a magnet to show the magnetic field (btw, why are compasses always encased in water or some other fluid?)

Assignment #22

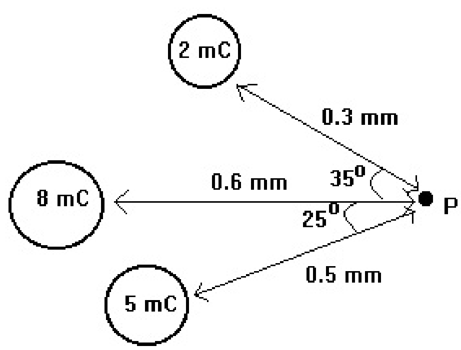
1.) A charge of 14e is placed in a uniform electric field with a strength of 11 N/C. What force does it feel ? (EL20)

2.) Consider the electron "gun" shown below. The electron leaves the filament, and is accelerated towards the plate by a field of 200,000 N/C between the plates. It then shoots out of the tiny hole in the positive plate. If the plate are 4 cm apart, with what velocity will the electron leave the gun? (E1*)



3.) Three electric charges are arranged in a line and have charges of 5 C, 7 C and 12 C (respectively from left to right). If the 7 C charge is 0.35 m to the right of the 5 C charge and 0.60 m to the left of the 12 C charge, what is a.) the force on the 7 C charge and b.) the value of the electric field at that point caused by the other charges ? (EL19)

4.) What is the electric field caused by the three charges at point P? What force would a 4 mC charge feel if placed at that point? (EL34)



5.) Consider the equation below. (a.) Given that x = 6, solve for an approximate value of y (first order approximation is sufficient) by expanding and approximating the terms on the left of the equal sign. (b.) Determine the percent error this would yield.

$$(x + y)^6 + (x + y)^3 = 49384$$

(E2)

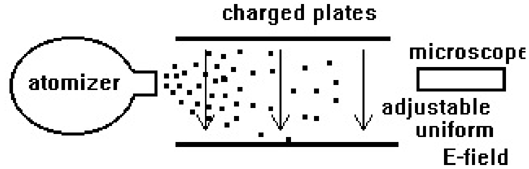
6.) Approximate and explain the field due to two positive point charges separated by a distance d at points along a line perpendicular to the axis (at the center of the dipole) at distances much greater than d. (E6)

7.) A test particle is immersed in an electric field created by a charge positioned somewhere on the x-axis of a coordinate system. The test particle feels the forces listed below at the corresponding positions on the x-axis. By graphing the data below, determine the position of the charge creating the field and the product of the two charges involved. Hints: Consult known equations in order to determine which graph to draw, use x as your dependent variable, and use $x-x_o$ as the distance from one charge to the other, where x_o is the position of the charge creating the field.

X	Force
6 cm	5.620×10^{10} N
6.5	3.596×10^{10}
7	2.497×10^{10}
7.5	1.830×10^{10}
8	1.400×10^{10}
10	6.240×10^{9}
12	3.510×10^{9}

(EL32)

8.) One of the most important experiments that determined that charge is quantized and measured the elementary charge was Millikan's Oil Drop experiment. A simplified version and explanation follows. The apparatus (below) consisted of an atomizer (basically a spray bottle) that sprayed tiny drops of oil into the field between the two charged plates. Millikan spotted one tiny drop in a microscope and then adjusted the electric field until the spot was balanced in mid-air between the force of gravity and the electric force. If he then knew the weight of the drop and the strength of the field that balanced the drop he could determine the charge on the drop. The charge on the drop must then be some integer multiple of the elementary charge.



Consider all the drops to have a radii of 1.64 micrometers and take the oil to have a density of 0.851 g/cm^3. Suppose Millikan found that the six drops were balanced by the electric fields listed below. What would he have concluded for the value of the elementary charge ?

E_1 = 163482 N/C
E_2 = 98089 N/C
E_3 = 75453 N/C
E_4 = 65393 N/C
E_5 = 51626 N/C
E_6 = 42647 N/C

(E9*)

9.) Decipher: "It is futile to indoctrinate a superannuated canine with innovative maneuvers." (DNCTHWG)