Chapter 17: Rotation

Torques and Non-Equilibrium Situations

Thus far we have been discussing only how to use torques in equilibrium situations and how the second condition of equilibrium allows us to solve more complicated and more sophisticated force problems. However, the usefulness of the concept of torques does not stop there. Just as forces explained accelerated motion, torques can also explain accelerated rotation.

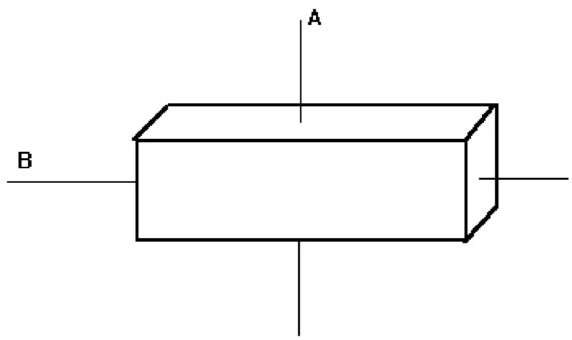
When $\Sigma T \neq 0$, the object is not in rotational equilibrium, meaning it is changing its rotational state (i.e. angularly accelerating). It is interesting to note that we can have cases where $\Sigma T \neq 0$ yet $\Sigma F = 0$. In such a case the object would be staying in one place and angularly accelerating. Any combination of the sum of the torques or force either equaling or not equaling zero is possible. A moments thought should be given to a physical interpretation to each case.

The question then arises as to what the sum of the torques equals if not zero. We would imagine that there would be an alpha (α) on the right hand side somewhere, but what else should be there? Recall $\Sigma F = ma$. In this case we have the summation of the forces equaling the mass times the linear acceleration. For rotation we have a similar equation:

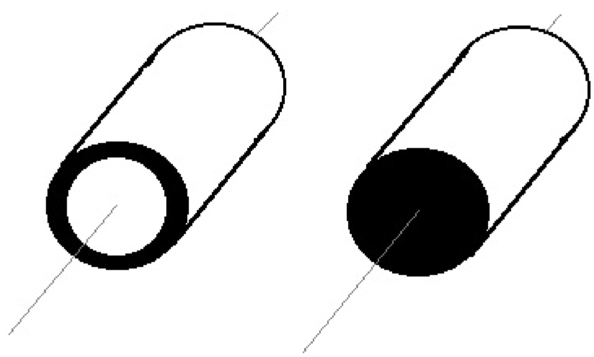
$$\Sigma T = I\alpha$$

The above equation is often called Newton's Second Law for Rotation (or Newton's Second Law in rotational form). We have the sum of the torques equaling I times α. The angular acceleration is not a new concept, but the moment of inertia (I) requires a definition and an explanation.

The <u>moment of inertia</u> (sometimes called rotational inertia or rotational mass) of an object is an objects resistance to a change in its state of motion. Notice how the definition very closely parallels the definition of the mass of an object. This is not mere coincidence, the moment of inertia <u>is</u> the objects rotational mass. Think for a moment about exactly what mass is. The inertial mass of an object is a measure of how hard it is to push an object (and cause it to accelerate). The moment of inertia is a measure of how hard it is to rotate an object. Just like mass, each different object will have a different moment of inertia. But unlike mass, one single object will have many different moments, depending on which axis is being considered. For example, think about a brick. It is harder to rotate the brick about an axis through the face (labeled A on the diagram below) than it is to rotate it about an axis through the side (labeled B).



Thus we would say that it has a higher moment of inertia around axis A than around axis B. Consider another example, two cylinders, one hollow and one solid. Which would have the greater moment of inertia about the indicated axis (imagine that both have the same mass and radius)?



The hollow cylinder has a greater moment of inertia, because the mass is distributed further out from the axis. Objects whose mass is further out tend to be harder to rotate.

When trying to understand the concept of the moment of inertia, always remember its parallel to mass. The moment of inertia is a measure of how hard it is to rotate an object. The following facts should also be kept in mind.

Notes Concerning Moment of Inertia

1.) The moment of inertia is axis dependent. That is, it will be different for different axes through the same object.

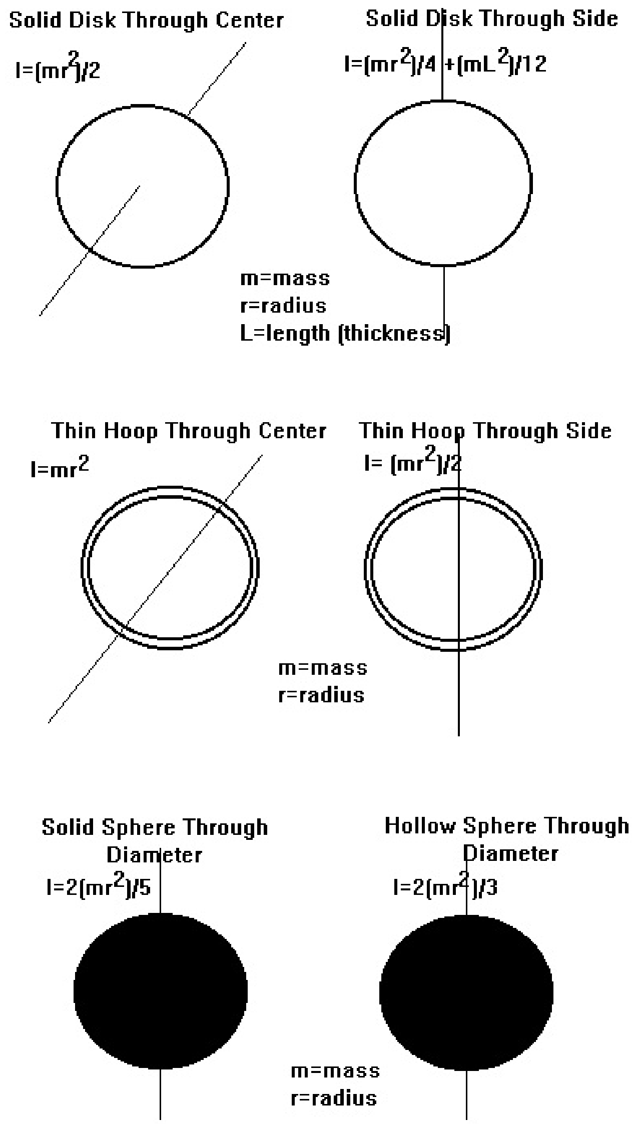
2.) The units for moment of inertia are $kg.m^2$.
3.) The moment of inertia for an object around a particular axis takes into account:
   a.) The mass of the object.
   b.) The shape of the object.
   c.) Most importantly, the <u>distribution</u> of mass around the given axis.
4.) The moment of inertia is not affected by length of an object in the direction of the axis (accept as this might affect mass).

When we say the moment of inertia take the distribution of mass into account, we mean that having more mass in one direction, or having mass far away from the axis will affect the moment of inertia. In fact, common sense will tell you (or experience) that it is harder to rotate an object with mass concentrated far away from the pivot (consider trying to rotate a barbell with the weights on the end versus one with the weights close in).

The fourth point above is rather cryptic and requires some explanation. What it means is that mass distribution along the axis is not a factor for the moment of inertia. An example will best explain this. Consider two solid cylinders, with the same mass, the same radius but different lengths (note how this would require different densities). The moments of inertia for the two would be the same. Length is not a factor. However, two cylinders of the same material with different lengths would not have the same moments since their masses would be different.

A few final words about the moment of inertia. First, remember that it is a measure of how hard it is to change the state of the objects rotation. Therefore, it not only measures how hard it is to rotate something, but also how hard it is to get something to stop rotating. Secondly, when comparing objects like we did with the two cylinders, it is important that only one factor is different. The comparison above is only valid if the two have the same mass and radius. In that instance we were comparing their distributions of mass. Thirdly, always recall that the moment will change depending on which axis your consider.

Although we will not actually derive the formulae for different moments of inertia, some representatives are shown below for common shapes:



Now that we have learned about the concept of the moment of inertia, we can get back to the business at hand, which is learning how to use Newton's Second Law for rotation. Simply put, a torque around a pivot will produce an angular acceleration according to:

$$\Sigma T = I\alpha.$$

Let us jump right into the problems.

EX. B)   Calculate the force necessary (applied at the edge) to cause a disk (r = 10 cm, m = 3 kg, l = 4 cm) to angularly accelerate at 1.5 rad/sec^2 around a.) an axis through its center and b.) an axis through its side.

The previous example was a relatively easy case to introduce you to these concepts. The example below is more qualitative, and shows you a very important fact about rolling objects. Very often, it is the friction between the object and the surface that provides the torque necessary for angular acceleration. Without friction, there would be no torque and thus no rotation. However, the friction involved in rolling is not kinetic friction, but rather static friction. This may seem strange at first, but in actuality, the point on the bottom of a wheel is not moving when the wheel is rotating (huh?! - besides that strange conclusion, the top of the wheel is moving at 2v!). Because of this, we cannot assume that the frictional force is μN. If you recall, static friction in not a constant, but is reactionary, only giving what it needs in any particular situation. Therefore, the force of friction in a rolling problem is oftentimes a variable or unknown.

EX C.) Imagine a disk, sphere and hoop, each of mass m and radius r. If all three were placed on an incline plane with friction, what would be the angular acceleration of each object?

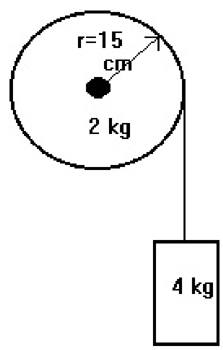
Besides the concept of using friction for the torque on an object, the above example also shows us that if the mass and radii of the shapes are equal, a sphere will always win the race, with the disk coming in second and the hoop in last place. It should also be noted that in the above example we only calculated the angular acceleration of each object. Only because the radii are the same can we say that the object with the greatest angular acceleration will reach the bottom first (why?). Consider the example below.

EX D.) Determine an equation for the time it takes a sphere to roll down an incline plane without slippage.

This particular example is interesting because we can notice that the mass and the radii both canceled. Think for a minute about what this means. All spheres will take the same time to roll down an incline regardless of mass or radius (but not regardless of air resistance). Combining this result with the previous example will lead to some interesting generalizations.

Let us do one last example using Newton's Second Law of Rotation.

EX E.) Consider a pulley made up of a disk with mass = 2 kg and a radius of 15 cm. If a string is wrapped around the disk and the other end is attached to a 4 kg mass, what is the a.) angular acceleration of the disk? b.) the acceleration of the block and c.) the tension in the cord?



Energy and Rotation

In our previous discussion of the conservation of energy, we had to leave out one very important aspect due to insufficient knowledge at the time of the discussion. That aspect was the energy of a rotating object. When an object is in rotation, it possesses a certain amount of extra kinetic energy due to its rotation. This energy is above and beyond the usual kinetic energy due to the objects linear motion ($T=(½)mv^2$) and needs to be included when we apply the conservation of energy. The reason we ignored this energy before was because the formula involves the objects moment of inertia. The kinetic energy of a rotating object is given by:

$$T_r=(½)I\omega^2.$$

Notice the similarity to the formula for the objects linear kinetic energy. To avoid further confusion, the linear, or <u>translational</u>, kinetic energy will be written as T_t and the rotational kinetic energy will be written as T_r. This new factor would be incorporated into our conservation of energy equation as:

$$T_i = T_f + \Delta U + \Delta H + \Delta E_d$$

or

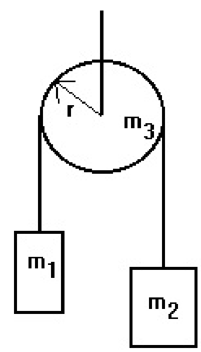
$$T_{ti} + T_{ri} = T_{tf} + T_{rf} + \Delta U + \Delta H + \Delta E_d$$

where the T_i was replaced by the initial translational kinetic energy and the initial rotational kinetic energy (likewise for the final). Using this can make many problems much easier, since there is often a relation between the linear (tangential) velocity (v_t) of an object and its rotational velocity (ω). Consider the example below:

EX F.) Using energy, determine the final speed of a solid ball of mass 2 kg and radius of 20 cm, released on the top of a 1 m long, 30^0 incline (assume it rolls without slipping).

As a further example of how this is useful, we will redo a problem that we have done twice already, this time adding a larger dose of realism.

EX G.) Write an equation for the acceleration of the system below, this time taking into account the mass and radius of the pulley.



By analyzing this equation, we can gain much greater realistic insight into how this system operates. We should also revisit another problem from our past which first appeared in the assignment on energy. Although it was impossible for you to correctly answer part e.) previously, it should be easy to do so now.

EX H.) Consider the following set-up: a 5 kg block is connected to a string and the other end of the string is wrapped around a flywheel (a cylinder on a free spinning axis with a radius of 0.1 m). The block and flywheel are then placed on an incline plane with the flywheel fixed in place as shown below. After the block is released, it slowly accelerates down the plane and when it reaches the bottom, the string comes off the flywheel, leaving it spinning at some angular velocity. The block reaches the bottom with a speed of 1.2 m/sec.

a.) What is the amount of energy given to the spinning wheel by the block?

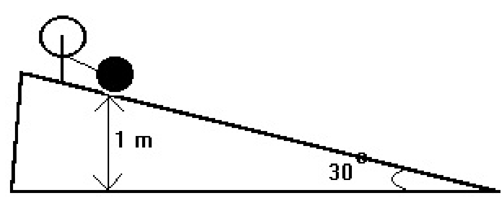
b.) What was the acceleration of the block down the plane?

c.) What was the angular acceleration of the flywheel?

d.) What was the flywheels angular velocity at the end of the run?

e.) What is the mass of the disk?

(E6)



Angular Momentum

Just as we did in energy, we also left out a very important topic when we discussed momentum. Rotational motion has a parallel concept to momentum, namely angular momentum.

To summarize briefly, if there are no external torques on a system, the angular momentum of the system must remain constant. Notice that this says no external torques, not no external forces. It is possible to have a force and exert no torque.

When discussing angular momentum, it is best to break the topic up into two categories, the angular momentum of a single particle and that of a rotating object.

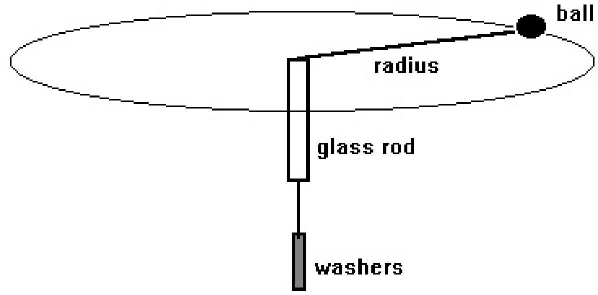
The angular momentum of a single particle is given by:

$$L=mv_t r.$$

Where L is the symbol for angular momentum. There are two stipulations for using this equation. First, this equation holds true only for particles orbiting a central point which is outside the object (for example, a planet orbiting the sun would fit in this category, but not the rotation of a planet). Secondly, this equation is an idealized approximation. In order to use this with accuracy, the orbital radius of the object must be very large in comparison to the diameter of the object. To put this another way, this equation is valid for a point mass orbiting some external point. Let us consider an example.

EX J.) We know that a planet's orbit is elliptical, not circular. What does the conservation of angular momentum tell us about this situation? Determine the speed of the earth at perihelion given the following information:

EX K.) On the apparatus below, imagine the weight on the bottom is greater than what is needed to provide the centripetal force to keep the ball in orbit. We know then that the weight will fall. Is angular momentum conserved? How does this result manifest itself?



The above problem is a good indicator of whether or not you know when angular momentum is conserved. There is an outside force affecting the motion of the ball (namely the tension in the string), but there are no outside torques affecting the object. A better way of saying this (albeit a bit more difficult to understand) is to say that there are forces in the r direction, but none in the θ direction which would affect its angular momentum.

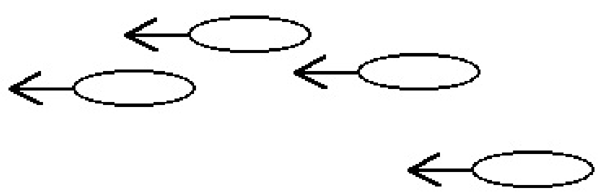
What is perhaps the most interesting aspect of angular momentum is the cases involving objects rotating about a central, or internal, axis. This is often called the case of a rotating, rigid body. If the object rotates about a central axis, the angular momentum of the system becomes:

$$L=I\omega.$$

Where I is the moment of inertia of the object and ω is the angular velocity of the object (we might note that this is equivalent to $L = mvr$ since the I for a point mass at some distance r is mr^2 and $\omega = v/r$). This angular momentum, like the angular momentum of a single particle, is conserved if no outside torques act on the system. It is worth rephrasing and underlining this last point: <u>If no outside torques act on an object, the combination of I times ω must remain constant</u>. This can lead to some very interesting results.

Consider a figure skater doing a spin on the ice. Often they will pull in their hands and this causes them to spin faster. Why? Since angular momentum is conserved, pulling in their hands will lower their moment of inertia and thus raise their angular velocity. The same effect can also be seen to some extent on divers as they tuck during a flip (notice in this case there is a force, but no torque about their center of mass - the same effect is also noticeable in gymnastics).

Another good example is the spiral on a football. When a football is thrown, it is not simply tossed, it is spun. This gives it angular momentum. If no outside torques act on it, that angular momentum must remain the same. This keeps the ball steady and facing the same direction (see diagram below).



Notice how in the diagram, the arrow representing the direction of the balls orientation (not motion) remains the same even though it is moved around. Recall that two vectors that point the same direction are the same regardless of their starting points. This means the orientation, the spin and the angular momentum are all in the same direction all the time. The exact same concept explains the stability of a Frisbee, a boomerang, and is employed to keep satellites stable as they orbit the earth (they are spun as they are released).

The above examples are all cases of giving an object extra stability (keeping its orientation the same) by giving the object a spin (thus giving it angular momentum). Once it has angular momentum, it requires an external torque to change it orientation. It should also be noted that the faster it spins the less effect a disturbance will create. Have you ever noticed that it is difficult to keep your balance on a bicycle that is stationary but easy once the bike is moving? When the bicycle is in motion, each wheel has angular momentum. The wheels will therefor keep their orientation and the faster you are going, the less effect disturbance (small bumps in the road) will have.

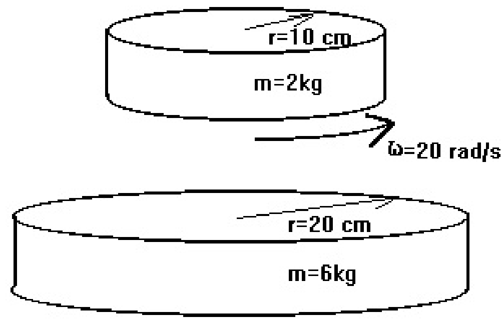
Another interesting application of this would be an astronaut trying to unscrew a screw on the outside of their space vessel. We have learned that they will rotate one way, but to conserve angular momentum (it stated at zero) the space ship must rotate the other (not at the same angular velocity though, why?).

Have you ever noticed that the front end of your car rises if you hit the gas suddenly? The reason lies in angular momentum. Before you hit the gas, you had no angular momentum, but after, the engine is turning and the back wheels are spinning. To conserve angular momentum, your car tries to rotate the other direction (so why don't you flip?).

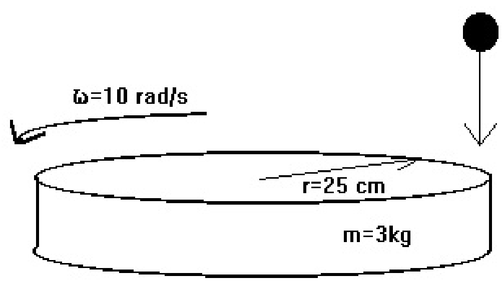
As a final, conceptual example of the conservation of angular momentum, consider a helicopter. All helicopters have a small, sideways blade rotating on the back. What is the purpose of this blade? Imagine a helicopter trying to take off without this blade. What would have to happen to conserve angular momentum? The blade provides an outside torque that allows the angular momentum of the blade to change without changing the rest of the system to compensate. It is interesting (but scary) to ponder what would happen if the helicopter lost its back blade in flight.

Now that we understand angular momentum, let us attempt some numerical problems.

EX. L.) Two wheels, assumed to be uniform disks, are arranged as shown. The top disk is spinning at 20 rad/sec and the bottom disk is still. The top wheel is dropped onto the bottom wheel and sticks tight, without slipping How fast is the combination moving after this occurs?



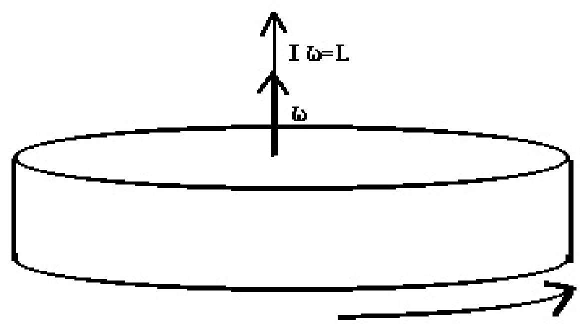
EX. M.) A disk (shown below) rotates at 10 rad/sec when a dense glob of clay is splattered on the top edge. If the clay has a mass of 0.5 kg, what is the final angular velocity of the set-up?



Before we leave this fascinating topic, one more comment should be made. Angular momentum is a vector, much like torque was vector. The direction of the angular momentum is seen in the equation where:

$\underline{L}=I\underline{\omega}$.

We see that I is a scalar and ω a vector, leaving L in the same direction as ω. You may recall that the direction of ω is actually perpendicular to the plane of rotation as we discussed earlier. Thus, for a rotating disk, the ω and L vectors look like this:



We see then that the arrows drawn in the football picture actually are the vectors for the angular momentum of a football spinning such that the top of the ball is going into the page.

Assignment #17

1.) Consider a disk of radius 15 cm and mass of 2 kg. What force would have to be applied to its edge in order to get it to rotate at 5 rev/sec from a dead stop in 4 sec?

2.) The crazy arch-villain, Brainlessman, attempts to stop the earth from rotating by attaching a huge rocket engine on the earth and firing its thrusters against the rotation. If the rocket engine has a thrust of 10 billion Newtons, how long would he have to fire it in order to stop the earth from rotating?

3.) Given that the mass of a baseball is 225 g and the radius is 3.5 cm, how much force does a pitcher put on the ball just to provide the torque for a fast ball to spin at 4500 rad/min after contact with his hand for only one tenth of a second? (assume the force was constant)

4.) A small model rocket engine is attached to the end of a 35 cm metal rod that is secured by a pin on its other end so it is free to rotate. When the engine is fired, the blast produces a force of 12 N for 2 sec. What is the velocity of the engine after the blast stops? If the system spins for 25 sec after the blast, what was the average force of friction acting on the pivot? Assume the friction acted on the edges of the pin at 0.5 cm from the center. (the mass of the rod is 400 g, the mass of the engine is negligible and the moment of inertia of a rod about an axis through the end is given by: $I=(1/3)ML^2$) (T14)

5.) Consider the old method of grinding wheat by using a large, horizontal, stone wheel and horses. If the wheel was 2000 kg and had a radius of 1.3 m, what force would each of four horses have to apply if they were attached at 1 m from the edge to get the wheel to accelerate to 1 rad/sec in 20 sec? Consider friction to be acting at r = 25 cm with a force of 3000 N. How much force will the horses have to give to keep the wheel rotating at the same speed? (T15)

6.) What is the kinetic energy of a car traveling at 55 mph? Assume the car to have a mass of 1500 kg, the wheels to have a mass of 10 kg each and a radius of 20 cm. Remember to include the rotational kinetic energy of the wheels (use the moment of inertia of a disk as an approximation)

7.) A ball is thrown up in the air with 60% of its energy in rotational spinning. If it rises to 30 m and then returns to the ground without its spin being significantly affected, how fast was it spinning? (mass=3 kg, radius=5 cm)

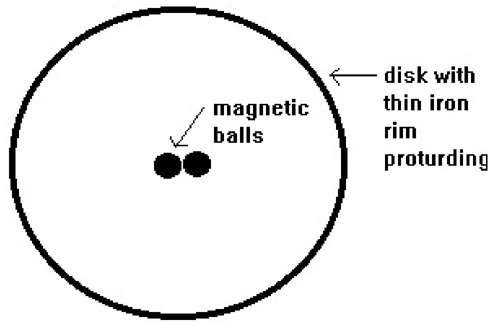
8.) A ball of mass 1.5 kg and radius of 10 cm is rolled up an incline at a linear speed of 2 m/s. If the incline is at a 40^0 angle, what distance up the incline does it travel? (no losses to friction)

9.) If a pitch is thrown in a baseball game at 80 mph and spinning at 5000 rad/min, what is the kinetic energy of the ball (mass = 225 g, radius=3.5 cm)? If this could be turned into pure linear kinetic energy, what would be the maximum range of the ball if thrown? (ignore the height of the pitcher)

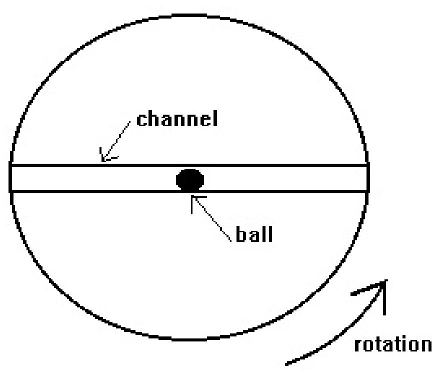
11.) Consider an astronaut of mass 80 kg that is standing on the outside of the cylindrical shell of a space station under construction (using magnetic boots). If the station has a radius of 20 m and a mass of 2000 kg and the astronaut attempts to run at a speed of 1 m/sec, (a.) how fast will the shell turn in the other direction and (b.) what will be the astronauts actual speed relative to the stations surface? (consider the station to be a hollow cylinder without ends)

12.) One method of causing a space ship to rotate is by having a heavy flywheel inside attached to a motor. When you want to rotate the ship one direction, you simply run the motor in the opposite direction and the ship will react by spinning the other way. If the ship has a rotational mass of 40 $kg.m^2$ and the flywheel has a rotational mass of 0.006 $kg.m^2$, how fast will the ship be rotating if the flywheel spins at 3000 rev/min? How do you stop this rotation?

13.) Consider the setup below. Two magnetic balls of mass m and radius r are positioned in the center of a spinning disk of mass M and radius R with a thin iron rim around its edge that sticks up over the face. While the disk and balls are spinning, a small explosion sends the balls outward to the edges where they stick to the rim. What is the ratio of the final angular velocity to the initial angular velocity of the apparatus? Ignore any frictional considerations but do not count the radii of the balls as negligible. (the moment of inertia of a sphere about an axis on its edge is: $I=(7/5)mr^2$).



14.) Imagine a disk of mass 1.5 kg and radius of 20 cm with a channel cut through its diameter is set in motion on a frictionless surface with an angular velocity of 7 rad/sec. If a 150 g ball with a radius of 1.5 cm is placed in the center of the disk, it will naturally stay there and rotate. However, imagine that a slight wind comes up and disturbs the ball. It will then shoot down the channel and fly out then end. What would be the angular velocity of the disk and the linear velocity of the ball after it left the channel? (hint: use angular momentum first, then conservation of energy)



15.) Consider a pocket watch of the following construction: watch body; 200 g, diameter of 3 cm, considered a solid disk for this problem, the hands are thin rods, 1.0 cm long, radius 1 mm, mass of 1.5 grams each, moment of inertia given by $(1/3)ML^2$. If the watch was placed on a frictionless surface, beginning with an angular momentum of zero, how far would the watch body rotate in one hour? In one day?

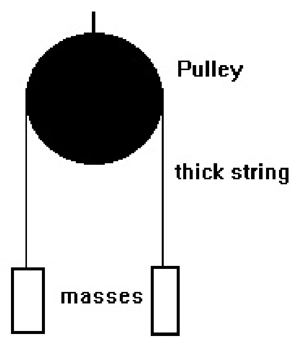
16.) Decipher: "The person presenting the ultimate cachinnation possesses thereby the optimal cachinnation." (DNCTHWG)

Lab #14 - Rotational Inertia

In this lab, you will use what is commonly (in physics, at least) called an Atwood's machine. It is simply a large pulley with different weights on each side. The pulley is large enough so that you cannot ignore its rotational inertia when the weights move. In essence, you are redoing the "Pulley" lab and factoring in the effects of a massive pulley.

Procedure:

1.) Set up the equipment as shown below, with 2 kg of mass on one side and 1 kg on the other. Be sure to place books under the masses to protect the floor and be sure the masses are securely fastened to the string (you don't want 2 kg to fall and hit you on the head). ALWAYS KEEP CLEAR OF THE AREA BELOW THE MASSES.



2.) Using a stopwatch, time how long it takes for the heavy mass to move from the pulley to the ground. Remember all the warnings and suggestions from the previous lab.

3.) Repeat the procedure with 4 times and determine an average time for this set of masses.

4.) Repeat the procedure 3 more times with other combinations of masses.

5.) Be sure to measure the distance traveled by the heavy mass.

5.) Using your equations of motion, Newton's Second Law and your knowledge of rotation, determine the moment of inertia for the pulley. You should do this once for each set of masses using the averaged time.

6.) Measure the mass and radius of the pulley.

Hints on Conclusions: How well do all of your different moments of inertia agree? Which do you feel is the most accurate? Why? Compare your moments of inertia against the theoretical for a disk and a hoop (using percent error)? Which one is the pulley closer to? Why? We know there is friction in the axis of the pulley. How would this affect the measured moment of inertia? Keeping this in mind, which was a better approximation, the disk of the hoop?