Chapter 20: Gravitation

The Four Forces

Up to this point, we have been discussing very concrete concepts, such as force, rotation and motion. It is now time to turn our attention to more general and more abstract notions. During the next few chapters, we will be focusing on the fundamental forces, the four forces that are the cause of all the effects in the universe. Instead of things like ropes and pushes, we will try to understand things like gravity and electromagnetism, which are the two most obvious of the four fundamental forces. As we progress, the concepts will become more general and harder to visualize, thus I implore the student to pay particular attention and be sure to understand each topic before going on to the next one. We are moving out of everyday life and into the most basic laws that govern the universe. We are beginning to look at the whole picture at once, instead of focusing on particular situations.

It was mentioned earlier that there are only four fundamental forces in the universe. It is believed that everything that occurs can be explained by the existence of the four forces. Thus far, at least, nothing has discovered that these four forces could not explain. This notion leads us to the conclusion that no other forces exist. It is important to keep in mind, however, that if something is discovered that is unexplainable through these forces, we would have to rethink our theories. Keep looking.

The four forces are: gravity, electromagnetism and the strong and weak nuclear forces. We will discuss gravity and electromagnetism in depth in the next few sections, but it remains to make at least a comment about the other two before continuing.

Everyone knows that the nucleus contains protons, or positive charges. In earlier science classes your were probably taught that like charges repel. How then, can the protons stay contained in the nucleus when they are repelling each other? It was this very question that led scientists to propose the existence of the strong nuclear force. This is an attractive force that only acts at very close range (dropping off very quickly to zero at distances further than the size of a nucleus). Thus, while the protons are being repelled by the electric force, they are also being attracted to each other by the (very) strong nuclear force. Much is still not understood about this force, and what is known is a very complicated mess of facts. For example: the nuclear force only affects certain types of particles (protons and neutrons) while other particles are immune to it's pull (electrons), protons and neutrons seem to be affected the same by this force, and other properties (such as atomic spin) seem to also play a role. Because of its complexity, we will not treat the strong nuclear force in detail. What you should remember for now is that the strong nuclear force is an attractive force that affects protons and neutrons only in close proximity.

The weak nuclear force is another force about which little is known. It turns out that there is one phenomena which the other forces cannot explain. This phenomena is called beta-decay (β-decay) and is a type of nuclear reaction where electrons or positrons are emitted from the nucleus of one element as it changes to a different element. The weak nuclear force was postulated to explain this event.

For the last few decades, physicists have been attempting to unify all of the four forces into one. They believe that there is only one force in the universe and what we are seeing is different manifestations of that one force. At the moment, they think that at very high temperatures, the four forces would "merge" back to one fundamental force and they believe this was the case when the big bang occurred. It seems that the weak force and the electromagnetic force do merge at high temperatures (it is then called the electro-weak force) and it also appears that the strong nuclear force joins them at even higher temperatures. Gravity still remains a mystery.

Newton's Universal Law of Gravitation

It is with this mystery that we begin. Gravity is the weakest of all the four forces (although after these sections, an astute student might ask how we can appropriately compare different forces) and it seems to be the strangest. It is unlike the other forces because it always attracts and appears to be linked into the fabric of space and time (!). However, it is the one force that most people are familiar with since it's workings are obvious to us.

Gravitation is the force of attraction between any two objects that have mass. Thus any two objects in the universe, no matter how big or small, will attract each other. Right now you are attracted to this paper, however, the force of attraction is drowned out by the other forces around you that are much stronger. If it was only you and this paper in the entire universe (a lonely proposition), you would see the paper float towards you. Gravity also has an infinite range, although it weakens with distance. Thus even if you and the paper were on opposite sides of that theoretical, lonely, universe, you would end up finding each other.

Gravity was first explained satisfactorily by Isaac Newton, when he (supposedly) saw the apple fall. It was then that he realized that the apple falling and the moon orbiting could be caused by the same effect, one of attraction between two masses. He later came up with a mathematical relation stating:

> "The force of attraction between two objects is directly proportional to the product of the two masses and inversely proportional to the distance between their two centers squared."

mathematically:

$$\underline{F} = -Gm_1m_2/\underline{r}^2$$

Where m_1 and m_2 are the two masses, r is the distance between the two centers of mass and G is a constant of proportionality called the Universal Gravitational Constant. Today we know G as having the value of:

$$G = 6.67 \times 10^{-11} \ \mathrm{N.m^2/kg^2}.$$

There is a negative sign in the above equation because gravity is attractive and because it is a vector, the equation needs a negative to yield the proper direction. Let us do one simple problem involving Newton's Universal Law of Gravitation (NULG).

EX B.) What is the force of attraction between the earth and the moon? Where is this force acting?

Once again, I would like to stress the importance of Newton's formulation of a theory of gravity, in terms of what it meant to the Aristotelian theory of motion. As was mentioned in a previous chapter, by developing <u>one</u> law for both the heavens and the earth, Newton dispelled the myth of there being separate, corrupted and non-corrupted universes. He united heaven and earth together for the first time by saying they are subject to the same laws. In doing so, it can be said that he was making a first tentative step towards a universe that is governed by reason, not superstition. If the heavens are a part of the earth, and no longer a spiritual realm, they are not to be feared. Notice that this means that religions whose deities preciously had their gods seated in the heavens, had to move them to another location. Nowadays, we have a different dichotomy, a separation of the physical and the spiritual world, where the spiritual world is no longer a part of "the universe" and thus no longer subject to the rules or investigations of science. Newton's Universal Law of Gravitation was just that, the first Universal law of the scientific age.

Before we proceed with our discussion of gravity, we should take a minute to discuss the measurement of the constant G. When Newton first formulated the Universal Law of Gravitation, he was only able to make a rough guess at the value of the Gravitational Constant. If we consider the equation $F=-Gm_1m_2/r^2$ for a moment, we can see that to measure G we would need to know all the other variables. We can easily measure mass and distance between two object in a lab, but what about force? Consider two object that can be easily brought into a lab, say masses of about 100 kg (roughly 220 lbs) and consider a reasonable distance of say about 1 m (remember that the distance is between their centers of mass). This gives a force of about 10^{-7} N (or on the order of 10^{-8} lbs or about a hundred millionth of a pound). Certainly we can see that this is not an easy measurement to make in a lab. After all, how would you measure a force this small? We can get better by increasing the masses or decreasing the distance, but it still remains on the order of millionths of a pound.

The first true and accurate measurement of G came in 1798 by a man named Henry Cavendish. He accomplished this by setting two small masses on the end of a long, thin rod and suspending them on a wire from the center of the rod. On the wire he attached a mirror and bounced a beam of light off the mirror to a distant screen.

Once the setup was established, he placed two larger masses which were fixed in place next to the small masses. This caused the rod to rotate because of the gravitational attraction between the masses and it caused the wire to twist until its internal forces balanced the gravitational forces. When the wire twisted, so did the mirror, sending the light beam to a different spot on the screen.

By knowing how much the beam moved, he could determine how much the wire twisted and by knowing the properties of the wire he could determine the torque caused by the gravity acting at each end. From this torque he could determine the force of gravity between the masses. The success of this experiment cannot be overstated. What Cavendish managed to do was to find a way to accurately measure forces as low as a millionth of a pound. Whenever I discuss this experiment, it impresses upon me the ingenuity and creativeness necessary for a good scientist. To me, thinking up, designing and carrying out this experiment was nothing short of brilliant. Later, this same design was used to measure other constants.

The Gravitational Field

The next step in understanding gravity is to understand the concept of a gravitational field. The idea of a field is very abstract and rarely understood by beginning physics students (and even some advanced ones). However, it is an essential concept in all of physics. Often I see students struggling with fields and just blundering their way through problems without any true understanding of what is going on. If you intend to be a serious physics student, I strongly suggest that you try your best to understand this concept and make sure each point makes sense before you move on (if you are not interested in being a serious physics student, I suggest you just blunder through, feebly understand what is going on and try to get a C-).

A field is a measure of the force an object can create. Often it is described as the area around the object where its force can act, although this is a little confusing. A field is not a space, it is an effect that could occur. Since we are dealing with gravity, we will use the gravitational field as an example. The student should remember that gravity is just an example and the concept of a field can be applied to other forces. What a field is is interwoven with the reason why we need the concept of a field to begin with. Imagine that we wanted to compare the gravity caused by two different objects, say the Earth and Jupiter. Using the force of gravity, we simply cannot compare them. Force is something that exists between two objects. One object cannot have a force. It is nonsensical to talk about the force of gravity of the earth. We can talk about the force of gravity between the earth and the sun or between the earth and a cat, but not "of the earth". Therefore, it is handy to have some concept related to the gravity of ONE object. This is a field.

Think back to our example of the Earth and Jupiter. We could compare their gravities if we put the same object on each planet and measured the force of the planet on the object. In fact, if we had one standard object, we could compare the gravity of any two objects in the universe. Although this is one way to accomplish this, it is not the easiest or most practical. Instead, we need a concept that divorces the second object from the first altogether. The field is found by finding the force between object 1 and the standard object and then dividing out the influence of the standard object. The influence of the standard object is given by the aspect of the object that is affected by the field.

Although what I have said above is fairly general, it may not make sense at first. Using gravity as our example, we would find the force in the following way: place a standard test object on the planet, measure the force between the test object and the planet and then divide that force by the mass of the test object. This would give us the field of the original object. Mathematically,

$$\text{Gravitational Field of Object 1} = \frac{\text{Force Between Objects 1 and 2}}{\text{Mass of Object 2}}$$

or,

$$g_1 = F_{12}/m_2.$$

Notice that we have used the small letter "g" for the gravitational field. This is no accident. It turns out that the gravitational field and the acceleration due to gravity are one and the same. However, this only works for gravity (the electric field is not the acceleration due to the electric force, for example) and this only works because gravitational mass and inertial mass appear to be the same (recall the discussion of the two types of masses earlier - the student should be able to figure out the reasoning).

There is one other nice thing about the field, and that is that it can be calculated theoretically without having to actually place an object anywhere. Consider NULG:

$$F_{12} = -Gm_1m_2/r^2$$

and consider

$$g_1 = F_{12}/m_2.$$

combining we get:

$$g_1 = -Gm_1/r^2.$$

This is the field of object 1. Notice, however, that the variable r has lost its meaning. It used to be the distance between the centers of mass of the two objects. It now means the distance to the point in question. In other words, the equation above give the field of object one at a distance r from its center.

Before we continue and do problems regarding this concept, there are four points I wish to discuss.

The first is simply a reminder about what a field is. A field is a measure of the gravity (or other force) the object could create if something was placed in its area of effect. It is the gravity of one object. It is debatable whether or not fields exist. After all, it is only a measure of what the object could do if something else were there.

Secondly, remember the defining equation of a field. It is one of the simplest and most often forgotten equations in basic physics. The field is the force divided by the mass.

$$g = F/m.$$

Thus, the force is the field times the mass.

$$F = mg.$$

The fact that the force is the field times the aspect of the object in the field that reacts to the field is true for all fields. For example, in electricity, $F = qE$. Where q is the charge and E is the electric field the charge is placed in.

The third point is rather theoretical. It has to do with the definition. We have referred to the object placed in the field to measure it as the standard object. There is another requirement we did not mention. This object must be very small and is called the test mass (or test object). The reason for this is that large objects will actually change the gravitational field they are placed in. Therefore, we imagine the test object to be infinitesimally small so that it has no effect on the original field.

Finally, I would like to take a minute to discuss the concept of a field equation. We have been talking about the field at a particular location, but it would be handier to discuss the field around an entire object. We do this with a field equation; one simple equation that mathematically describes the field all around the object. Then, if we wanted to know the field value at a location, we could just plug in the numbers. Obviously, the only variables in such an equation should be the location of the point in question. All the other quantities should be known. For example,

$$g = Gm/r^2$$

is the field equation for a planet. G and m must be known, then we can plug in a value for r to find the field at point r. In physics, often we are in search of a field equation. Once that is found, the rest is assumed to be easy (it often isn't). The field equation for a planet is simple (actually, the above equation only applies to a uniform, spherical planet, with noting else in the near vicinity), but imagine the field equation for the solar system. At each point there would be nine planets plus the sun and numerous comets and asteroids all pulling at the test object. Moving the test object around would greatly change the force in a complicated (non linear) way. Besides that, the planets are moving at different rates. Such an equation would me monstrous indeed (and essentially out of reach to imagine it as one formula). We will discuss field equations in more detail when we discuss the electric force, since there exist many different combinations of charged objects that can create complicated electric fields.

With the proper substitution of variables, this equation above should give us the value of g at the surface of the earth (9.8 m/sec^2). However, we should point out a few obvious problems with that value. First, we would need to use an average value for r, since different locations are different heights above sea level. In fact, since the earth is not actually spherical, but rather bulged around the equator, further adjustments would be needed to give an accurate response. Secondly, the equation above works only for a sphere of uniform density. Gravity at a specific location can vary due to the density of rock beneath the spot where you are standing. Thirdly, since the earth is a non-inertial reference frame, fictitious forces are needed to properly achieve an answer. After saying all that, the average value of g at the surface of the earth is 9.8 m/sec^2, and while it does vary slightly, for most common problems, the variation is negligible.

Let us take some time to practice using these simple equations.

EX C.) Given that the gravitational field strength at sea level is 9.803 m/sec^2 and at Java it is 9.782 m/sec^2, what is the distance from the center of the earth to (a.) sea level and (b.) Java?

EX D.) Find the gravitational field strength on the surface of Jupiter. What would be the force of gravity on a 70 kg object on the surface of the planet? How long would it take that object to fall 10 m?

The last thing to mention about gravitational fields is that they are vector quantities. The actual equation for a gravitational field is:

$$\underline{g} = \underline{F}/m.$$

Which shows you that the gravitational field, $\underline{g}$ is found by dividing another vector ($\underline{F}$) by a scalar (m). This means that the gravitational field is in the same direction as the force (which means pointing towards the center of mass of the object causing the field). It is interesting to note that one single equation can give the field strength at any location, but the direction must be figured out logically. There is a method involving vectors that will give the direction, but for now we will gloss over it.

Since fields are vectors, they add as vectors. Consider the example below.

EX OOPP.) Find the field strength and direction for the gravitational field at point P caused by the two masses shown.

Gravitational Potential Energy

There is one final topic related to gravity that we should discuss: gravitational potential energy. In the past we have discussed gravitational potential energy and defined it as $\Delta U = mgh$. However, you may recall that we stated that this equation was only valid if g remained constant. We would like to have some way of determining the potential energy in a situation that does not have a constant g. Using calculus, we can derive the following result:

$U = -Gm_1m_2/r$.

Where this equation gives the gravitational potential energy of an object (m_2) in the gravitational field of another object (m_1) at some point r. Notice how this equation is not a change of potential energy, but rather the potential energy itself. When we first discussed potential energy we said it was only possible to discuss it in terms of changes and that comment is still true. The above equation is actually the change in potential energy from the point in question out to infinity. This technique is common in physics, when we have a concept that involves changes, we often standardize it using an infinite distance as one of the reference points. The above equation should actually be written as:

$$\Delta U_r = -Gm_1m_2/r$$

Which says that the change in potential energy between the point r (a distance r from the center of the object causing the field) out to infinity (where the gravity can be taken to be zero) is equal to negative G times the masses divided by the distance r.

EX OOPQ.) Find the potential energy of the space shuttle (mass = 50000 kg) when it is in orbit 300 miles above the surface of the earth.

Physically, this potential energy is the energy stored in the bond between the two masses. By finding the energy between r and infinity, you have actually calculated the amount of energy necessary to remove the object from its orbit and send it out to infinity. If we wanted to send a space probe from the surface of the earth out to deep space, this is the energy we would have to supply it (not counting the extra energy needed to overcome air resistance). Thus the energy you have just calculated is the energy needed for the space shuttle to leave orbit and reach deep outer space. This same concept is also used in electricity, it is the amount of energy stored in the bonds between nuclei and electron, or as it is called, the ionization energy of an element. Another interesting aspect of this concept is that it can be used to give you the escape velocity of a planet. The escape velocity is the velocity at which you would have to throw something up in the air and have it escape the gravitational pull of the planet. If we simply imagine throwing something up in the air, we would expect it to eventually fall back down. The faster we throw something, the longer it would take to fall back down and the higher it would go. But as it rises, the gravity of the earth would get weaker. Imagine that you could throw something fast enough so that it would rise to the point where the earth's gravity went to zero. It would never come back down. Since gravity is infinite in range, that height would be at infinity. The speed you would have to throw an object for it to reach infinity is called the escape velocity. What goes up at that speed need not come down! In other words, the gravity of the planet would get weaker faster than the object would slow down, causing to never stop and having it end up at infinity. Think of it this way: finding the energy of an object at the surface of the earth would tell you how much energy is needed for it to leave the surface and escape the planets pull of gravity. That energy could be supplied in the form of kinetic, thus there is an associated velocity.

EX E.) Determine the escape velocity of the earth.

One last comment about escape velocity should be made before we move on. This is the velocity an unpowered object would need to break free of the earth. Rockets are not launched at escape velocity, since they are powered along their trip.

The concept of potential energy between a point and infinity can be extended to find the change in potential energy between any two points in a field, since energies are additive. For example:

$$\Delta U_{AB} = \Delta U_A + \Delta U_B = \Delta U_A - \Delta U_B$$

schematically:

An example of this is shown below.

EX OOPR.) How much energy is needed for the shuttle to change from a 300 mile orbit to a 400 mile orbit? How much energy was needed to put the shuttle in its 300 mile orbit in the first place?

Understanding the connections between energy and gravity and being able to read a problem like the one above and determine what is being asked, then determining how to solve it are very important physics skills.

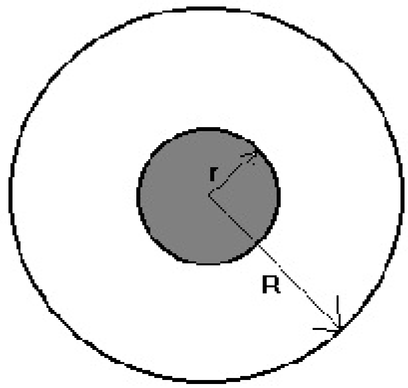
The Shell Theorem

Before we close this section, let me make a rather extended comment on one assumption that has gone (probably) unnoticed throughout this discussion. When we discussed gravity, we have been looking at it as seen in the diagram below:

When in reality, we know that this is not the case. The masses are each made up of many tiny masses, all of which pull at each other. The diagram below shows what is meant by this in a simplified way.

Here, the small object is being pulled by each of the eight "parts" of the larger object. Notice how the parts to the left pull with less force than the parts to the right, since they are further away. What we have been doing in this section is to simplify this by saying that all the mass of the object is concentrated at the center of the object (the center of gravity). In other words we have been using point masses (an infinitely small object that contains all the mass) instead of evaluating the entire object. This original assumption bothered Newton quite a bit as he was formulating his theory of gravity. Eventually he proved that it was accurate (provided that the object was a perfectly homogeneous sphere) by proving what is now called the "Shell Theorem".

The shell theorem gave us two important and interesting results. The first is that a uniform sphere behaves exactly like a point mass. The second is that the gravity <u>inside</u> a uniform sphere is governed only by the mass between the point of consideration and the center. The mass "above" it has no effect.
This effect can be best seen by use of a diagram:



Suppose you wanted to calculate the force of gravity on an object inside a planet at radius r. The shell theorem tells us that standing <u>inside</u> a planet at radius r is the same as standing <u>on</u> a planet with radius r (provided the densities are the same). The mass between r and R has no effect on an object located at r. Actually it does have effects, but they all cancel out.

The student should ponder the two consequences of the shell theorem and see if they make good logical sense (don't bother to try to prove them mathematically, just see if you can explain why they are true).

<u>Why Bother With Fields at All?</u>

Before we close the chapter, some time should be spent articulating the reasons that we bother with the concept of fields at all. Fields are important for a number of reasons, and many of the problems in more advanced physics deal with fields. The reason is that the field is the simplest concept that can lead to complete information about a situation. If we know the field, then it should be easy to determine the force on another object($F = mg$). Once we have the force, the acceleration can be found ($F = ma$). Once the acceleration is found, the motion of the object can be determined. Since a field describes the gravity around an object, it can be used to determine the motion of any other object placed in the area. Although this sounds direct, it is often difficult since fields themselves can be complex. However, it does allow for a method to arrive at an answer.

The student should remember that a field describes two things: 1.) the "gravity" of one single object and 2.) what the effect would be on an object placed at that location (without ever having to actually go and place an object there to determine it). Although sometimes hard to understand, fields are essential and useful concepts.

Assignment #20

1.) What is (a.) the force of attraction between the Sun and Saturn? (b.) the value of the Sun's gravitational field at Saturn, (c.) the value of Saturns's gravitational field at the Sun, and (d.) the acceleration due to gravity on the surface of Saturn?

2.) Suppose you had two cars (assume a mass of 1500 kg each) and wanted to position them so that the gravity between them was measurable (say 0.1 N). How far apart would you have to place them? Comment on this distance.(G9)

3.) At some point between the earth and the moon the combined gravitational field between them is zero. How far away from the eath is this? Express the answer in meters and in percent of the total distance. Draw a scale diagram showing this location.(G1)

4.) Which exerts a stronger combined pull on the earth from the sun and moon, a solar eclipse or a lunar eclipse? How much stronger is the strongest of the two? Express you answer in terms of percent (the difference divided by the weaker).

5.) Imagine that an object was released from a point half way between the earth an the moon and allowed to fall to earth.

a.) What would be its speed on impact, neglecting air resistance?
b.) If it arrived at the surface with only half of the speed found in part a, how high would its temperature have risen provided it was made of iron and had a mass of 1000 kg? (You must also assume that all the energy went into heating the object and very little into heating the air.)

6.) Using formulae, show how the gravity on the surface of a planet would change if the radius was increased and the density was kept constant.

7.) How far above the surface of the earth must you go for your weight to be one half of its value on the surface? Answer both numerically and in terms of the earths radius r_e.

8.) It is possible, using the sum of the forces and Newton's Universal Law of Gravitation, to derive Kepler's Third Law of Planetary Motion. Do so.

9.) Using the results of the shell theorem, prove that the gravitational force on an object inside a planet of constant density is proportional to r, the distance from the center.

10.) If you stand on the earth, and consider yourself as stationary, you have not only gravity acting on you, but also a centrifugal force from the rotation of the earth. What percent of your weight is the centrifugal force acting on a person at the equator? (G13)

11.) Consider the earth-moon system.

a.) What is the force of gravity from the earth acting on the moon?
b.) What is the force of gravity from the moon acting on the earth?
c.) What is the value of the gravitational field of the earth at the position of the moon?
d.) What is the value of the gravitational field of the moon at the position of the earth?
e.) What energy is stored in the bond between the earth and the moon?
f.) How much energy would it take to increase the moon's distance from the earth by 100 km?

12.) Decipher: "Insisting that 2 + 2 = 5 is a commonplace behavior of the genus Homo, but the ability to bear less than acceptable behavior is an attribute of a transcendental being." (DNCTHWG)

Lab #16: Determining the Earth's Gravitational Field

The purpose of this lab is to make as accurate a reading of the value of "g" (the earth's gravitational field) as possible in the class room. To do this, we need to focus on two areas: accurate measurements and reduction of friction. We will measure g by finding the length and period of a pendulum and using that information to determine the field.

The period of a pendulum is given by: $T^2 = 4\pi^2 l/g$ (this is an approximation for small angle oscillations).

Materials: Heavy pendulum bob, strong, lightweight, inextensible string or line, stop watch, meter stick.

Procedure:

1.) Set up as long a pendulum as possible in the classroom using the lightest string or fishing line available and the heaviest bob that is practical.

2.) Measure the length of the pendulum from the top where it is attached to the center of mass of the bob. Be as accurate as possible and play close attention to significant figures.

3.) Set the pendulum swinging, with a small but significant amplitude (no more than 15°). At the same time, start the stop watch.

4.) Time how long it take the pendulum to complete ten full swings (a full swing is back and forth).

5.) Repeat this procedure two more times and take the average of the three trials (remember: significant figures!)

6.) Repeat the entire above procedure for seven full swings and then again for five full swings.

7.) From each of the three results, determine the period of the pendulum (time for one swing).

8.) From this, determine the acceleration due to gravity for each of the three trials. Average these for one final answer.

	10 Swings	7 Swings	5 Swings
Trial 1			

Trial 2		
Trial 3		
Average		
Period		
Grav. Field		
Average g:	Pendulum Length	

9.) For comparison, get a 500 g mass and a mass hanger. Determine the exact mass of the two by using a vernier scale balance. Hang the two on a force sensor connected to the computer interface. From these two measurements, determine "g" (remember: sig figs!!). Repeat with three other masses.

Trial	Measured Mass	Measured Force	Value of "g"
1			
2			
3			
4			

As you draw your conclusions, keep the following questions in mind: Why did we use the longest pendulum length possible? Why did we use the heaviest bob possible? Why did we do so many trials? Which trial do you think was the most accurate? Why? How accurate would you expect this lab to be? How many significant figures are allowed in your answer? If your timing was off by either plus or minus 0.5 sec, how would this change the value of g for the 10, 7 and 5 swing trials? How did the pendulum experiment results compare to the other set of results?