Chapter 19: Return to Energy

Work

Many chapters ago we introduced the concept of the conservation of energy and we found that we could apply it to many situations in order to explain things and make predictions. However, when we discussed energy, we did not define it and we promised that we would return to it when we had the proper tools to give a correct definition. The reason that we never defined energy is that the true definition involves forces. In order to understand this, we must first introduce one other force-related concept: work.

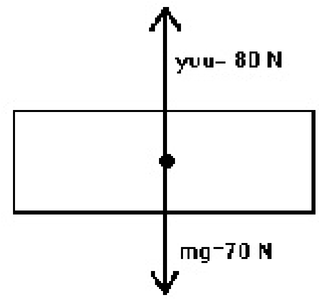
Work is defined as the product of the force applied to an object times the parallel distance through which the force acts. In short, work is force times a distance.

$$W=Fd$$

We will further explain the meaning of the "parallel" distance in a short while (although those of you that remember torques should immediately realize it is the opposite of the perpendicular distance discussed in that section), but for now let us just concentrate on work as being force times distance.

It should also be noted that work is very object specific. In other words, one object does work on another. Work is not something that an object possesses, like energy, instead it is something that an object (or field, such as gravity) does to another object. Because of this we need to watch our wording very carefully. There may be five forces acting on an object, but if you are asked to calculate the work that one of these forces does, you must ignore the other forces. The example that follows will illustrate this.

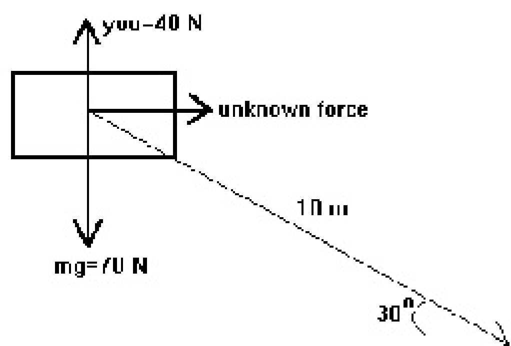
EX R.) Suppose you lift a book with a force as shown below. If the book rises 3 m, how much work did you do? How much work did gravity do?



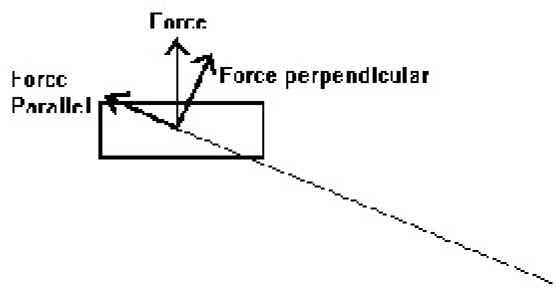
The above example, however simple, shows how we individually distinguish each different work done by a different object. It also shows how we must assign a sign to the work done according to whether the distance is with the force or against it. It also shows us that the units of work are N.m, the same units as energy.

The next example shows us exactly what is meant by parallel distance. Parallel distance means that we only count the distance in the direction parallel to the force. For example, if we push an object up but the object moves sideways (because some other forces are acting on it), the parallel distance for our force is zero, since the object never moved parallel to our push. If the book moved up, we would count the entire distance, since all of the distance is parallel to the force. If the object moved at an angle, then we must trigonometry to determine just how far the object moved in a direction parallel with the force.

EX. S.) Three forces act on the object below: you push up, gravity pulls down, and another unknown force pushes to the side. How much work do you do? How much work does gravity do? The path it travels is marked by a thin line.



Oftentimes, just as we did with torques, it is easier to use the parallel force than the parallel distance. Consider the example above. We could have broken the force that you gave to the book up into components parallel and perpendicular to the distance. We would have then calculated the work by multiplying the parallel force times the full distance. Such a situation would have looked like this:



The two situations are equivalent. This is because work is actually two vectors (force and displacement) multiplied together by the scalar method of multiplying vectors. The is written as:

$W=\underline{F}\bullet\underline{d}$ (the scalar product of $\underline{F}$ and $\underline{d}$)

As you can see, the answer to this multiplication is a scalar, not a vector, as in the case of the cross product (which we learned earlier). Often this method of multiplying vectors is also called the dot product. The magnitude is given by:

$W=|\underline{F}||\underline{d}|\cos\theta$

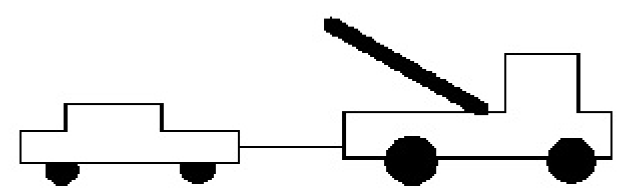
where θ is the angle between the two vectors as shown below:

Because work is actually a dot product of force and distance, the easiest way to calculate the work is to remember the equation: $W = Fd\cos\theta$. You will find that by using this equation, you will always get the appropriate sign for the work done. As a way of keeping things straight, it is suggested that you draw both the force and the distance as vectors so you can see which way they point. Often students will use the wrong angle if they forget to draw an arrowhead on the displacement vector indicating direction.

With those things in mind, let us run through a series of exercises that are designed to give us practice with these concepts.

EX T.) A tow truck pulls a car weighing 20,000 N from rest with a constant force of 10,000 N for 100 m. Friction also acts on the car with a force of 6,000 N.

a.) How much work does the tow truck do on the car?
b.) How much work does gravity do on the car?
c.) How much work does friction do on the car?
d.) What is the total work done on the car?
e.) How much work does the car do on the tow truck?
f.) By using the forces and our equations of motion,



determine the final kinetic energy of the car at the end of 100 m.

The Work-Energy Theorem and the Definition of Energy

This last problem should have produced an interesting result. The total work done on the car is the same as the change in kinetic energy for the car. What we have just demonstrated is what is called the Work-Energy theorem.

The Work-Energy theorem states that the change in kinetic energy of an object is equal to the total work done on the object by all outside forces.

$$W=\Delta T.$$

When using this, it is important that you include all the work done on the object by all the forces involved (including gravity and friction).

Before we apply this new formula, we should now define energy in its correct format.

> <u>Energy</u>: the ability to do work; the ability to apply a force over a distance.

Thus we see that the amount of energy an object has is a measure of the amount of work it could do. When we say something has 10 J of kinetic energy, we mean that if it hit something, it could do 10 J of work on that object (by exerting a 5 N force for 2 m for example).

It is important to remember that the amount of work something can do is the energy of the object. The student should begin to think of work and energy as interchangeable. Work is energy and energy is work. When a problem asks you to find the work done, if you find the change in energy, you have answered the problem (One warning: they are not <u>exactly</u> the same thing, as can be seen from the different definitions. However, since they are so closely intertwined, you can often substitute one for the other.). Consider the example below.

EX. T.) Meteor Crater in Arizona is thought to have been formed by the collision of a 5×10^{10} kg meteor with the earth at a speed of 7200 m/sec.

a.) How much work did the earth do on the meteor stopping it?

b.) If the crater is 600 ft deep (182.9 m), what average force did the earth apply to the meteor to stop it?

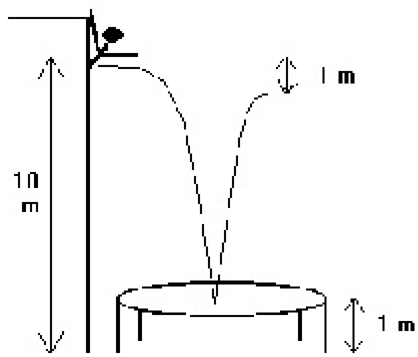
EX. U.) A 60 kg woman jumps out of a burning apartment building and lands on a trampoline set below her window (see diagram).

The trampoline extends 0.5 m to stop her and when she leaves the trampoline she bounces up to 1 m below the level of the window.

a.) What was the work the trampoline did to stop her?

b.) What was the average force exerted by the trampoline?

c.) What was her velocity when she left the trampoline on the way back up?



Before we continue with further examples, there is a serious clarification that must be made. We have been dealing with the work energy theorem ($W = \Delta T$) and the student should be reminded that the left hand side of this equation is the TOTAL work done on the object by all outside forces, including gravity. This can be at times confusing. There is another way of expressing this same idea that might be a tad bit easier for the student to apply:

$$\text{Work} = \Delta E$$

This says that the work done by outside influences (other than gravity and heat) must equal the change in energy that occurs. Another way to say this is that the left hand side (Work) includes all the categories and effects that we do not have energy formulas for (friction, pushes, etc.), or things that involve forces, while the right hand side (ΔE) involves kinetic energy, gravitational energy, and heat changes, for which we do have specific energy formulas. An astute student might immediately see that there is yet another way to write this equation:

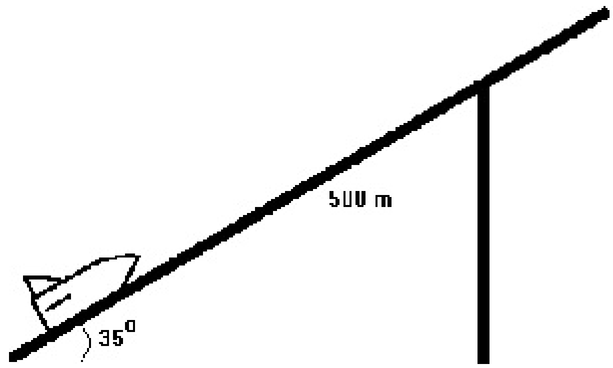
$$T_i = T_f + \Delta U + \Delta H - W$$

Where W is the work done <u>ON</u> the object by other forces. In a much, much later chapter the student will see a variation of the above equation and it will have a special name: the first law of Thermodynamics.

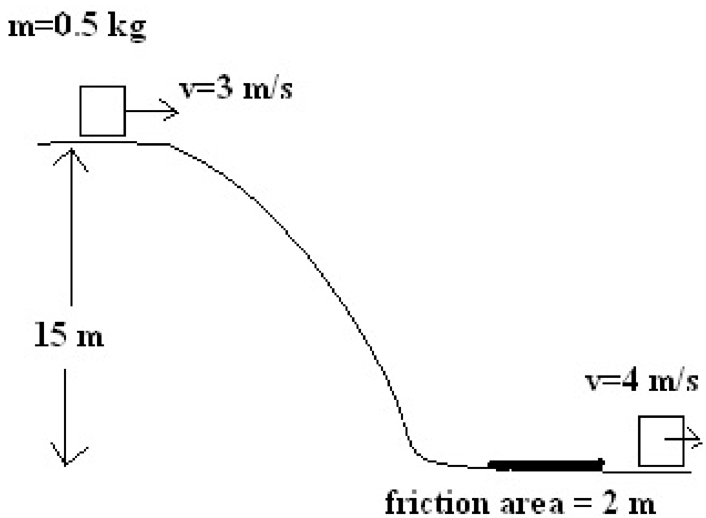
When using this formula, sometimes students get confused about what to include in the work section and what to included in the other sections. A good, simple way to think about this is that every effect on the object should only be counted once. For example, if gravity is counted in the total work done on the object, then you do not include gravity in the change in potential section.

Two examples will help the student to understand how to apply any of the three forms of the same idea listed above.

EX W.) An experimental "sling shot" launch pad launches a rocket as shown below. It applies a force of 10,000 N to the 800 kg rocket as it is accelerated up the launch pad. With what speed does the rocket leave the top?

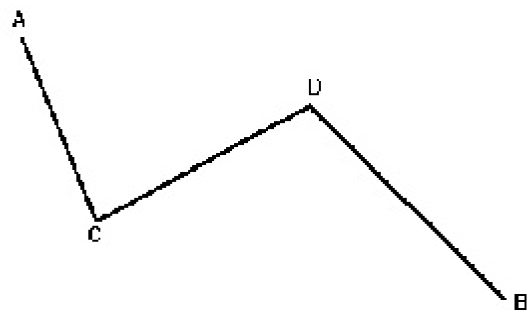


Ex WEEE.) A block is moving on a frictionless surface as seen below. It slides down the hill and then encounters an area of friction as shown. Calculate the force of friction involved and the coefficient of friction in the area.



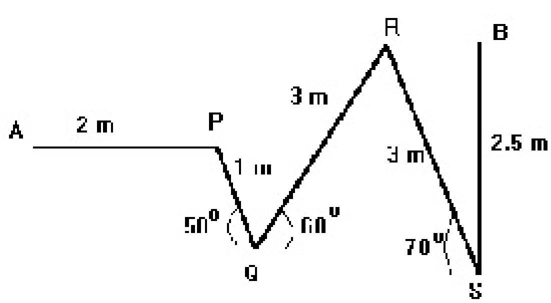
The above examples should have given you an appreciation for how to use the work-energy theorem. Before we leave this section, three notes should be made.

First, the forces that were calculated or used in the previous problems were all average forces. Certainly the earth does not give a continuous force to a meteor as it collides. But since the force is not constant, we can use an average value in the work equation. Secondly, work is additive. If you wanted to find the total work done on a particle going from point A to point B in the diagram below, you could calculate the work from A to C then C to D then D to B and add them all together.

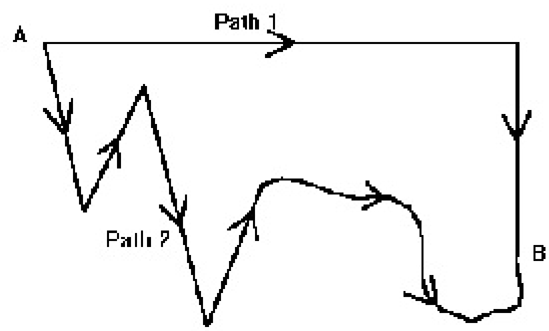


After mentioning the above fact, it might be useful to discuss how to calculate work done in a gravitational field (although an astute student will have already figured this out). Near the surface of the earth, we find the work done by using $W = Fd\cos\theta$, where $F = mg$. An example will clarify this immensely.

EX REEE.) Find the work done by gravity as a 20 kg object moves from A to B on the path below, assuming it is near the surface of the earth and that it follows the entire path with a constant speed.



Finally, work in a gravitational field (or with any constant force) is path independent. Regardless of what path is taken, the same amount of work will be done. In the diagram below, it requires the same amount of work to travel from A to B along either path 1 or path 2.

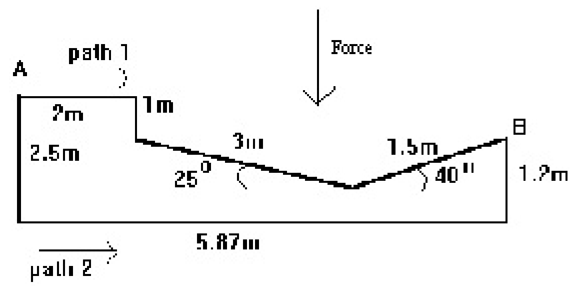


Because of this, you can see that the work done all the way along any closed path must be zero. Consider following path 1 to point B and then following path 2 back to point A. The work done along path 2 would be equal and opposite (why) to the work done along path 1. Your total work would be zero. Whenever you return to the spot from which you started, the total work must be zero. If this is true (as it is for gravity and constant forces) the thing causing the forces is called a <u>conservative field</u>.

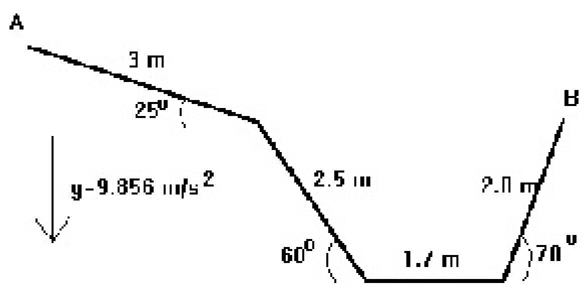
Assignment #19

1.) It is possible, using the formula W = change in Kinetic Energy, to derive one of our equations of motion. Show how this is done. Hint: Use Newton's Second Law. (WO1)

2.) Consider moving a 10 N object near the surface of the earth as shown below. If the object moves from position A to position B, how much work does the field do on the object if (a.) it follows the path marked #1 and (b.) it follows the path marked #2 ? Assume that the object begins and ends at rest. (WO3*)



3.) How much work is done moving a 15 kg object from point A to point B along the path below through the gravitational field given? If it gets to point B with a speed of 16 m/sec and it started with a speed of zero, how much work was done on the object ? (G11*)



4.) A 200 N box is pushed up a 3 m ramp with a force of 160 N of force. If friction exerts a force of 50 N on the box and the ramp has an angle of 30^0, what is:

a.) the work done pushing the box?

b.) the work done by gravity?

c.) the work done by friction?

d.) the work done by the normal force?

e.) the total work done on the box?

f.) the final speed of the box, assuming it started at rest?

5.) A 1500 kg car, traveling at 35 mph hits a large path of gravel in the road and slows down to 32 mph. If the patch was 3 feet wide, what was the average force exerted on the car by the gravel? (WO9)

6.) A winch uses 800 J of energy per second to lift a 10 kg mass 1.5 m vertically in 3 sec (at roughly a constant speed). What was the efficiency of the winch? If all the extra energy was lost to heat in the device, how high would its temperature rise if it was made of 5 kg of iron? (Need some info? Go look it up.) (WO10)

7.) Imagine that you had a frictionless bicycle and disregard air resistance. If you could give 15 J of energy to the bicycle every second by pedaling, how fast would you be going in one minute? In ten minutes? Translate both answers to mph and comment on the validity of the assumptions (Assume a bicycle/rider mass of 70 kg and a flat road).

8.) Decipher: "Scintillate, scintillate asteroid uinific." (DNCTHWG)

Activity #19 - Work and Forces

In this activity, a computer interface will be used to test the work energy theorem. By using a force sensor and a motion sensor, the student will measure the force on an object and the speed imparted and compate the work done to the change in kinetic energy.

Procedure:

1.) Place a cart on the lab table with large weights on top of it (if the cart is too light, a slight force will make the velocity too high and thus inaccurate). Attach a force sensor to the cart and set up a motion detector to measure the position and speed of the cart.

2.) By holding the force sensor, pull the cart across the table with a small force.

3.) Since the force will probably not be constant, using Fd would be difficult without calculus. We will thus use a different technique for measuring the work done. Since W=Fd, it will also equal the area under a force versus distance graph.

4.) Before proceding with this, a time interval should be chosen to consider. Pick an initial time before the cart started moving and a final time when the measurements are still consistent with the experiment (before you stopped pulling, or the cart fell off the table, etc.). If possible, discard all other data points.

4.) Produce a force versus distance graph and find the area under the curve. This technique is called integration (a calculus term). This the work done.

5.) From a velocity versus time graph, find the final time in your interval and determine the velocity.

6.) Find the ΔT for the cart and compare it to the work done. You will, of course, need the total mass of the cart for this calcultaion.

7.) Repeat the above procedure two more times, varying the way the cart is pulled across the table.