Chapter 16: Torques

Torques

In our discussion of motion and forces up to this point, we have focused entirely on concurrent forces, which you may remember are forces that act on only one point on the object. However, as you know, there are often times when forces act at different places on the object and this must be taken into account if we are to present a realistic picture of the effects of forces. For example, consider two people pushing a desk. If they push as shown in figure A below, there will be a different effect on the desk than if they push as shown in figure B (even though the forces are identical).

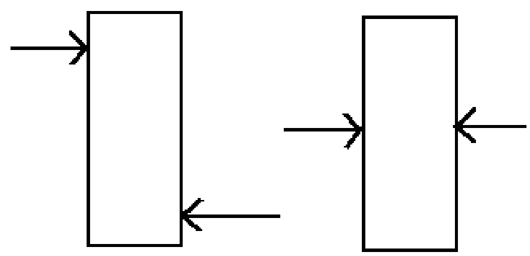


figure A figure B

Our previous treatment of forces required that we draw all forces from the center of the object, but we see that this is not realistic. We must now begin to deal with non-concurrent forces. By imagining the situation shown above, we can see that if both forces are equal, the $\Sigma F_x=0$ for both cases. However, in figure A the desk will begin rotating. This tells us two things: first that $\Sigma F=0$ is not a sufficient condition for total equilibrium in and of itself and that we must investigate the connection between forces and rotation.

If we intend to discuss rotation, we must first choose what is called a pivot point. This point will be considered the center of rotation, but what is not obvious is that the choice of a pivot point is arbitrary. In the example above, the most logical pivot point would be the center of the desk, however, since the point is not a physical thing, but rather a mathematical abstraction, we are free to call one of the corners the pivot. Although the choice is arbitrary, choosing an illogical pivot will often make the mathematics in the problem either incredibly difficult or impossible.

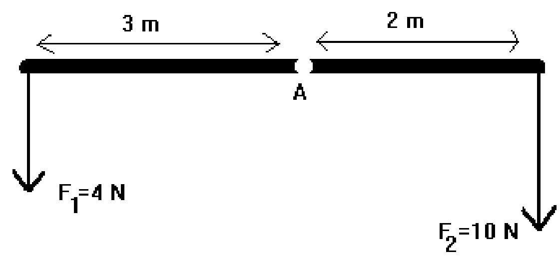
Forces do not cause rotation, but they are part of another concept that is responsible for rotation. Torques cause rotation.

A Torque (abbreviated T, the capital Greek letter tau) is the product of the force times the torque arm of the force. The torque arm is the perpendicular distance from the pivot point to the place where the force is applied. Mathematically,

$T = Fd_{\perp}$

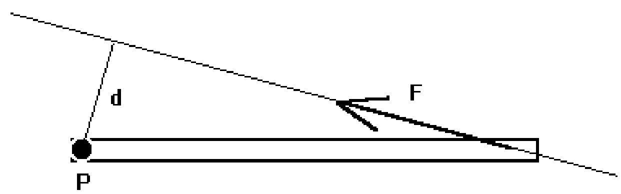
where F is the force and $d_{\perp}$ is the perpendicular distance between the force and the pivot. We will discuss what is meant by perpendicular distance in a short while. For now, let us look at the units of torques, which are Nm or Newton-meters. The best way to understand these things is to consider an example:

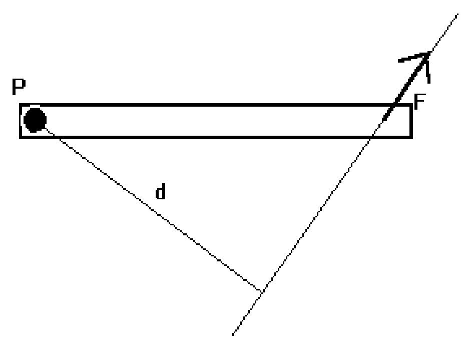
EX EY.) What are the torques around pivot point A on the bar below?



The above example shows how torques are calculated and it also shows that it is necessary to always choose a pivot point before calculating torques. In fact, without a pivot point, the concept of torques is meaningless. Torques also have a direction, since they are vector quantities. For now, we consider the choices for directions to be either clockwise or counterclockwise, just as we did for angular velocity (in reality, the directions for torques are actually perpendicular to the plane of the force and the distance, but that is another story...). In determining the direction for a torque, it is usually most helpful to consider what direction the object would rotate without the torque present. You then know that the torque must have been working in the opposite direction to counteract this effect. It is usual to assign a positive to counterclockwise and a negative to clockwise.

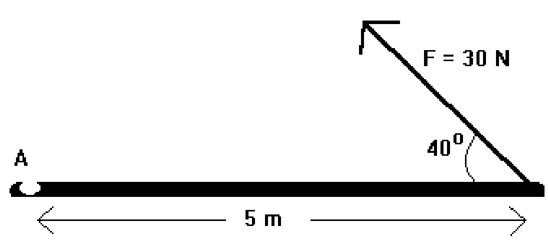
Before we do any further examples, we should return to the concept of perpendicular distance versus regular distance. The perpendicular distance from the pivot to a force is not always the distance given. We find the perpendicular distance in the following manner: First, a line is drawn in the direction of the force, extending both in front of and behind the force. Second, a perpendicular is drawn from the pivot to this line. This perpendicular is the perpendicular distance. Two examples are shown below. In each case the force is labeled F, the pivot P and the perpendicular distance labeled d.





Thus we see that the perpendicular distance will always be either equal to or less than the straight line distance from the force point to the pivot. With this concept in mind, let us do the next two sample problems.

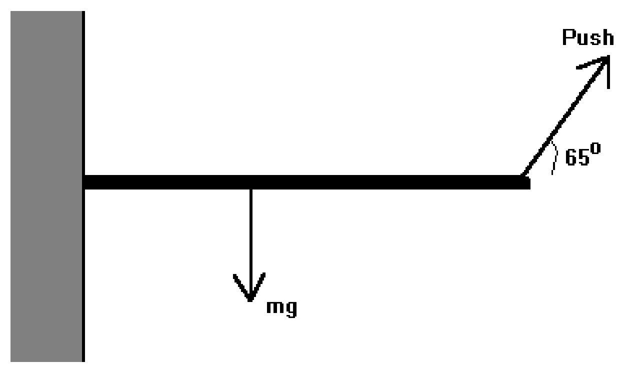
EX EZ.) If a bar is pushed as shown, what is the torque around pivot point A caused by the force F?



X FA.) What is the torque around pivot point A caused by force F?

This last example shows us that forces that pass through the pivot point produce no torque around that pivot. Notice how the force is not exactly <u>on</u> the pivot, but is simply pointed towards the pivot (a similar situation would arise if the force pointed away from the pivot).

EX FB.) Evaluate the sum of the torques around the point shown below where the bar is attached to the wall. The bar is 1.0 m long with a center of mass 40 cm from the wall and is being pushed up with a force of 6 N. The bar weighs 3 N.



In the example above, you had to take into account the weight of the rod itself, something that was ignored in problems in the past because we couldn't deal with it at the time. Notice (although it should go without saying) that the weight of the bar is acting at it's center of mass. Thus, the weight of an object will create a torque around any pivot point that is not located at the center of mass. This is an important concept.

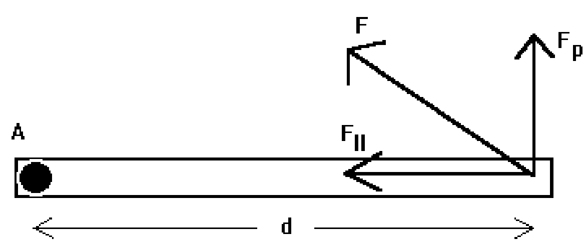
Calculating torques requires that you know the force applied and that you solve for the perpendicular distance. There is, however, another way to determine the torques, which is equivalent, but perhaps a little more understandable (or perhaps not, it depends on which way you see things). Besides a torque being equal to:

$$T=Fd_{\perp}$$

we can also use:

$$T=F_{\perp}d$$

which says that the torque is equal to the perpendicular force times the distance. Consider the situation below. Instead of calculating the perpendicular distance we can break the force up into two components, one headed straight towards the pivot and one perpendicular to a line from the pivot to the force. The first component, $F_{\parallel}$ (for force parallel) does not contribute to the torque around A. The second component, F_p (for force perpendicular) supplies the only torque around A and its perpendicular distance is simply d.



In short, we break the force into parallel and perpendicular components and then throw out the parallel part since it does not contribute. In reality this is the same process as the perpendicular distance process.

EX FC.) Redo the last problem using the method of perpendicular forces.

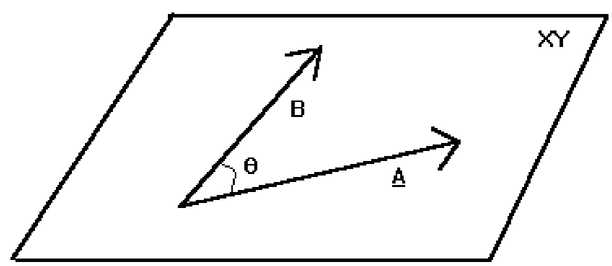
The True Nature of Torques

The reason the two methods are equivalent is due to the fact that neither one is the true method for determining torques. Torques are actually our first real exposure to vector multiplication. In reality, a torque is defined by:

$\underline{T}=\underline{r}\times\underline{F}$

where $\underline{T}$ is the torque vector, $\underline{r}$ is the distance vector running from the pivot point to the point of action for the force and $\underline{F}$ is the force vector. x stands for one type of vector multiplication called the cross product.

The vector cross product is defined as a method of multiplying vectors that yields a new vector as the product of two vectors. The magnitude of the new vector is given by a mathematical equation and the direction of the new vector is given by the Right Hand Rule (to be explained shortly). To understand this, consider two vectors as shown below, lying in plane XY. The angle marked θ is the angle between the two vectors (in the case of the force and the distance, you must move the force vector - remember we can do that - so that it shares an origin with the distance vector).



If we have the case where:

$\underline{A}\times\underline{B}=\underline{C}$

then the new vector, $\underline{C}$ will have a magnitude given by:

$|\underline{C}|=|\underline{A}||\underline{B}|\sin\theta$.

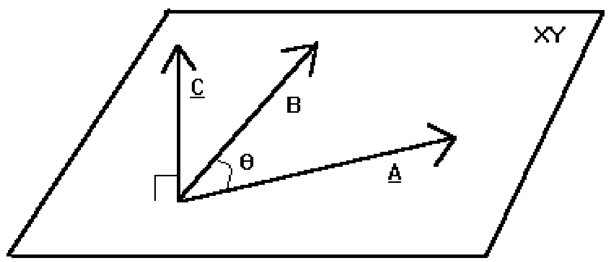
All that remains is to determine the direction of the new vector. It turns out that the cross product produces a new vector that is perpendicular to both the original vectors, in other words perpendicular to the plane that contains them. The direction is given by the right hand rule, which has many variations. Below are two equivalent statements of the right hand rule:

The Right Hand Rule

1.) To determine the direction of the cross product, imagine turning a regular screw with a screwdriver in the direction from the first vector to the second (from A to B). The product is in the direction the screw would naturally move.

2.) To determine the direction of the cross product, lay your pinky of your right hand along the first vector in the product (in this case A) in such a manner that you other fingers can curl towards the second vector (B). The direction of the product is the direction your thumb is pointing.

Although these rules seem hard to understand, you should try to familiarize yourself with them so that you can use them. Both rules give the result shown below for the new vector C. Both rules also show you that A×B≠B×A (why?).



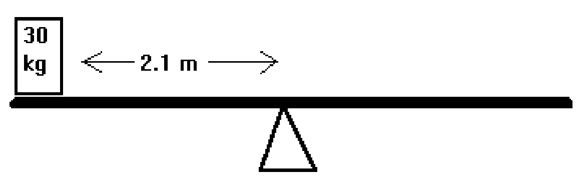
One thing to notice about the cross product is that it is a way to multiply two vectors together such that it returns to you an answer indicative of how perpendicular the two vectors are. For example, if the two vectors are exactly perpendicular, it returns the maximum value of $|A||B|$ or simply the length of A time the length of B. If the two vectors are parallel, it returns a zero (if they are neither, it returns to you a fraction of the value of $|A||B|$ depending on how perpendicular they are). This is an important fact to remember about the cross product since the other form of vector multiplication will do just the opposite.

Now that you know how the cross product works, you will not need to necessarily use it in this section. All of the torque problems can be done by using our simplified method involving either the perpendicular force or distance. However, the cross product will return to haunt you in a later chapter.

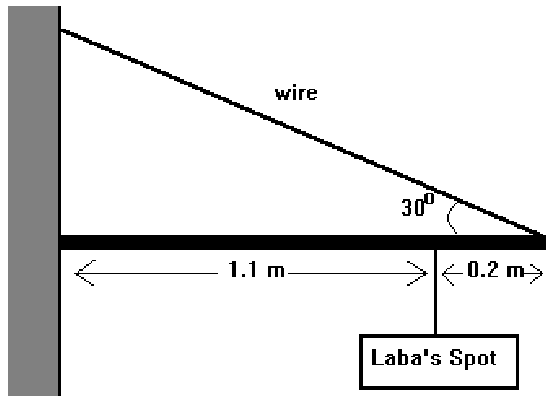
Rotational Equilibrium

In previous chapters, we have discussed the concept of equilibrium and forces and we stated that the condition for equilibrium was $\Sigma F=0$. You may remember, that we stated that this was only the first condition for equilibrium. The sum of the forces equalling zero only guarantees that the center of mass of an object is not moving linearly. It is possible to have the sum of the forces equalling zero while the object rotates (or even angularly accelerates). Thus, we need a second condition for equilibrium; $\Sigma T=0$. This second condition for equilibrium, the sum of the torques equalling zero, insures us that the object is in rotational equilibrium. That is, the object is either at rest or rotating at a constant angular velocity (ω). In other words, the object is not angularly accelerating. If we say an object is in equilibrium, we are implying both $\Sigma F=0$ (the object is in translational equilibrium) and $\Sigma T=0$ (the object is in rotational equilibrium). This second condition allows us to solve many more problems than possible with only $\Sigma F=0$. Since $\Sigma F=0$ broke down into $\Sigma F_x=0$ and $\Sigma F_y=0$, we had two equations to work with, thus we could solve for two unknowns. Now, $\Sigma T=0$ gives us another equation, allowing us to solve for three unknowns. This condition also allows us to solve many problems more easily than in the past. If we consider $\Sigma T=0$, we might ask, around what pivot does this apply? The answer is around any and all pivot points. Since the choice of a pivot point is entirely optional, a judicious choice can make the problem much easier, by eliminating forces that are of no concern to us in a particular problem. Doing a few examples will make this much clearer.

EX FD.) Consider the see-saw below. What force is necessary to balance the see-saw by pressing down at a point 1.4 m to the right of the center? What is the normal force on the see-saw from the support? (assume the see-saw is massless)

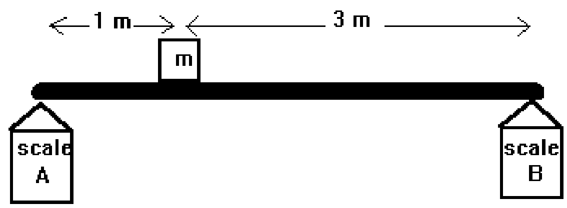


EX FE.) What is the minimum amount of tension that the wire in the set-up below should be able to withstand? The bar weighs 20 N (with the center of mass 0.7 m from the wall) and the sign weighs 60 N.



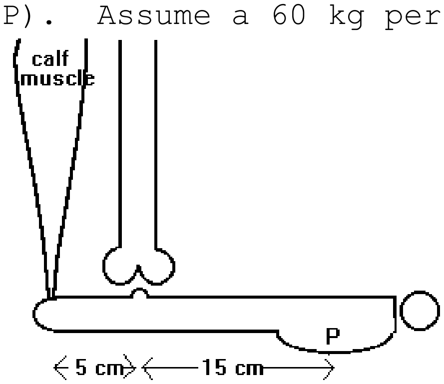
Doing the last example shows us that often there are forces in a problem that appear to be hidden (consider the forces from the wall). When these situations arise, it is often helpful to imagine what would happen if the object causing the forces was not present. In this case, the bar would fall and swing to the left. Therefore, the wall must be exerting a force up and to the right. Using this type of logic will help you to identify all the forces in a problem. It also shows us that there are problems, of which this was one, which could not be solved by using Newton's Second Law alone. A judicious placement of the pivot point eliminates those pesky forces and makes the problem relatively easy.

EX FF.) Suppose we had a 15 kg mass (m) resting on a uniform plank of mass = 10 kg as shown. What would each scale read?

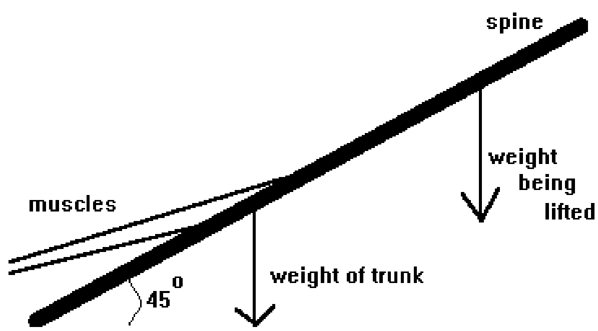


Torques, and rotational equilibrium, have many applications in the human body. The following two examples are meant as extremely rough approximations that not only demonstrate how to solve torque problems, but also demonstrate how incredible the human body is. Please note, these are rough approximations of what goes on in your body, do not take them too seriously and do not use them to try to perform an operation on a person.

EX FG.) Below is a schematic diagram of the bones in persons foot. Calculate the force that the calf muscle must exert so that the person can stand on tip-toes (when the entire weight of the person is balanced on point P). Assume a 60 kg person.



EX FH.) Assume the rough diagram below illustrates the forces acting on your back when you pick up a 44 N object by bending over. The back muscles will be approximated by using just two muscles, one attached at 20 cm from the base and making an angle of 20° and the other attached at 40 cm with an angle of 15°. Also assume that the center of mass of the trunk (441 N) is located at 30 cm from the base and the arms are attached at 50 cm.



Before we leave the topic of torques, it is worthwhile to spend a few minutes discussing a few of the many ways we use torques to our advantage in everyday life. The idea of torques is one of the most easily recognizable and common physics concepts we can see in use all around us. Let us examine these ideas by answering a few questions.

EX. FHA.) Discuss how torques are used in screwdrivers. How does the length of a screwdriver affect its torques? How does the thickness of a screwdriver affect its torques?

EX. FHB.) Discuss how torques are used in wrenches. What is a torque wrench? What is the benefit (and drawback) of using a breaker bar?

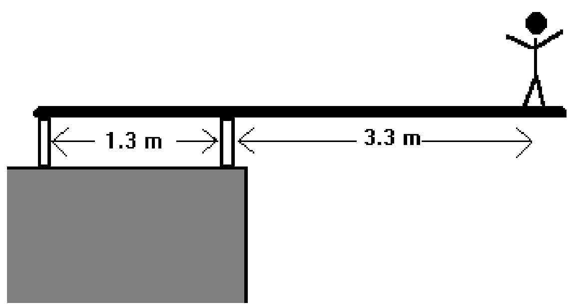
EX FHC.) How are door knobs positioned to provide adequate torque? There are two parts to this answer, one for the door and one for the lock. Discuss both.

EX FHD.) How are levers used to manipulate force and distance (torques) to better effects. Three examples include: an old fashioned car jack, a bottle opener and a broom.

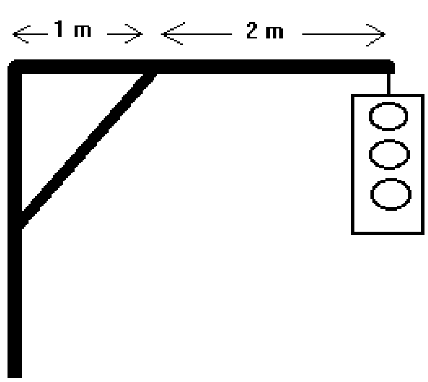
Assignment #16

1.) Suppose you wished to raise your car using the lever shown below. If the cars mass is 1500 kg, its distance to the pivot point is 0.75 m, and you can only exert a force of 700 N (average body weight), how long should the lever be ? Disregard the weight of the plank and assume the forces act at 90° to the lever. (T3*)

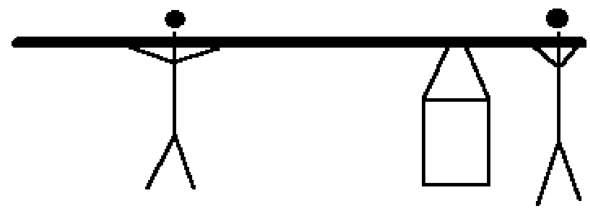
2.) A 50 kg person stands on the end of a plank, supported as shown below. What forces (magnitude and direction) do the two supports exert on the plank (assumed massless)? (T11)



3.) On the traffic light below, what force must the brace, positioned at 45^0 to the upper beam) exert on the upper beam? (consider the beam to be massless and the light to have a mass of 90 kg) (T6*)

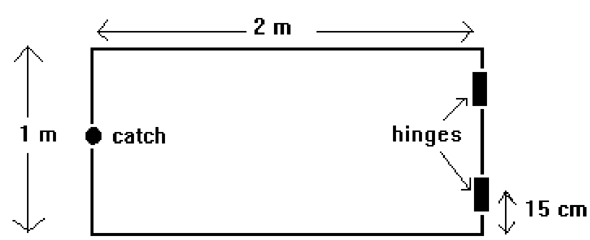


4.) Two people want to carry a 600 N object and share the load equally. They will use a 6 m, 200 N iron bar. Where should the second person hold the bar if the first person is at one end, 1.2 m from the object? See diagram. (T7*)



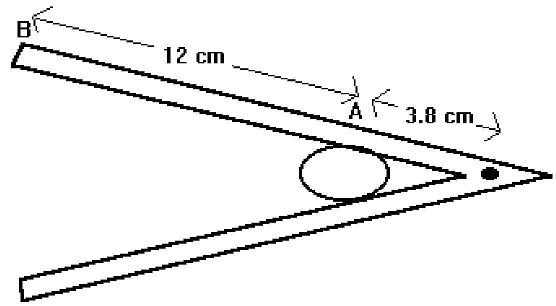
5.) A person wishes to carry a 3 m bar that weighs 100 N perfectly horizontal with a 500 N load on one end and a 300 N load on the other. Where should he or she hold it ? (T8*)

6.) Consider the following trap door:

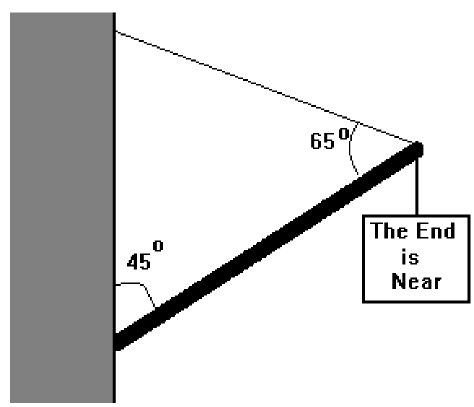


Suppose the door weighs 10 kg (the center of gravity is at the center) and it is set up so that when a 70 kg person (walking from right to left) lands at a point 0.5 m from the left edge, the catch will break (<u>not before</u>). What maximum force should the catch be able to sustain ? What is the force on each hinge at the instant before the catch breaks ?(T2*)

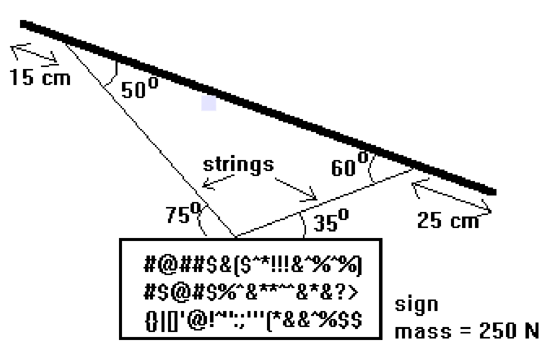
7.) Using the nutcracker shown below, how much force must you apply at point B (on both sides) in order to crack the nut placed at point A if the shell can withstand up to 70 N of force on both sides (assume the nut acts back on the nutcracker at 90^0)? (T12)



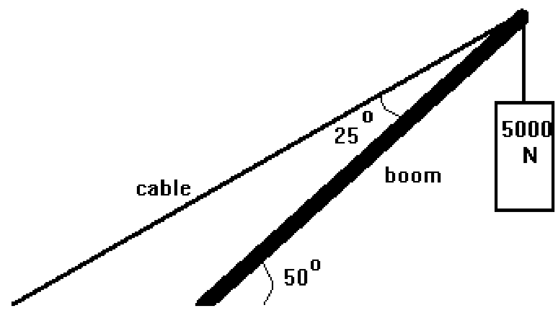
8.) A 20 kg sign is hung according to the diagram below. Determine the tension in the cable and the horizontal and vertical forces that the wall exerts on the rod. The rod weighs 25 N and is 0.75 m long. (T10)



9.) A piece of modern art is to be hung in a gallery as shown below. The bar from which it hangs is 2 m long and weighs 100 N. The artist insist that it be hung from one string connecting the bar to the ceiling and that it must be balanced as shown or his commentary on the angst of modern life will be ruined (he says as he strokes his goatee). Exactly where on the bar must the shop keeper position the string? Note: diagram should say sign weight = 250 N.



10.) Below is a diagram of a crane. The boom (solid support) weighs 1000 N and is uniform. Find the tension in the cable. There is no information missing. (T9)



12.) Decipher: "Refuse to verbalize the cessation of life."
(DNCTWHG)

Activity #16 - Levers

In a previous exercise, we learned about the concept of mechanical advantage of a pulley system. Another simple machine that we should discuss is the lever. A lever uses the concepts of torques to either multiply or reduce a given force ("Why would you want to reduce a force?" you ask, well I leave it to you to answer that question. Hint: observe the distances covered in this activity). Levers are simply bars that are fixed to pivot at some point and by lifting at one position, you can lift a weight at some other position.

The previous concept of actual mechanical advantage can carry over from pulleys to levers without modification. Thus:

AMA = Weight of object lifted/effort needed to raise object

The IMA, however, poses difficulties. The distances traveled by your hand and the weight when using a lever are actually arc of a circle. Instead of measuring these, we measure what is called the effort arm and the resistance arm. Below are the necessary definitions.

Fulcrum: The fixed pivot point on a lever.

Effort Arm: The distance on a lever from the fulcrum to the effort force.

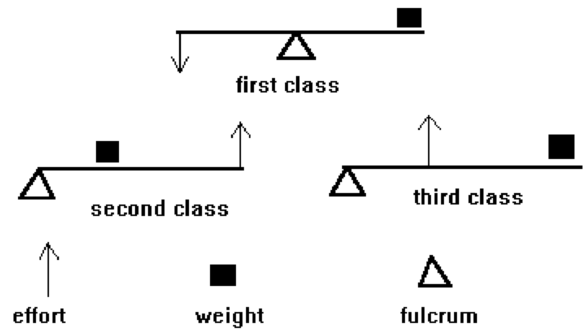
Resistance Arm: The distance on a lever from the fulcrum to the weight being lifted.

We then define the IMA of a lever to be found by:

IMA = Effort Arm/Resistance Arm

Please note that the arms of a lever are measured from the fulcrum to the force. Students often mistakenly measure from one force to another on some levers. I should also note two things before we proceed. First, we assume that the forces will always be at 90^0 to the lever, and secondly we are assuming that the formula above is equivalent to the previous formula for IMA. An astute student should be able to prove that both of these assumptions are valid (although the first is not exactly valid, it is equivalent to calculating the angles and factoring them in, as long as the weight and effort are parallel).

Levers are usually classified into one of three categories; first, second or third class. On a first class lever, the fulcrum is in the middle, on a second class, the weight is in the middle and on a third class, the effort is in the middle.



A few examples in everyday life are listed below:

First Class: Car jacks, see saws.

Second Class: Wheel barrows, can openers.

Third Class: Brooms, elbow joints.

Before we begin, I should mention something else for the astute student to consider. There is something special about the IMA of second and third class levers (a little thought will give you the answer).

<u>Procedure:</u>

1.) Construct a first class lever with an IMA of 1. Measure the mass of the object and the effort force and determine the AMA.

2.) Repeat the above procedure for two more first class levers, one with an IMA greater than one, and one with an IMA less than one

3.) Repeat procedure for three second class levers with varying IMAs.

4.) Repeat procedure for three third class levers with varying IMAs.

Draw any general conclusions you can.

IMA	Effort Arm	Resist. Arm	Effort Force	Object Weight	AMA
First	Class	Levers			
Second	Class	Levers			
Third	Class	Levers			

Lab #13 - The Second Condition of Equilibrium

In this lab you will construct three different set-ups, each in equilibrium. After you have constructed them, you will evaluate the sum of the torques around a pivot point and determine how far off of the experimental value your theoretical value is for each trial.

Procedure (part I):

1.) Construct the set-up shown in figure 1.

2.) Determine the angle between the torque bar and the string that hangs from the top bar to point A.

3.) Determine the mass of m, using a spring scale.

4.) Determine the distance from m to pivot A.

5.) Determine the distance from A to the string labeled T_1.

6.) Record the value of the scale and label it T_1.

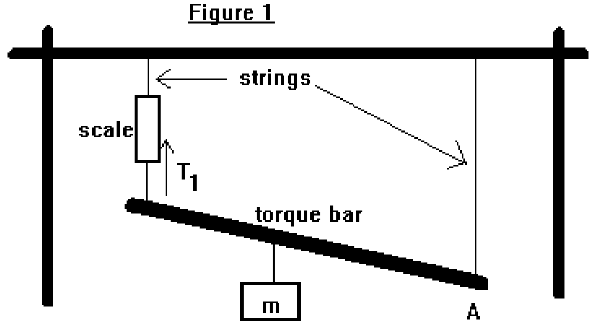
7.) Using T_1 as your unknown, evaluate the torques around pivot A.

8.) Add another mass (m_2) to the bar (at some random position) and repeat the procedure.

9.) Compare T_1 as measured experimentally to the theoretical value.

10.) Measure the mass of the bar (for a later calculation).

Figure 1



Procedure (part II):

1.) Construct the set-up shown in figure 2. Be sure the torque bar is horizontal, this might take some maneuvering.

2.) Determine the mass of m, using a spring scale.

3.) Determine the distance from m to A.

4.) Determine the distance from A to B.

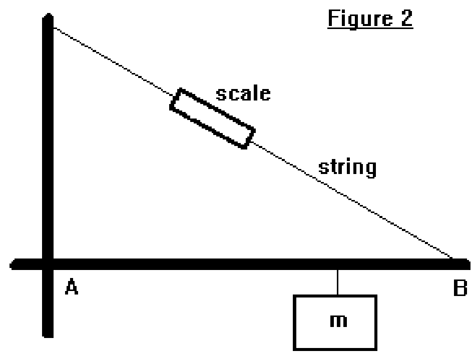
5.) Determine the angle between the string and the bar.

6.) Record the reading in the scale.

7.) Using appropriate pivot points, determine the tension in the string, as well as the horizontal (H) and vertical (V) forces on the bar at point A.

8.) Compare the tension in the string found experimentally to the theoretical value.

Figure 2



Conclusions: In your conclusions, be sure to do a percent error between the actual and the calculated values of your unknowns. Comment on reasons for discrepancy. To attempt to determine your discrepency, evaluate the sum of the torques around point A in the first set up with your measured vaule for the tension of the string. It should be zero, but might come out to have some value. This should give you a rough idea about the expected error. Why do we not calculate a percent error for this value? Secondly, we have left off the weight of our bar from our calculations. Redo this last exercise and factor in the weight of the bar. Determine a percent error between this and the first value for the sum of the torques.