Chapter 18: Simple Harmonic Motion

Simple Harmonic Motion

There is one type of motion which has yet to be discussed, yet plays a vital role in the physical world. Any observant student has probably noticed that if you disturb a motionless swing, it will oscillate back and forth. Such motion is common in nature and certain types of repeating motion is called Simple Harmonic Motion.

One of Galileo's most famous discoveries was in this area. While listening to a sermon given by the priest in The Duomo of Pisa, Galileo noticed that a chandelier above him was swinging after being disturbed by a cleric who had lit the candles. He used his pulse to find the time the lamp took to make one complete swing and noticed that as the swings amplitude decreased over time, the amount of time it took to make one swing remained constant. In short, he concluded that the period of a pendulum is independent of the amplitude of the swing. Today, in the Duomo of Pisa there hangs a chandelier called "Galileo's Lamp" which was supposedly the one he observed. Although it is true that Galileo made this discovery, it is doubtful that he did so in the method that the legend describes. For one thing, the church's records show that the lamp was installed ten years after Galileo died and for another, I have never know anyone who does not pay full and complete attention to a sermon. Galileo's discovery was incredibly important, since it led to the improvement of clock construction. Without accurate clocks, neither science, commerce nor athletics would have advanced at the rates that they did.

That type of motion, the motion of a pendulum undergoing small oscillations, is what is called Simple Harmonic Motion. Simple Harmonic Motion is repeating motion that follows a predictable, trigonometric pattern. Thus not all repeating motion is simple. There are other types, but because of their difficulty, we will not be discussing them.

In order to execute SHM, a system must meet three requirements:

Requirement for Simple Harmonic Motion

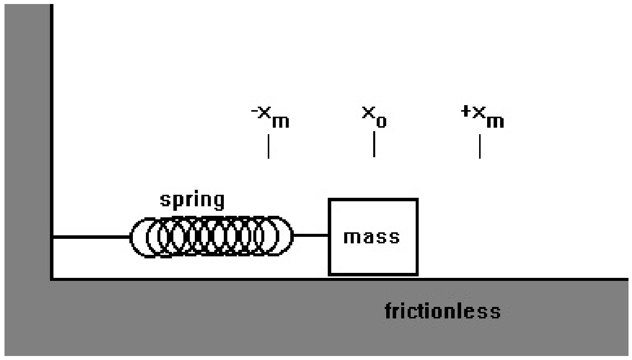
1.)   The system must contain some inertial component (a mass).

2.)   The system must possess stability at some point (there must be a stable equilibrium point).

3.)   The system must have a force that returns the object to equilibrium when disturbed (a restoring force).

Olencik, in his Mechanical Universe series, describes SHM as "natures response to disturbing a stable system". Such a definition is very accurate in describing when SHM occurs.

It is best to talk about SHM by using an example. Consider the set up shown below, consisting of an ideal spring connected to a mass m on a frictionless surface. Points labeled $+x_m$ and $-x_m$ are for future reference.



The above example is very similar to a hanging mass on a spring, however, for a certain reason (the astute student might want to keep this question in mind), we cannot use a vertical situation, since it is not is SHM.

Suppose we pulled the mass out to position $+x_m$ and then released it. What would happen? It would swing back and forth between $-x_m$ and $+x_m$ forever. The distance it was disturbed, x_m is called the <u>amplitude</u> of the disturbance. Notice how it is half the overall distance of motion. The definition of amplitude is actually the maximum displacement from the equilibrium position. In SHM, the equilibrium position is generally labeled x_o.

If we were to do a force analysis on the mass, we would find that the only force that is unbalanced is the force of the spring. We know the force of the spring from a previous chapter is given by: $F = k\Delta x$. But we should also consider the direction of the force. When the mass is on the positive side of x_0 (to the right) the force is in the negative direction (to the left). When the mass is on the left, the force is to the right. Thus the force is always opposite to the displacement. Our force is actually given by:

$$\underline{F} = -k\Delta\underline{x}.$$

It is important to notice that the force depends on which side of x_o the mass is on, <u>not on which direction the mass is moving</u>. When the mass is first released, its force and motion are to the left. When it passes the equilibrium point, it is still moving to the left, but the force begins pointing to the right. After it stops, the force continues to point to the right and the object begins moving to the right. Notice how we have been discussing that the force changes as the object moves (the equation above shows that very clearly since the position determines the force). Thus it follows that the acceleration will change along with the force. Our previous treatments of motion were only appropriate for constant acceleration. <u>SHM is not constant acceleration</u>. We will need to develop some new equations to handle SHM (of course Newton's Laws and all of our definitions still work, we just cannot use our beloved equations of motion).

Two other notes should be made regarding the force shown by the equation above. The formula $F = -k\Delta x$ is called Hook's Law and gives the force from a spring. Any force, be it from a spring or anything else, that obeys this law is called a Hook's Law Force. In actuality, Hook's Law is one of the better definitions of SHM. Simple Harmonic Motion is the motion that results from an object influenced by a Hook's Law force.

To continue our quantitative discussion of SHM, we need to define a number of other quantities. The first is the period of the motion. The period (abbreviated by the letter T) is the time it takes the system to perform one complete oscillation. Notice it is one complete oscillation, not the time from one end to the other (it is measured from when the mass leaves $+x_m$ to when it returns to $+x_m$, not to when it reaches $-x_m$). Obviously it is measured in seconds.

The next quantity is the frequency. Frequency (abbreviated with the Greek lower case letter nu: ν) is the number of oscillations completed in one second. If the oscillator completes four oscillations in one second, it has a ν = 4 oscillations/second. Notice that oscillations are not true units (in much the same fashion that radians are not true units), thus the units are actually 1/seconds. The units 1/s are pronounced either "one over seconds" or "inverse seconds" or "seconds to the negative one" and have a special name. They are called Hertz.

$$1\ 1/s = 1\ \text{Hertz}.$$

This is a very common unit, so always remember what it stands for. You might often hear this unit associated with waves, where it represents the same thing. A radio wave with a frequency of 95 MHz means that the wave undergoes 95,000,000 cycles per second. The unit is named for Heinrich Hertz who first proved the existence of radio waves.

An astute student might already realize that there is a relationship between the period and the frequency of an oscillator. If the oscillator takes two seconds to complete one oscillation (period) then the number of oscillations per second is 0.5 (frequency). If the oscillator takes 0.25 seconds for one oscillation (period) then the number of oscillations per second is 4 (frequency). Therefore, we see that the period and frequency are inversely related:

$$\nu = 1/T$$

and

$$T = 1/\nu.$$

The next quantity to discuss is very important, but requires a little explanation to achieve complete understanding. It is called the angular frequency. The definition of angular frequency (abbreviated with an ω) is the number of oscillations per second the system undergoes, represented in radians per second. What the student needs to do to understand this is to think of a complete oscillation as 2π radians from start to finish. Thus half an oscillation is π radians, etcetera, etcetera. Thus if the oscillator goes through one and a half oscillations in one second, we say that it has an angular frequency of $3\pi/2$ radians per second. Technically, the units are still Hertz, but historically (and common sensically) we still say rad/sec. It is easy to see that:

$$\omega = 2\pi\nu = (2\pi)/T$$

As mentioned, it is very important that the student begin to think of oscillations in terms of radians, since that is the prevailing unit used in physics to describe them.

Another radian related concept that is often used in describing SHM is called the phase shift. The phase shift (abbreviated with the Greek lower case letter phi {pronounced phee} φ) is the position of the oscillator at time zero in radians. Generally, the $+x_m$ position is called $\varphi = 0$ radians and the $-x_m$ position is called $\varphi = \pi$ radians. Imagine that you started the oscillator moving and began timing when the oscillator passed x_o on its first swing. The φ would be equal to $\pi/2$. Now imagine that you started the clock after the mass had already reached $-X_m$ and was on its way back, passing X_o going to the right. The φ would now be $3\pi/2$. Notice how each position on the x axis has two φs, one for each time it passes the point (think about it: in one complete oscillations, the mass passes each point, except the ends, twice.). The concept of phase shift can be confusing, but a little practice will make it clearer to the student.

We have already discussed the forces acting on an oscillator, now we should take a look at how the forces relate to the acceleration. We know that:

$$F = -k\Delta x$$

and

$$F = ma.$$

These two give:

$$-k\Delta x = ma$$

or

$$a = -(k/m)\Delta x.$$

Looking at this equation, we see the following things:

Conclusions About Acceleration in SHM

1.) The acceleration is not constant.

2.) More mass means less acceleration.

3.) The acceleration is always in the opposite direction of the displacement.

4.) Acceleration is proportional to displacement, meaning it is greatest when Δx is greatest (at the ends) and zero at x_o.

5.) Acceleration is proportional to the spring constant. The stronger the spring, the stronger the acceleration.

Most of the above simply make good logical sense.

There are a few other relations to mention, which arise from the fact that $\omega^2 = k/m$ (a result which we will neither explain nor prove). Knowing this, we get:

$$a = -\omega^2 \Delta x$$

and

$$\nu^2 = (k/m)/(2\pi)$$

or

$$T^2 = 4\pi^2(m/k)$$

It perhaps might be helpful to summarize all this information in a table form.

<u>The Equations, Relations and Variables of SHM</u>

Quantity	Symbol	Units	Relations with other quantities
Period	T	seconds	$T=1/\nu$, $T=2\pi/\omega$, $T^2=4\pi^2(m/k)$
Frequency	ν	Hertz	$\nu=1/T$, $\nu=\omega/(2\pi)$, $\nu^2=k/(4m\pi^2)$
Angular Frequency	ω	rad/sec	$\omega=2\pi\nu$, $\omega=2\pi/T$, $\omega^2=k/m$, $\omega^2=-a/\Delta x$
Phase	φ	rad	
Position	Δx	m	$\Delta x=-F/k$, $\Delta x=-a/\omega^2$
Force	F	N	$F=-k\Delta x$, $F=ma$
Acceler.	a	m/sec^2	$a=F/m$, $a=-(k/m)\Delta x$, $a=-\omega^2\Delta x$
Spring Constant	k	kg/sec^2 or N/m	$k=-F/\Delta x$, $k=\omega^2 m$, $k=-ma/\Delta x$

The student can see that there are many equations in SHM and they are all intertwined. It is important that you know that these relations exist (although it is not necessary to know all of the above, since many are combinations or rearrangements of a few, core equations).

<u>The Mathematics of SHM</u>

Although our mathematical abilities do not allow us to solve for the equations of motion for a simple harmonic oscillator directly (that would require a thorough knowledge of calculus), we can arrive at them indirectly through logic.

Let us begin by trying to graph the equation for the position of an oscillator that has a phase of 0 radians (time begins when the oscillator is at $+x_m$). Using our logic, we know that it will begin at a very slow speed and gain velocity until it passes zero, then it will slow down and come to a stop, repeating the procedure in reverse.

EX OSC.) Graph the motion of a simple harmonic oscillator on the graph below.

What we have here is a classic cosine curve. Our equation for such a curve is:

$$X(t) = x_m\cos(\omega t)$$

or, if a phase angle was present:

$$X(t) = x_m\cos(\omega t+\varphi).$$

This equation is very important, since it allows us to locate the particle at any given time.

Now let us turn our attention to the velocity of the particle. We know that it is at its maximum (v_m) when the particle is passing the equilibrium position and at zero at either end. In fact, it seems to do just the opposite of the position graph.

$$v(t) = -v_m\sin(\omega t+\varphi).$$

One note should be made, and that is that there is a relation between the maximum velocity and the maximum position, namely,

$$v_m = \omega x_m.$$

This gives:

$v(t) = -\omega x_m \sin(\omega t+\varphi)$.

Another important relation.

We have already discussed the accelerations relation to the position, this is:

$a = -\omega^2 x$

and substituting to create an equation yields:

$a(t) = -\omega^2 x_m \cos(\omega t+\varphi)$.

It is interesting to graph these equations and draw some conclusions:

EX IUY.) Graph the velocity and acceleration of an object undergoing SHM.

<u>Some Conclusions About SHM Velocity and Acceleration</u>

1.) All graphs have the same general shape.

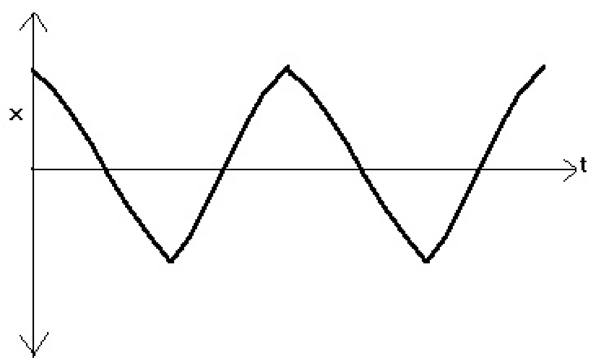
2.) The acceleration graph is the same shape as the position graph, only inverted.

3.) The velocity graph is one quarter cycle out of phase with the position graph.

4.) The acceleration graph is 3/4 of a cycle out of phase with the position graph.

5.) The acceleration graph is one quarter cycle out of phase with the velocity graph.

Before we jump into numerical problems involving these formulas, let us answer a question that the astute student might have. Such a student might ask, "How do we know that the position graph exactly matches the cosine curve. Why can't it look like this?"



The curve suggested fits most of our descriptions (although there are at least two major problems {what are they?}) yet does not match a cosine curve. The answer to the students question comes from calculus. Calculus tells us that without a doubt, any object that fits a Hook's Law force ($F = -k\Delta x$) will always trace out exactly a cosine curve. Calculus also tells us that there is another function that will fit Hook's Law, but it is a complicated, imaginary function. This other function is actually of more use in physics than the cosine representation, but we will not cover it in this class.

EX GGT.) A block of mass 2 kg is set in simple harmonic motion on the end of a spring. If it has an amplitude of 10 cm and repeats its motion 20 times every 5 sec,

a.) What is the period?
b.) What is the frequency?
c.) What is the angular frequency?
d.) What is the spring constant?
e.) What is the maximum speed the object undergoes?
f.) What is the maximum force it experiences?
g.) Write the equation for the oscillator, assuming it starts at position zero at time zero.
e.) Find its location at t = 3.13 sec.

The first example simply served as an opportunity to practice some of the simpler calculations in SHM. Let us go further.

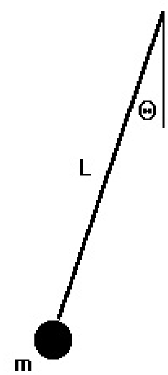
EX TRT.) An object undergoes SHM with an amplitude of 8 cm and a period of 0.2 sec. If the object begins at position x = -3.6 cm at time zero,

a.) write the equation for the oscillator.
b.) write the velocity equation.
c.) write the acceleration equation.
e.) determine its acceleration and velocity when it is at x = 4 cm.

Simple Pendulums

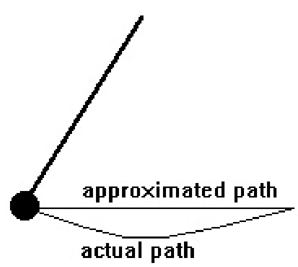
Now that we have some of the major concepts under our belt, we should turn our attention to some other examples of oscillating systems.

The first example is that of what is called a Simple Pendulum. By rigid definition, a simple pendulum is "a particle of mass m suspended by an inextendable massless sting of negligible mass and length l.



We are all relatively familiar with such a pendulum, but the question is, does it undergo SHM? We would decide that by determining the force on the mass m and seeing if it matches Hook's Law. If we did so, we would find that it does not exactly match the law and therefore does not undergo SHM. However, for small displacement angles (less than 20^0) we would find that it does approximate SHM very well. Therefore we will use an approximation technique to solve the motion of the pendulum for small angles.

The way we do this is to imagine that the bottom arc of the pendulum is our x axis and is straight, not curved (see below).



This approximation allows us to define our pendulum's position in terms of x and Δx, as we have done with oscillators.
In this case, a free body diagram gives (it is useful to remember that

although our path is "straight" it is supposed to be slightly curved, so that a component of gravity lies along it):

$$mg\sin\theta = ma_x$$

$$a_x = g\sin\theta.$$

Since $\sin\theta = \Delta x/L$,

$$a_x = g\Delta x/L$$

or

$$a = (g/L)\Delta x.$$

An astute student will recognize this to be the acceleration of a particle in SHM (since it matches the form $a = -\omega^2\Delta x$) and thus for a pendulum we get:

$$\omega^2 = g/L.$$

Rearranging and substituting we find that for a simple pendulum at small angles:

$$T^2 = 4\pi^2 L/g$$

A very important result. There are a number of conclusions that can be drawn from this result.

<u>Conclusions Regarding Simple Pendulums</u>

1.) The period of the pendulum does not depend on the mass of the bob.

2.) In this approximation (small angles), the period does not depend on the beginning displacement (remember Galileo's Lamp?). The pendulum moves faster through long arcs, slower through small arcs.

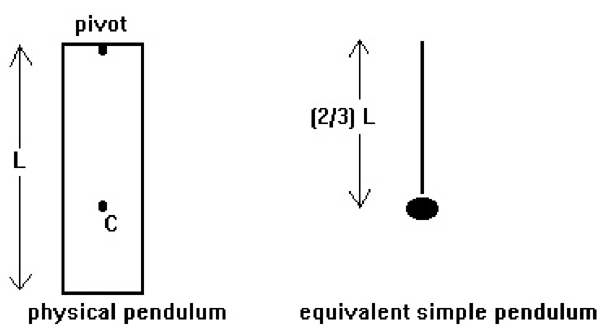
3.) The period is proportional to the square root of the length. Doubling the length will not double the period.

4.) The period depends on gravity (inverse square root). Thus pendulums will have different periods at different locations on the earth.

A few warnings on these conclusions: recall that this was arrived at by an approximation only good for small angles (less than 20^0. Also, these are simple pendulums, meaning the masses are point masses, there is no air resistance or friction and the string will not stretch and is massless.

EX EZZZ.) A simple pendulum one meter in length is set in motion by pulling the mass 10 cm to the right to start it off. What is the a.) position, b.) velocity, and c.) acceleration 8.75 sec later?

A second type of oscillating system that is closes linked with simple pendulums is the physical pendulum. A <u>Physical Pendulum</u> is any solid object that can vibrate like a simple pendulum. Take a solid piece of wood, for example. If we fixed the center of one end, as shown below, it would swing back and forth like a simple pendulum. In fact, an analysis of the time of its period would show that it is vibrating at the same rate as a simple pendulum with a length equal to 2/3 the length of the solid wood. The point C, located at 2/3 the length, is called the <u>center of oscillation</u> or <u>center of percussion</u> and is a very important point.



Two other interesting things about the center of percussion should be known. First, if the board above is flipped and mounted so that it is fixed at point C and allowed to oscillate, the center of percussion will be the previous pivot point. Thus the pivot and the center of percussion make a pair of corresponding points. Secondly, striking the object at the center of percussion will cause the object to try to rotate about the original pivot, even if it is not fixed, or fixed at some other point. This interesting effect is seen (or used to be seen before the advent of aluminum bats) in baseball. Every bat has what is called the "sweet spot" (where the old Louisville Slugger logo used to be branded in the wood). Legend (and experience) have it that if you hit the ball there, with the brand facing up,

1.)   You will get the best hit possible.

2.) You will not crack the bat.

3.) Your hands will not sting.

It seems that legend is based in physics. The sweet spot is the center of percussion for the bat. By hitting it there you ensure that the bat will oscillate around the end and give you the best swing possible. Think of the bat like a lever. The resistance is the collision of the ball, the effort is your hands and the fulcrum is the pivot (third class, no less... I wonder why...). Where you hit the ball will determine where the fulcrum is placed. You want it as far from your hands as possible and certainly not in between them (ouch!). By hitting on the sweet spot, you guarantee the best results.

There are two other aspects of pendulums that are worth mentioning before we go off to the next topic. They are "ghost" pendulums and resonance pendulums. The ghost pendulum is simply a name given for an effect that occurs often when precision experiments are done with a pendulum. No matter what you connect your pendulum to, that support will be affected by the swing of the pendulum. Even if it is connected to a heavy steel beam, swinging the pendulum will cause the beam to swing in the opposite direction each pass. The effect is more pronounced the larger the pendulum is. Considering that to reduce the effects of air pressure, you would need to a large pendulum for a precise experiment, this could be a problem. When experiments of this sort are done, they are usually done with two pendulums, swinging oppositely to counter act the effects. Measurements are take on both and the results averaged.

A resonance pendulums is a system that shows a unique and interesting transfer of energy in a resonant system. We will discuss resonance later, but you will investigate this in the upcoming activity.

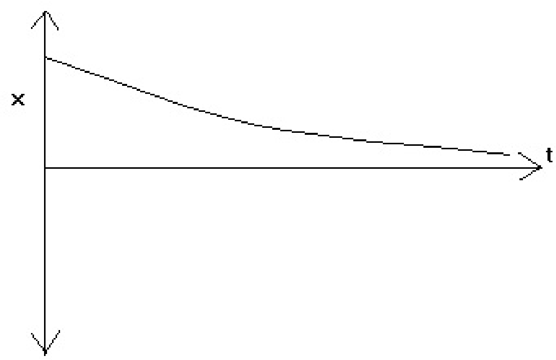
Damped Harmonic Motion

We have discussed simple harmonic motion, but an astute student might have noticed that SHM only occurs without friction. Any time that friction is involved, the motion is not simple, we call it damped harmonic motion. Damped harmonic motion can either be a pain in the rump or beneficial. In a grandfather clock, the damped motion of the pendulum (which is keeping time) is an annoyance that must be counteracted with the use of hanging weights. In a car's shock absorbers, damped harmonic motion is beneficial so that the disturbance caused by a bump will die out quickly and not continue on as you drive away. If the damping is small, the period of the oscillation does not change, so our equations are still valid, except for the amplitude.

DHM gives us a good chance to examine an important concept, combining two functions to meet an expected result. For example, we have said that our equations are still valid except for the amplitude. Therefore, let us propose that the equation below will be used for DHM:

$$x(t) = x_m f(t)\cos(\omega t+\varphi)$$

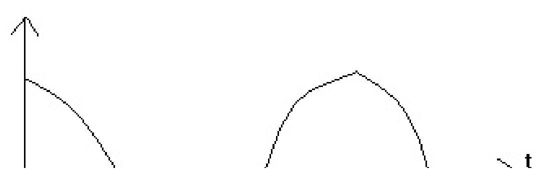
where f(t) stands for some function of time that will cause the amplitude to decrease. We would imagine that f(t) would look something like this:

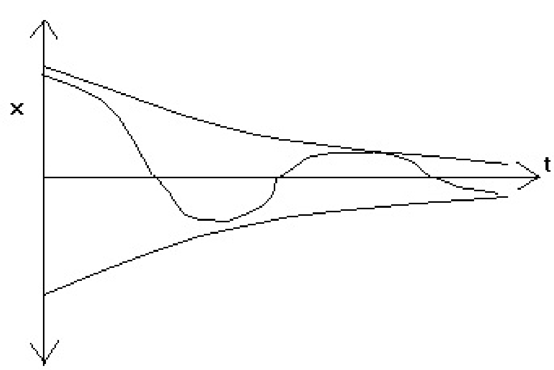


Our mathematician friends tell us that an equation that fits this form is the exponential equation $f(t) = e^{-ct}$, where c is some constant determined by the physical situation. Now our equation for DHM becomes:

$$x(t) = x_m e^{-ct}\cos(\omega t+\varphi).$$

We know our original equation looked like this:





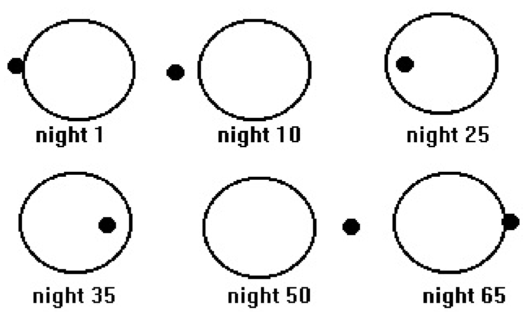
Thus combining them should give this:

From a realistic and engineering standpoint, damped harmonic motion is a very important topic.

Circular Motion and Simple Harmonic Motion

The anal-retentive student might have made a comment such as "why do they use ω for angular velocity and then use it again for angular frequency? I like all my symbols to stand for only one thing so my neat and ordered world does not get confusing." There are two ways to answer that protest. First and foremost, there are more things in the world that need symbols than there are Greek letter. Secondly, and more importantly, there is a connection between the two angular quantities. A historical anecdote will illustrate this.

Galileo (not him again, was there anything this guy didn't do?) was credited to be the first person to use a telescope to make detailed observations of the heavenly bodies. When looking up a Jupiter, he noticed at over a series of nights, an object (actually a number of objects) appeared to move in the neighborhood of the planet. It looked something like the series below.



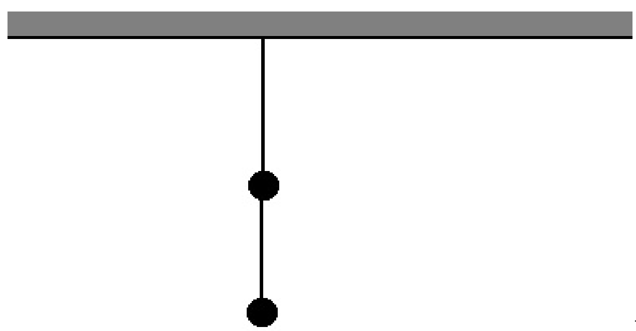
A quick glance tells us that what saw was SHM. However, he reasoned that it was actually the circular motion of a moon around the planet. Thus Galileo was the first person to recognize that other planets have moons, just like the earth does (yet another thing to show the church that the earth was not the center of the universe). The importance of this to us in this chapter, is to recognize that when viewed from the side, circular motion appears to be simple harmonic motion. The Mechanical Universe video series shows this by putting a pencil on the rim of a bicycle wheel and shining a light from behind it while viewing the wheel edge on. You will actually see the pencil executing SHM as the wheel spins around.

Thus the angular velocity of a particle in circular motion is related to the angular frequency of the motion when viewed from the side. An interesting thing to think about is what is the exact relation?

A Side-Track into Chaos Theory and the Double Pendulum

In physics and mathematics, formulae and laws are either linear or chaotic. As more and more research is done, we are finding that much of the world appears to be chaotic. I would like to take a side track using pendulums to show you the difference and to give you a glimpse into what chaos theory is all about.

Imagine a normal, simple pendulum. If you pull it back, say 6 cm and release it, it will swing back and forth very nicely. Imagine timing it for 3 seconds and noticing where it was located. Suppose it was at position 3 cm. Now suppose you pulled the pendulum back 6.0001 cm and released it. Where do you suppose it would be after 3 sec? You would probably guess that is located somewhere very, very close to 3 cm. Now imagine the double pendulum shown below.



It consists of one pendulum attached to the end of another. If this were pulled back 6 cm, we might imagine that it was at -4 cm (all measurements from the bottom bob) at the end of 3 sec. If we pulled it back 6.0001 cm, however, it would not end up anywhere near -4 cm at the end of 4 sec. This is an example of a chaotic system. It is important to note that chaotic does not mean that it is unpredictable or random, it simply means that the slightest little change in initial conditions can lead to very different results. Note that if we pulled it back to exactly 6 cm again, it would be at -4 cm at the end of 4 seconds once again. Also note that it is possible to predict where the bob will be (although much more difficult in than the linear case). A chaotic system is one that is very highly affected by slight changes in the initial conditions. Often times you hear people try to explain chaos theory by saying that "If a butterfly in Japan flaps its wings on Tuesday, that will determine whether or not it will rain in New York on Friday". Although we are unsure whether or not this is true, it is a poetic way of describing the sensitivity to initial conditions present in a chaotic system.

Assignment #18

1.) An object undergoes simple harmonic motion, moving 15 cm each direction from equilibrium 18 times per second. The object is at position -8 cm at t = 0. What is the equation for the oscillator ? (S5)

2.) If a piston in your car undergoes SHM and moves a total distance of 8 cm from the bottom of the cylinder to the top at 5000 RPM (rev/min), what is a.) the period, b.) the maximum speed (in m/sec), and c.) the maximum acceleration (in m/sec^2 and "g"s) ? d.) When does the maximum speed occur ? e.) When does the maximum acceleration occur ?
(S2)

3.) A duck floating on a lake undergoes SHM as the waves pass beneath it. If it follows the equation below, a.) how high does the duck rise with each wave, b.) how many waves pass by each second, and c.) how high is the duck when t = 9 seconds ?

$$\text{Duck} = (8\text{ cm})\cos(3.14t + 1.2)$$

(S4)

4.) The equation below represents an object in SHM.

$$y(t) = 9\cos(12t + 1.3)$$

a.) mathematically determine which direction (+ or -) the object is moving at t = 4 sec and t = 4.6 sec.
b.) make an accurate d vs. t graph of the motion.
c.) make an accurate v vs. t graph of the motion.
(S3)

5.) As the tides rise and fall in a harbor, a ship rises and falls with it, returning to the same level in 12 hours and 10 minutes (this is the time for a full cycle). How long does it take the water to rise from its lowest level to one half of its maximum level (from equilibrium)? (S10)

6.) Imagine that a trapeze artist swings on a 25 m rope. What is the period of her oscillation? If she stands up on the seat and her center of mass raises 65 cm, what is her new period? What is the percent change? Assume this problem can be simplified to a simple pendulum. (S11)

7.) Consider a clock that runs off a pendulum. How long should the pendulum be if the clock is to be accurate at a location where g = 9.800 m/sec? (To be accurate, the period must be one second.) if it is moved to a higher altitude where g = 9.7800 m/sec, how far will the clock be off after one day? one week? Will it be fast or slow? (S13)

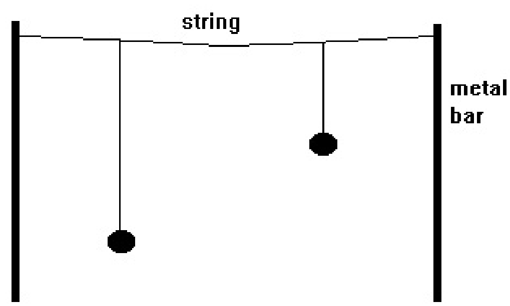
8.) Decipher: "He failed to have a single femur, tibia, or fibula available to support his bulk". (DNCTHWG)

Activity #17 - Strange Pendulums

In this activity, you will investigate and learn about two different types of pendulums; the resonant pendulum and the complex pendulum.

Part I: The Resonant Pendulum

1.) Construct the set up as shown below, where the shorter pendulum is exactly 0.5 times the length of the longer pendulum.



2.) Pull the longer pendulum back and observe the results.

3.) Repeat the procedure, replacing the short pendulum with other pendulums that are 0.75, 1, 1.5 and 2 times the length of the original pendulum. In each case begin swinging the original pendulum and let the other begin at rest.

4.) Draw any conclusions that you can.

Part II: The Chaotic Pendulum.

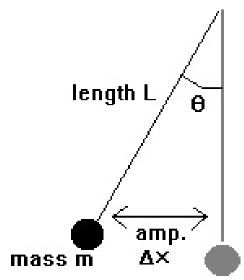
1.) Begin by taking a regular pendulum, swinging it from a set location and noticing where it is located at 3 sec. Do this a few times to get an idea of how accurate you can be in timing. Longer pendulums work best for this experiment.

2.) Set up the chaotic pendulum as shown below and repeat the procedure. Your inaccuracy in measuring the release point should be enough to see a difference, if not, change the position slightly each time.

3.) Draw any conclusions you can.

Lab #15 - The Simple Pendulum

In this lab you will attempt to investigate what factors affect the period of a pendulum. The period of a pendulum is defined as the time it takes to complete one full swing. A full swing occurs when the pendulum goes from point A on the diagram below back to point A, not when it goes from A to B. The factors you will check for in this lab are: mass of the bob, length of the pendulum, and initial amplitude. The initial amplitude is the distance from the equilibrium point that you pull the bob to in order to start it swinging (see diagram).



In this investigation we will only deal with small amplitudes (i.e. the angle theta should not be greater than 40°).

Procedure:

N.B. In the procedure I do not tell you what it is necessary to write down. You must determine that yourself.

1.) Begin by setting up the pendulum with some initial mass and length.

2.) Pull the bob back to some set amplitude and release. Using a stopwatch, time how long it takes to make five complete swings.

3.) Divide the time by five to calculate the period.

4.) Determine which factor to change first. Change that factor by some integer increment (i.e. double it, triple it, half it, quarter it, etc.).

5.) Determine the period of the new pendulum using the method described above and taking care not to change any of the other factors.

6.) Change the chosen factor at least four or five times.

7.) Repeat the procedure for the other two factors.

8.) From your data, determine what factors affected the period of the pendulum. Use graphs to determine the exact relationship, if one existed.

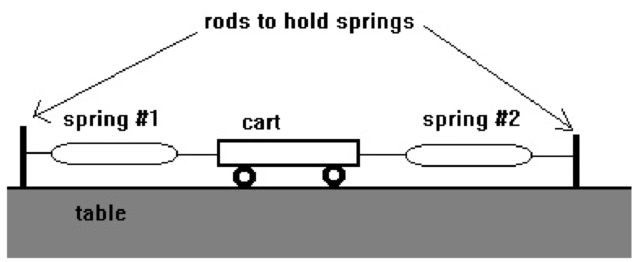
Conclusions: What factors affected the period of your pendulum ? What was the relationship between the period and these factors ? (i.e. was period proportional to $mass^2$ or to 1/mass ?)

Activity #18 - SHM Variables

In this activity, a computer interface will be used to measure all the variables involved in simple harmonic motion: force, acceleration, velocity, position and the spring constant. The measured and graphed values will be compared to theory.

Procedure:

1.) The setup will be the same setup used in lab #7 (pictured below), but with the addition of a motion sensor to measure the motion of the cart.



2.) Set the cart in motion and produce graphs of position, velocity and acceleration versus time.

3.) Use the mass of the cart to change the acceleration versus time graph to one of force versus time.

4.) Using a mass and a mass hanger, determine the spring constants for each spring.

5.) Compare and comment on the correlation between the four graphs compared to what you expect from your knowledge of simple harmonic motion.

6.) Use the force and position graph to find the overall spring constant for the setup. Be careful about how you use the position graph, remember that the equation calls for Δx, not just x.

7.) How does the overall spring constant compare to the individual spring constants measured earlier? Is it a combination of the two? Do spring constants in this situation simply add together?