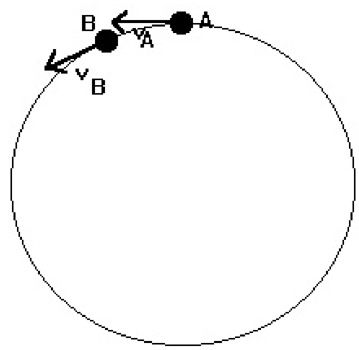
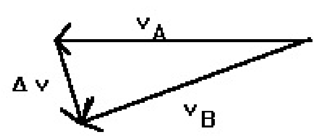
Chapter 15: Circular Motion and Newton's Second Law

Centripetal Forces and Accelerations

If we consider an object in circular motion, such as a planet or a ball on the end of a string, we might ask if such an object is accelerating. The answer of course is yes. Even if the object is moving around the circle at a constant speed, its velocity is constantly changing direction. We also know that if an object is accelerating, there must be a net force acting on the object in the direction of the acceleration. But what direction is that and what is the value of the acceleration?
Consider an object moving in the circle below. At point A the velocity is in the direction indicated and a moment later at point B the velocity has changed to the direction shown.



Remembering that we can move vectors to any location without changing them, we see that the direction of the change in velocity is as indicated by Δv in the diagram below.



When we look at the diagram, we should look at it as representing a vector equation of $v_A+\Delta v=v_B$. Since acceleration equals $\Delta v/\Delta t$, this should also be the direction of the acceleration and the force. However, to be more precise, we should look at the velocity at one time and the velocity an instant later. Taking this to the limit of an infinitely small change in time, we see that the direction of the acceleration is actually towards the center of the circle. This also makes sense if you look at the object as trying to move in a straight line, but being pushed in towards the center a little bit every instant. You can see that this type of push would result in the object moving in a circle (note, however, that a tiny bit too much of a push or a tiny bit too little will not keep the object in a perfect circle. The student should consider what would happen in those cases).

Such an acceleration is called a centripetal acceleration (centripetal means "center seeking"). And since the force is always in the direction of the acceleration, we have a centripetal force associated with the acceleration. Take a moment to notice how this is different than both the angular acceleration of the object and the tangential acceleration (important distinction).

If the object is moving in circular motion at a steady speed, then we know that the change in velocity (Δv) vector must be just the right length so that the new velocity vector is exactly the same length as the old one (simply with a new direction). Calculating that change needed is not an easy task, but the results give:

$$a_c=v_t^2/r.$$

And since F=ma,

$$F_c=mv_t^2/r.$$

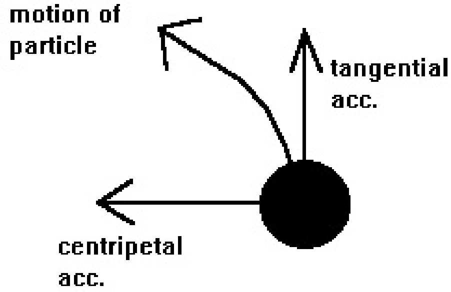
The above equations give the centripetal acceleration and centripetal force for an object moving in a circle. It is important to remember that these equations give "just the right amount" of centripetal force. In other words, if an object is moving in a circle, then it must have exactly mv_t^2/r Newtons of force acting on it directed towards the center. No more and no less. Notice also that the equations above involve the tangential velocity and would thus change over time if there is any tangential acceleration. To recap, we see that any object in perfect circular motion must have a centripetal force keeping it on the circle and this force must be of the value mv_t^2/r, directed towards the center.

It is important to keep in mind here that centripetal force is not some new force in its own right, instead, it is simply a special name we give to whatever force happens to be keeping the object in its circular path. In other words, something must supply the necessary centripetal force in each and every case. In the case of a ball on a string, we say that the tension in the string is supplying the centripetal force, for a planet it is gravity and for a car rounding a turn it is friction.

EX EP.) Consider a 0.5 kg ball on the end of a string, swinging in a circle with v_t = 1 m/sec and radius of 0.25 m. What is the centripetal acceleration? What is the centripetal force? What is supplying this force?

EX EQ.) If a 0.5 kg ball moves in circular motion making 4 complete revolutions in 3 sec and the radius of the circle is 2 m, what force is needed to keep the ball in this motion?

When we attempt to use this new concept in our force analysis, we need to look at the object at one instant in time (i.e. a snapshot) and evaluate the forces in the x and y directions. In actuality, what we are doing is evaluating the forces in the r and θ directions, since using x and y for a circle is not convenient. With this in mind, we need to remember that in the r direction the object will always have a centripetal acceleration and in the θ direction there may or may not be an acceleration (depending on whether or not there is an angular acceleration. Usually there will not be, otherwise the problems gets fairly sophisticated.). The direction of the centripetal and tangential accelerations are shown below.



These two accelerations mean very different things for the motion of the object. The tangential acceleration tells us if the object is speeding up or slowing down as it moves around the circle. It is certainly possible to have circular motion without having a tangential acceleration. However, it is not possible to have circular motion without having a centripetal acceleration. The centripetal acceleration is the one responsible for keeping the object moving in a circle (and not going off on a straight line path).

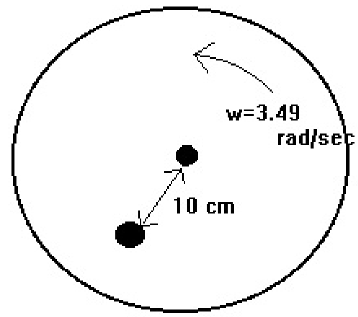
The standard set-up then is:

$\Sigma F_r = ma_c$ or $\Sigma F_r = mv_t^2/r$ and $\Sigma F_\theta = 0$ or $\Sigma F_\theta = ma_t$

Once again, we will not usually be dealing with cases of angular acceleration. When you use these equations, be careful not to fall into the usual trap that students encounter. What they often think is this: "Well, the object is not moving either back of forth in the r-direction, it is always at the same location, therefor, the sum of the forces is zero in the r-direction." Absolutely not! The object is accelerating in the r-direction, even if it is not moving that way. Thus $\Sigma F = ma_c$.

Let us attempt to use these equations in some problems that involve friction supplying the centripetal force.

EX ER.)   A penny (mass = 3 g) is placed on a phonograph turntable, 10 cm from the center. If the phonograph is rotating at 3.49 rad/sec, what coefficient of friction is needed to keep the penny in place?



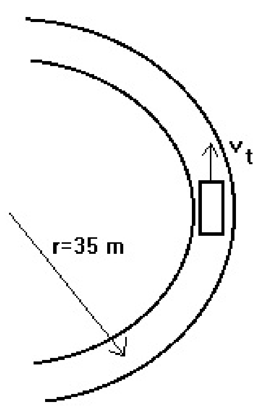
Did you fall into the trap of putting another force opposite to the friction to balance it out? Remember, the sum of the forces equals mass time acceleration not zero. This problem also showed us an equivalent, yet slightly different formula for the centripetal force. By substituting ωr for v_t, we get:

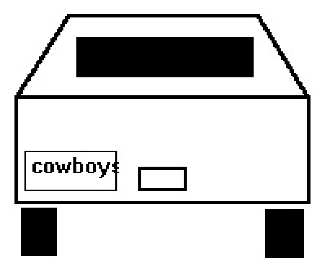
$$F_c = m\omega^2 r.$$

This can be used just as our original formula was used.

The penny example is illustrative of how to use the centripetal acceleration to deduce the friction constant. This same situation can be applied for the case of a car rounding a turn. In this example, it is important to remember what was said about the direction of friction in the previous section in order to get the free body diagram correct.

EX ES.) If the coefficient of friction between a car and a dry road is 0.7, what would be the maximum safe speed on an exit ramp with a radius of 35 m? (The maximum safe speed is the speed at which the car can round the turn and still be held in place by friction.)



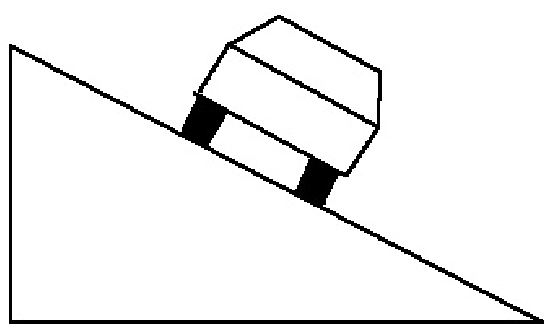


The result here is fairly general, and worth typing out, since we have already seen it used in two different problems. For a object in circular motion where the centripetal force is supplied by friction,

$$v^2 = \mu g r.$$

In the previous example, we saw how friction between the road and the tires acts as the centripetal force necessary to hold the car in its circular path. From experience, however, you may know that often exit ramps are designed with a slight angle (especially on race tracks). The example below shows you why this is beneficial.

EX ET.)   Derive an equation for the maximum safe speed on an angled exit ramp as shown below (use θ for the angle of the ramp and assume it was built with a radius of r).



These formulas are again useful and should be noted. The maximum safe speed for a car on an angled ramp with friction is given by:

$$v=\sqrt{(rg(\sin\theta+\mu\cos\theta)/(\cos\theta-\mu\sin\theta))}$$

Note how in the above example both friction and the normal force are supplying the centripetal force to hold the car in its path, but also note that not all of the friction is in the necessary direction to provide the force. Thus although you get an assist from the normal force, you lose some of the effects of friction. It should also be noted that the above example is more complicated than it might appear at first. One stipulation to consider is the direction of the friction. If the car is going fast enough, the friction is down the bank. However, if the car is going slowly, the friction will be up the plane. It might also be useful to consider how to build an exit ramp so that friction is not necessary at all (provided the car is moving the appropriate speed). In this case we use the above result, setting $\mu=0$ and arrive at:

$$v^2=rg\tan\theta.$$

Before we go back into a conceptual discussion of centripetal accelerations, let us try one more example of a very common type of centripetal force problems.

EX ETA.) Suppose a 60 kg child is swinging on a swing of length 2.5 m. If the child is moving at 2 m/s at the bottom, what is the tension in the chains holding the swing?

EX ETB.) Consider an airplane making a banked turn. Draw the forces on the plane and determine the centripetal acceleration.

Centripetal versus Centrifugal

Often, before taking Physics, students hear about something called the centrifugal force. Centrifugal force is a concept that is used to explain such things as water staying in a bucket as it is spun in a vertical circle, or the ride called the Rotor or Spindletop at amusement parks (This used to be a common attraction, but recently they are being phased out. In this ride you stand against the wall in a circular room that is spinning and the floor drops out). I believe this concept is often used because it is simpler to understand than centripetal force when explaining things to young children and it implants itself in their minds. Usually it is explained by saying that when something goes in a circle, a centrifugal force arise that pushes the object to the outside.

In actuality, centrifugal force is a concept used in physics, but it is what is called a fictitious force, or a force used to explain what is an illusion created by viewing motion from a certain reference frame.

We have already explained that objects moving in a circle require a centripetal force to keep them in their circular path. But now I ask you to stretch your imagination and envision yourself inside a bus with no windows. Also imagine that this bus is your entire universe, you don't realize that anything else exists (what I am actually saying is to imagine the interior of the bus to be your frame of reference). Furthermore, imagine that for some reason the bus was perpetually driving in a circle at high speeds. In the bus, it would feel as if a force was pushing you to one side of the bus constantly. As seen from above, however, it would appear as if your body as attempting to move in a straight line but was constantly being pulled in. If you were trying to determine the laws of Physics inside this bus, you would probably have two kinds of gravity, one down and one to the side. This illusional force that is pushing you to the side is called the centrifugal force.

Notice the difference in the two perceptions of the same motion. In one case, in the bus, you appear to have an extra force pushing you outward from the center. When viewed from above (frame of reference of the ground), you only have an inward force (perhaps friction) holding you in (note: in the bus you would have the friction as well). Obviously the difference arises from the choice of reference frames. What is going on here is that Newton's Laws of

Motion are only applicable in reference frames that are inertial. Inertial reference frames are defined as reference frames that are not themselves accelerating. A second definition is simply a reference frame where Newton's Laws work (an astute student might ask a question here that would be incredibly difficult to answer and would show that the entire empire of physics is based on pure speculation...). Thus Newton's Laws work fine when we look at things from the point of view of the ground being stationary. However, when we look inside the bus, since that reference frame is accelerating, we find that Newton's Laws do not work properly. In order for them to work, we must add fictitious forces (forces that aren't really forces). Once we have added these, Newton's Laws work fine. The two most common fictitious forces are the centrifugal force and the coriolis force (the force that causes storms to spin in one direction in the northern hemisphere and in the opposite direction in the other). We say that these are not true forces for two reasons. First, their purpose was to make laws work in a situation where the laws were not supposed to work. Thus they are added mental constructs, not actual forces caused by nature. Secondly, these forces have no reaction forces, they are not part of an action-reaction pair, and thus they violate Newton's Third Law.

In order to use the centrifugal force in a problem, you would set up the sum of the forces as:

$$\Sigma F_r = 0$$

instead of the usual

$$\Sigma F_r = mv_t^2/r$$

because in your reference frame there is no acceleration. However, when you sum the forces in that frame, you must include an extra force (the centrifugal) that goes out from the center and has a value of mv_t^2/r (this is worth repeating and underlining: In an accelerating reference frame, you must add the centrifugal force as an extra force in order to use Newton's Laws. In a non-accelerating frame, you do not add any forces, you simply call the force holding the object in the circle the centripetal force). Thus the centrifugal force is equal in magnitude and opposite in direction to the centripetal force.

It is important to notice that since they are used in different reference frames, they will never arise together. Let us do one simple example.

EX EU.) Imagine a penny sitting on a table in the bus as described above. Draw and explain the forces on the penny in a.) the reference frame of the bus and b.) the reference frame of the ground. Also write out Newton's Second Law for each of the situations.

Vertical Loops and Loop-the-Loops

There are two special problems in this section that are worth looking at in detail before the homework. They are vertical loops and loop-the-loops.

The vertical loop is confusing for some students because the force providing the centripetal force is changing over time. Notice that the last sentence was worded "the force providing...is changing" and not "the centripetal force...is changing...". Take a moment to absorb the difference. The centripetal force only changes if the speed changes. Let us then look at what is going on in a vertical loop.

EX EV.) Draw a free body diagram for a ball on the end of a string that is swung in a vertical loop at a constant speed v. Draw the diagram four times, once at the bottom, once at the top and once at each half way point. Write down Newton's Second Law for each position and determine what is providing the centripetal force in each case.

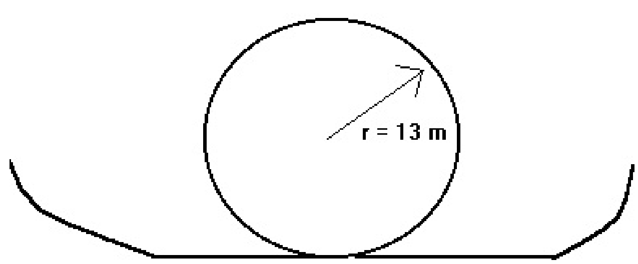
A few notes should be made regarding these equations. First, I hope you didn't fall into the trap that many students do when they attempt these problems. I hope that when you wrote the equation for the bottom of the loop that you did not write $T - mg = 0$. Remember that the object is not in equilibrium at that point. From these equations you can see that at times gravity works against the centripetal force (at the bottom), with the centripetal force (at the top) and at other times it simply has nothing to do with it (on the sides). This should also show you that the tension in the string is varying as the object swings around (at what point in the circle would a string be most likely to break as an object was swung in a vertical circle? Why?). The tension in the string will change to make up for the centripetal force needed to hold the object in a circle. Because of this, it is actually pretty hard to swing an object in a circle at a steady speed.

EX EW.) If a 1 kg ball is swung in a vertical circle of radius 1 m, what is the minimum speed it can have at the top of the circle if it is to maintain its perfectly circular path?

The key to this problem is knowing how to interpret the question in mathematical terms. The minimum speed necessary is the speed at which the weight of the object will provide all the necessary centripetal force and the tension in the string can then be zero.

Vertical loops have become common on amusement park roller coasters, and a few notes should be made. The above discussion holds true for any loop-the-loop situations, in roller coasters, the track will provide a normal force to supply the centripetal force, instead of a string providing tension. The normal force of the track takes the place of the string. It should also be noted that for technical reasons, most loops at amusement parks are not circles, but actually a different shape called a cycloid.

EX EX.) Imagine a roller coaster that contained a perfectly circular loop-the-loop as shown below. If the coaster reaches the top of the loop traveling at 15 m/sec and has a total mass of 1200 kg (this is an approximation made for a number of sundry reasons), what is the normal force the track must apply? We know that the track supplies this, put what ultimately supplies this force and in which direction is it pointing?



Satellite Motion: Circular Motion as Projectile Motion

It is interesting to think of circular motion around the earth (satellite motion) as a type of projectile motion. Consider a mental exercise first devised by Newton. It is often called Newton's cannon. Imagine that a cannon is placed on the highest mountain in the world and it fires a cannon ball. Naturally that ball will fall to earth. However, if the ball is moving very fast, it will move further simply because the earth curves away underneath it. Now imagine that the cannon fires faster and faster. It would go further and further and the curvature of the earth would play a greater effect. If you take this example out to the extreme, there would be a speed at which the cannon would be going so fast that the curve of its path would match the curve of the earth. In this case, the cannon ball would continue to fall, but never hit the earth. It would simply stay the same distance away from the earth forever (or at least until it goes all the way around and hits the cannon in the back!). This is exactly what is going on in circular motion. Satellite motion is made when an object is thrown fast enough that its projectile path matches the curve of the earth. In short, satellites are continually falling into the earth, but since they also move over, they never get any closer. The moon does the same thing. The moon is actually accelerating towards the earth at about 3 mm/sec^2, but since it is also moving to the side, it falls but never gets any closer.

This can explain, in a way, a common misconception that students have regarding the forces on an object in circular motion. Often students ask, "What is the force going out from the center that causes objects to fly off the circle?" This question is understandable, but it shows that the student is missing the point of Newton's first law. NO FORCES ARE REQUIRED TO MAKE THE OBJECT 'FLY OFF' THE CIRCLE. OBJECTS WILL MOVE IN A STRAIGHT LINE UNLESS

DISTURBED. There is no force making it fly off, there is only a force pulling it in to keep it on the circle. Think of the object as wanting to move in a straight line, but being pulled (accelerated) in from that straight line to force the arc of the circle. Just as the satellite falls in and over, it doesn't need a force to push it out.

Assignment #15

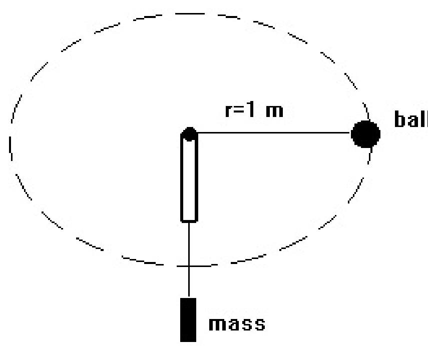
1.) Suppose a 5 kg object makes 38 revolutions in 60 seconds around a circle with a radius of 3 meters. What is (a.) the centripetal force and (b.) the centrifugal force on the object ? (C8)

2.) Determine a.) The centripetal force on the earth as it orbits the sun and b.) The centripetal acceleration of the earth. c.) Then do the same for the moon's orbit around the earth. (C2)

3.) A tire (r = 26 cm) on a car rotates at 10 rev/sec. What is the centripetal acceleration of a point on the edge of the tire ? (C15)

4.) On a dry, sunny day the maximum speed on a flat exit ramp is 30 mph. If it is raining the maximum speed is 20 mph. If the radius of the ramp is 80 m, how much did the coefficient of friction change (in percentage) ? (C10)

5.) Consider the set-up shown below. A ball (mass = 125 g) on a string revolves in a circle of radius 1 meter. It completes one revolution in 0.75 sec. How much mass (m) is needed to keep this in equilibrium ? (C13*)

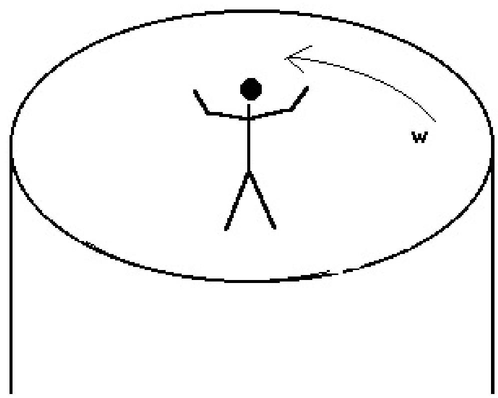


6.) A ball of mass 200 g is attached to a string that can only withstand a tension of 50 N before it snaps. Imagine the string is threaded through a pipe of length 1.2 m, with the loose end tied to one end and the ball resting on the other. The entire contraption is swung in a horizontal circle, beginning at an angular acceleration of 2.1 rad/sec^2. How long will it take for the string to snap? How fast will the ball fly off the end of the rod? In what direction will it travel?

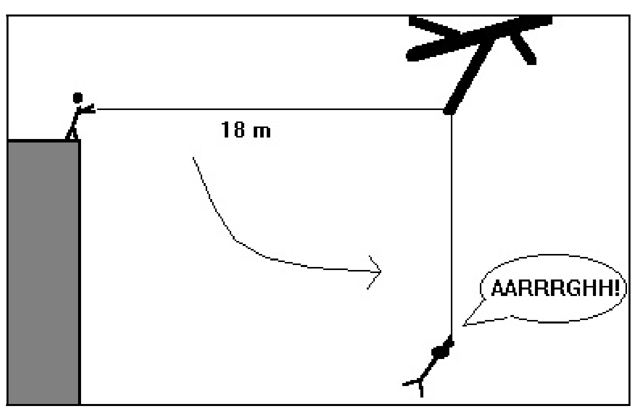
7.) Pilots can black out at accelerations of as little as 4 "g"s. With this in mind, consider a pilot flying at mach 1 (the speed of sound, assume a value of 330 m/sec) who attempts to make a 180 degree turn. What is the minimum time the pilot should allow to make the turn (this problem can be done without considering the effects of banking or the contribution of downward gravity on the pilot)?

8.) As inspector Colombo sifted through the wreckage of the Ferrari on the side of the angled exit ramp he noticed that the posted speed limit was 30 mph and he estimated that the radius of the ramp was 60 m. Back at the station a quick phone call to the Pirelli tire company told him that the average coefficient of friction for the tires on the car was 0.95. This gave him all the information he needed to figure out the minimum speed the car was traveling when it left the road. What was that speed (hint: the recommended speed is the speed at which a car can take the turn without relying on friction)? (C)

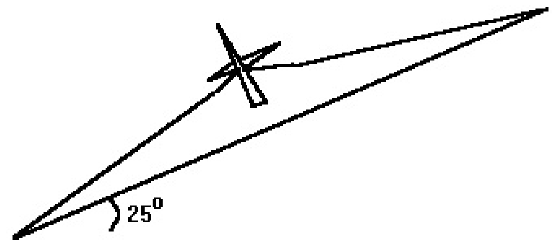
9.) A popular ride at Six Flags used to be the Spindle Top. This ride consisted of a cylinder where you stood against the inside walls and the ride angularly accelerated up to some speed and then the floor dropped and you were stuck up against the walls. If the cylinder had a radius of 2 m and the coefficient of friction between the person (assume a child of 50 kg) and the wall was 0.5, what angular velocity must the ride have attained before the floor was dropped in order not to hurt the poor kid?



10.) Tarzan, the King of the Jungle, jumps out of a tree and swings on a vine, making a semi-circular path of radius 18 m. If he has a mass of 70 kg and jumped from the tree when the vine was perfectly horizontal, what is the tension in the rope at the bottom? (Hint: to get started, consider conservation of energy to give you the speed at the bottom.) (C22)



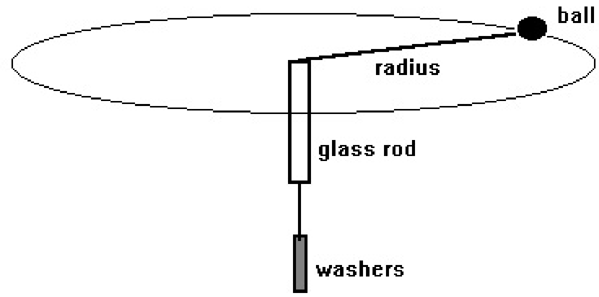
11.) The plane below has a mass of 2000 kg and is making a banked u-turn. The only forces acting on it are gravity and lift (which acts perpendicular to the bottom surface of the wings). If the plane completes the turn in 3 minutes, what is the radius of the turn? (C24)



12.) Decipher: "Rejection on conspicuous consumption prevents penury." (DNCTHWG)

Lab #12 - Centripetal Force

In this lab we will be examining the relationship between centripetal force and tangential velocity. Using the apparatus below, the ball will be swung in a horizontal circle above your head. When the ball is moving at just the right velocity so that the weight of the washers supply the correct centripetal force, the paper clip will remain motionless. When this occurs, the speed of the ball can be determined using the time it takes to complete 10 revolutions and the radius of the circle, and the centripetal force determined by weighing the washers. These two quantities can then be graphed and the relationship determined.



Centripetal Force Apparatus

1.) Place six washers on the free end of the string (designated as weight above). Measure out enough string so that the ball will swing in a circle of radius 0.75 m and then mark the string with a small piece of tape at the point where the string leaves the bottom of the glass rod.

2.) Place an paper clip on the string about 4-5 cm. below the tape and attach a piece of tape to the clip so that it is easily visible.

3.) Swing the ball in a horizontal circle above your head so that the paper clip remains stationary and the tape is just barely visible. Once you have achieved this, have your lab partner use a stop watch to time the amount of time it takes to complete 10 revolutions.

4.) Using this information, determine the velocity of the ball. Record this information.

5.) Use a triple beam balance to determine the mass of the washers and the mass of the ball. Record this information.

6.) Repeat steps 3 through 5 nine more times with differing numbers of washers (maximum of 20). If you find that 10 revolutions is occurring too fast to measure accurately, you should use 20 or 30 revolutions.

<u>Notes on Conclusions:</u> Plot a graph of centripetal force versus velocity and determine the relationship between the two variables (this may take more than one graph). The slope of your final graph should have an accepted value. What is it ? Determine the percent error for this value.

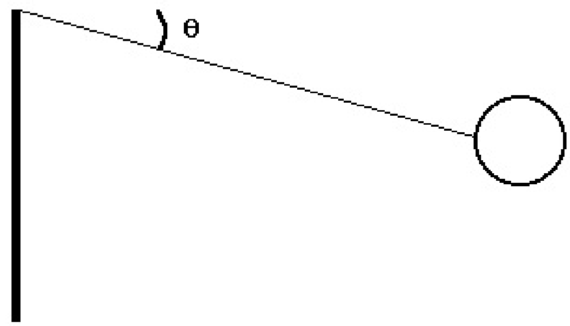
<u>Common Mistakes:</u> Please note that you are graphing centripetal force versus velocity. In each instance, the centripetal force is supplied by the weight of the washers (DO NOT CALCULATE IT USING YOUR FORMULA - THIS IS WHAT YOU ARE TRYING TO PROVE!). The centripetal force formula should never be used in this lab. If you find yourself using it, you are doing something wrong.

Trial	# of washers	# of revol.	time	velocity	centrip. force
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					

mass of ball	
mass of one washer	
radius of circle	

Lab Extension - Adding Realism

In reality, the ball can never be swung perfectly horizontal. The actual weight of the ball will always cause the string to make some angle with the horizontal, shown as θ in the diagram below.



This dip causes two sources of error to occur in your lab. First the centripetal force is no longer equal to the weight of the washers, it is equal to Wcosθ. Second, the radius that was used to calculate your velocity and your slope is no longer a constant for each trial (r), but instead is equal to rcosθ, which is different for each trial (r is the string length).

To make the lab more realistic, do a force diagram and then determine θ for each trial, then redo all the calculations. From this new data table, graph F_c versus v_t^2/r' and get a slope value (r' is the new radius, rcosθ). What is this value? Find a percent error and compare to the percent error in the original lab. **Note: the graph in the extension and the graph in the lab are different. They do not have the same accepted slope - be careful and make sure you understand what you are doing and what you are comparing.**

Trial	θ	velocity	v_t^2/r'	Weight of washers	Centrip. force
1					
2					
3					
4					
5					