Ch. 13: Forces and Newton's Second Law

Forces

In the chapter regarding Newton's Laws of Motion, we introduced the concept of a force and did a few problems that used forces. In this chapter we will look in greater detail at how to use Newton's Second Law to solve force problems and thus it would be helpful for us to once again define forces, this time in a more formal and detailed presentation.

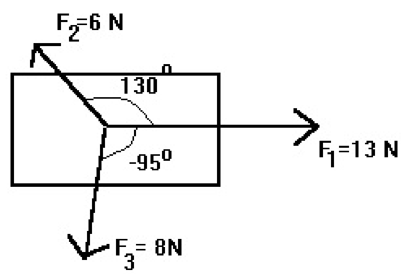
A force (as mentioned previously) is loosely defined as a push or a pull, or more accurately defined as something that has the ability to change the state of motion of an object (cause it to accelerate). Forces are measured in Newtons ($1\ N = 1\ kg.m/sec^2$). There are two types of forces, forces at a distance and contact forces (in reality, all forces are forces at a distance, but in some cases the distance is so small that we can better understand things if we consider them in contact. On the other hand, modern physics theories present forces as all contact...). Forces that act at a distance are gravity, electromagnetic, nuclear and the weak nuclear force. Gravity, for example, is the force responsible for holding the earth in orbit around the sun, thus we see that some forces can have effects at very large distances. Contact forces are forces that only arise when objects are in contact with each other, as in the case of an object resting on the floor, or a string holding a pendulum bob to the ceiling. It is the belief of physicists that there are only four different forces in the universe, gravity, electromagnetic, strong nuclear and the weak nuclear. All the forces we see in everyday life (a car driving up a hill, the thrust of a rocket, the collision of one boulder with another) are actually manifestations of these four fundamental forces. We will examine this theory in detail in a later chapter, for now we will call forces by more common names (thrust for a rocket engine, tension for the pull of a rope, etc.).

As Newton's Third Law tells us, forces always exist in pairs (as an action-reaction set) that act on both objects involved in creating the force. It is important to recall that every force has a subject and an object, so that when we describe a force, we do not just say "the force of gravity", but instead we say "the force of gravity acting on the earth from the sun". Very often, since we will be talking about only one object, it will only be necessary to discuss one of the forces from an action-reaction pair. If we are dealing with the earth, we will only consider the force of the sun on the earth, not the force of the earth on the sun. The reason for this was stated earlier, if we are interested in only the motion of one object, we need only consider the forces that act on that object. The forces the object creates on others have no direct effect on the object itself.

Forces have a direction, and when we add them together we must take that into account. We cannot treat forces as just numbers, we must treat them as vectors. When two or more forces act at the same point on an object (at the same time) we call those forces concurrent. In this chapter we will assume that all force that act on an object are concurrent and we will usually assume that they are all acting at the center of mass. One other vocabulary term that we will use is the resultant or net force. This is the single force that would have the same effect as two or more concurrent forces. The net force is the one force that we could replace the two or more forces with and still produce the same motion. As you can probably guess, the net force is simply the vector sum of all the forces acting on a particular object (for example, if two children pulled on a wagon, one east at 10 N and one west at 6 N, the net force would be 4 N east).

As we begin to learn how to solve force problems, there are a number of preliminary steps the student must master if they intend to correctly solve these types of problems. What we are about to embark on in this section is an explanation of the steps for solving force problems. Students should pay close attention, for once the problem is set up correctly, solving it is simple.

When using forces in a problem, it is essential that you label all the forces acting on a particular object. Very often we draw these force in what is called a "free body diagram". This is simply a thumb-nail sketch showing the forces and their directions. For example, if three children pull on a wagon in three different directions, the free body diagram (from above) might look like this:



Notice how all the forces are drawn from the center of the object and all angles are measured relative to the horizontal (+x) axis.

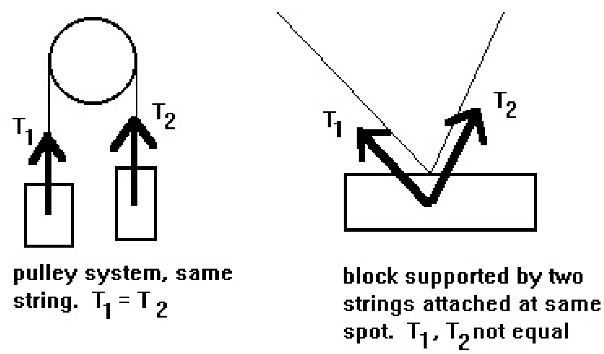
Below is a list of the different types of forces that you might encounter in a force problem. It tells you when to draw a force.

How to Determine if a Force is Present on an Object

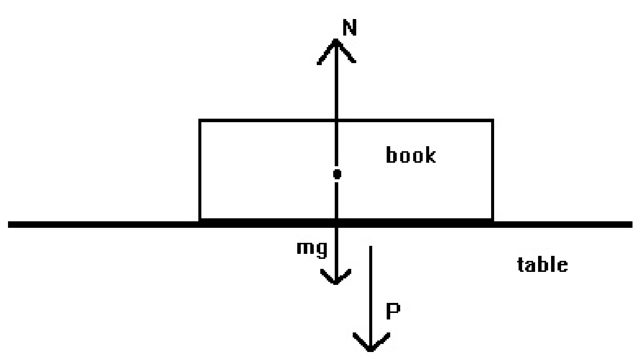
1.) Any time the problem says explicitly there is a force. For example, if the problem says the child pushes on the wagon, there is a force from the child acting on the wagon.

2.) All objects have weight (if they are near the earth or any other planet), thus every time you do a force problem, the first force usually labeled is the weight (or the pull of the planet on the object). The weight of an object is given by mg where m is the mass and g is the acceleration of gravity on that particular planet.

3.) If an object is attached to a rope or a spring, it will have a force from the rope acting on the object. Usually we label this force T for tension. This force is drawn in the center of the object, not on the string, since we are interested in the motion of the object, not the string. The direction of the tension is the direction the string is holding the object (i.e. from the center towards the string itself). Also, the tension throughout one string must be equal (if one string is attached to two objects, as in a pulley system, the force of the string on each object must be the same). Notice however, that the tension is not the same on two strings that are connected to the same object. The diagram below shows both of these cases.

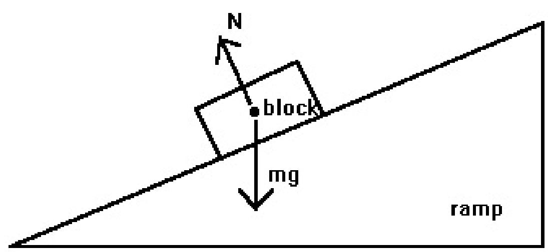


4.) All objects that are being pushed against a surface or another object have what is called the Normal force acting on them. In our section on Newton's Laws, we did an example of a book resting on a table. Since gravity pulls the book down (this force is labeled mg in the diagram below), and the table is in the way, the book will push up on the table (force P) and by the third law, the table will push back on the book (force N). This "push back" is called the normal force.



In is important to note a few things in the diagram above. First, the forces N and P make up an action reaction pair, not the forces N and mg. Although common sense will tell you that mg=N, it is <u>not</u> because they are an action-reaction pair. They just happen to be equal and opposite (where is the reaction force to mg?). Also, the force P is drawn off to the side for clarity, it should be directly below N. In most force problems, the P force is not drawn. Once again, the reason is that we are only interested in the forces acting on the book. P acts on the table and is caused by the book, thus it will likely never enter the problem.

The student may be wondering why this contact force is called "the normal force". The answer lies in the mathematical definition of the word "normal". Normal is a term used in math to mean perpendicular. Whenever you have a normal force arising from an object being pushed against a surface, it will push perpendicular to the surfaces in contact. An example is shown below, where a block is placed on a ramp.

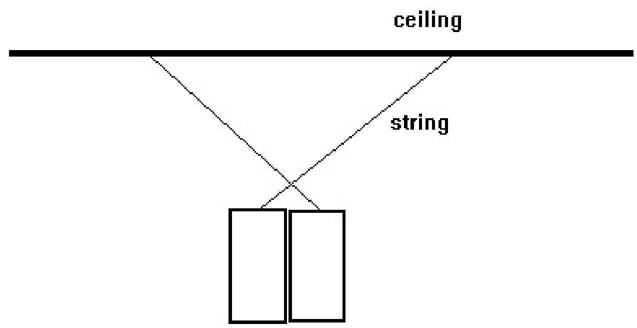


One final comment should be made about the normal force. In many cases the normal force will turn out to be equal to the weight of the object (as in the book diagram), but it is not necessarily so (as in the block diagram). Students have a habit of associating the two forces and jumping to the conclusion that N=mg always. There is no direct way to tell the value of the normal force, it must be taken as an unknown and solved for.

5.) Friction is another force that we will dealing with in the next chapter. The student should note that there is no need to include friction in a problem unless it is stated. If there is no clue regarding friction, we can consider the problems to be frictionless. More on friction after we master the basic concepts in this section.

As an example of using these concepts, consider the problem below:

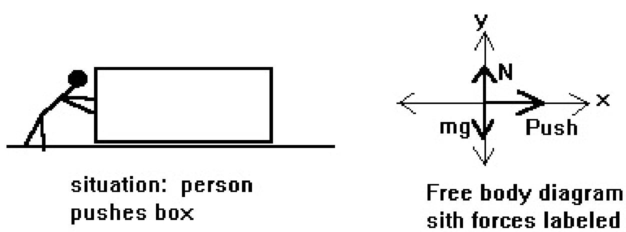
EX DS.) Two wooden blocks are attached by strings to the ceiling. The blocks are then arranged so that they are pushing against each other at their midpoint as in the diagram. Draw a free body diagram for the block on the right.

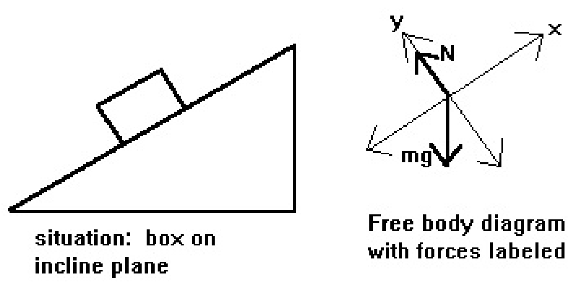


EX DSA:) Imagine two wooden blocks resting side by side on a frictionless table. If one block is smaller than the other and you push the large block into the smaller block. Draw a free body diagram for each block.

You should have drawn three forces; gravity, tension and a normal force from the block on the left.

Once the forces have been drawn, the next step in a force problem is to resolve the forces along some axis system. Each individual problem will need its own axis system, which is an arbitrary choice made by the person doing the problem. Very often that axis system will be the usual horizontal/vertical system that we are so used to working with, however, the choice can be different for different problems. It is often most convenient, for example, to position the x axis along a line of motion (if the object is moving in a straight line NE, for example, the best choice is to rotate the usual axes 45^0 so that you now have motion in one dimension instead of two). Once you have the axes, you resolve the force vectors along these axes, so that you are left with only force components that are along either the x or y direction. Two examples are shown below. The first is a case where there is no preferred direction of motion, and thus the axis is positioned normally (as seen on the right hand side of the diagram). The second picture shows a cart about to roll down a hill. In this case it is more sensible to tilt the axis so that the motion is one dimensional along the x axis.





Newton's Second Law - Equilibrium Situations

Now that we know all about what forces are and how to set up our problems, we can turn our attention to using Newton's Second Law and how to use it to solve force problems.

Consider Newton's Second Law, $\Sigma F=ma$. If we deal with a situation where there is no acceleration, the law tell us that the sum of the forces on that object must equal zero ($\Sigma F=0$). Please remember that this law does not tell us that there are no forces on the object, it only says that the forces must add up (as vectors) to zero.

The sum of the forces equalling zero is called The First Condition of Equilibrium. Equilibrium is defined as the a condition where the state of motion of an object remains unchanged. In other words, there is no acceleration (either linearly or angularly, but more on the second part in another chapter). Thus a bridge is (hopefully) in equilibrium, a book on a desk is in equilibrium, and a marble dropped into maple syrup is eventually in equilibrium. Note in the last example that the object is moving. Equilibrium does not mean no motion, it means no acceleration. In maple syrup the marble will sink at a steady speed after the first few moments.

The other thing about the first condition of equilibrium is that it is a vector equation, thus it can be broken down into two (or three, if you are doing a three dimensional problem) other equations (Side note: All vector equations imply two or three other equations that can be considered as scalar equations, by resolving them into components. Since the components are all in straight lines, we only have to deal with positive and negative as choices of directions, thus we can deal with scalars.)

In other words, $\Sigma F=0$ (the First Condition of Equilibrium) implies that:

$\Sigma F_x=0$ and $\Sigma F_y=0$.

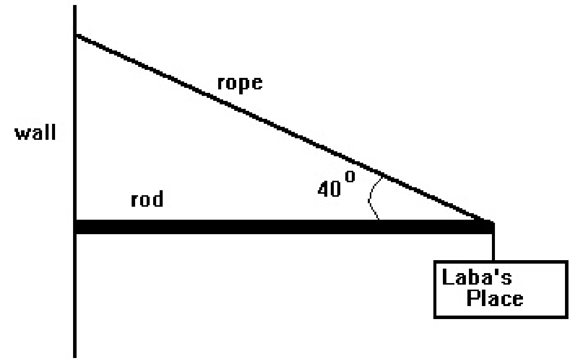
The sum of the forces being zero implies that the sum of the forces in the x direction equals zero and the sum of the forces in the y direction equals zero. This makes logical sense if one thinks about it. If we add five vectors together and the sum is zero, that means that the x and y components must have each added up to zero independently. If we were to operate in the z direction (for a 3-D problem), we would also have $\Sigma F_z=0$. Later, when we deal with non-equilibrium situations, we will use an identical method, and show how things can be in equilibrium in on direction but not another.

Two other technical notes should be made before we begin these problems. When there is a scale in a problem, it will read certain forces. A spring scale attached to a string, for example, will read the tension in the string. A regular scale (of the bathroom variety) will read the Normal force of the object sitting on the scale (in other words, when you stand on a scale in the bathroom, it will read not your weight, but actually the normal force acting on you.

Now we can begin the problems. In each problem, the method of attack is the same:

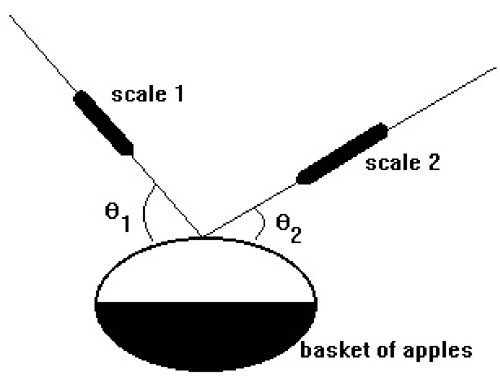
1.) Draw a free body diagram.
2.) Resolve the forces.
3.) Write out scalar equations for $\Sigma F_x=0$ and $\Sigma F_y=0$.
4.) Use those equations to find the unknowns.

EX DU.) Consider the sign below. What is the tension in the rope and what is the horizontal force the rod exerts on the wall? (the mass of the sign is 3 kg and the mass of the rod is negligible)



This example shows us that not all forces are obvious from the wording of the problem, and some thought is needed to identify all of them.

EX DT.) A grocer is selling fruit by weighing it with two scales as shown. He claims that often people by more fruit than one scale can hold. If he adds the two scale readings as pure numbers, is he overcharging, undercharging or is his logic correct?



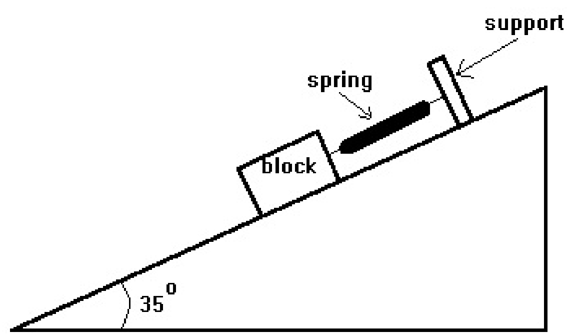
This example shows us how to break the vectors up to solve for a single unknown. It also shows us that we can use non-reference angles provided that we insert the correct signs ourselves.

One other type of force that needs to be mentioned is the force created by a spring. When a spring is stretched or compressed, it exerts a force equal to:

$$F = -k\Delta x.$$

We have discussed springs previously and noted that K is the spring constant and Δx is the compression or stretch of the spring. The negative sign is included in order to make this a proper vector equation. The force will always be opposite in sign to the displacement of the spring. If you push it in, it will push out and visa versa.

EX. DV.) On the incline below, a spring is attached to a block and the block is released. It bobs back and forth for a while then settles into equilibrium. If the block has a mass of 3 kg, and the spring has a spring constant of 300 kg/sec^2, how far did the spring stretch? What is the normal force on the block?

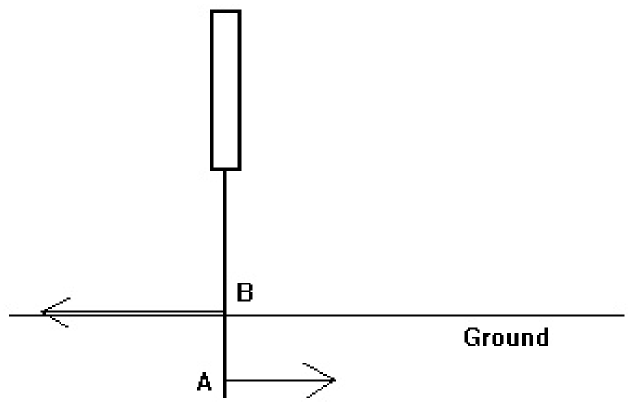


Before we move on to non-equilibrium situations, we should introduce another aspect of forces. Forces create pressure. Pressure is defined as force per unit area, or:

$$P = F/A$$

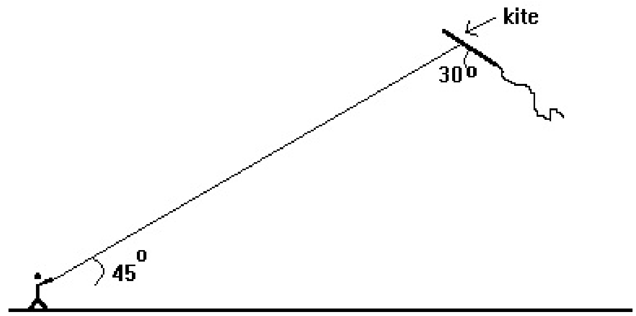
Where P is the pressure, F is the force and A is the area over which the force acts. The units of pressure, therefore, are N/m^2, which are called Pascals (Pa). Consider the following problem below:

EX DW.) A highway sign is shown below. If the wind is blowing heavily on the sign, the ground must exert a force of 500 N to the right at point A and a force of 1000 N to the left at point B to keep the sign from falling over. If the sign has an area of 0.3 m^2, what pressure is the wind exerting on the sign?



This next example simply gives another chance for practice at force problem.

EX DX.) On a nice, windy day, a person flies a kite as shown below. What is the magnitude and direction of the force of the air on the kite? The kite has a mass of 0.5 kg and the string can be considered massless. The angle between the string and the face of the kite is 105^0 and the tension in the string is 30 N.


kite
30o
45o

Non-Equilibrium Situations

All of the previous examples demonstrated how to use Newton's Second Law in equilibrium situations. The Second Law also is appropriate in non-equilibrium situations as well. In these cases, ΣF does not equal zero, it is equal to ma. Thus we will be using:

$$\Sigma \underline{F}=m\underline{a}.$$

Once again, we are reminded that this is a vector equation, thus it automatically breaks down into:

$$\Sigma F_x=ma_x$$

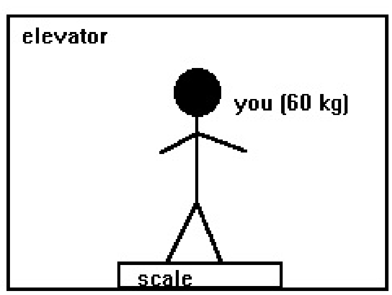
and

$$\Sigma F_y=ma_y.$$

This allows us to solve for the components of the acceleration in any case and then to combine them back together to give us the full vector form of the acceleration. Normally, it is easier and more conceptually visible if you rotate your axes so that all the motion (and acceleration, if they are in the same direction) lie along the positive x axis. Then you can solve the problem by saying $\Sigma F_y=0$ and $\Sigma F_x=ma$.

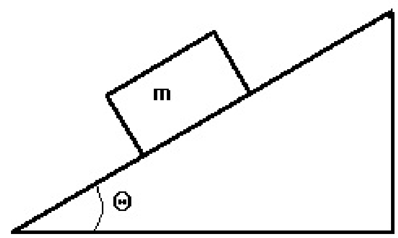
This form of Newton's Second Law provides us with an important method of solving more complicated problems. Once the acceleration is solved for, it can be used in our one dimensional motion equations (provided that the motion meets the criteria for those equations). With that said, let us begin to look at some examples:

EX DY.) What is your apparent weight (the reading on a typical bathroom scale) in an elevator that is a.) accelerating at 3 m/sec^2 upward? b.)accelerating at 3 m/sec^2 downward? c.)moving upward at 3 m/sec?



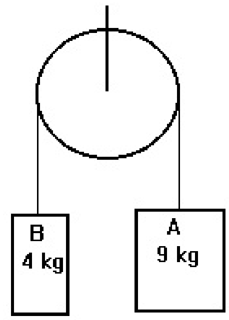
The previous (very simple) example shows us how to use $\Sigma F=ma$ and explains that strange sensation you get in an elevator when it takes off and stops (remember, your body is an accelerometer). The results can also be viewed as two forces being applied to your body: one to counteract your weight (gravity) and an extra force to accelerate you.

EX DZ.) Determine an equation for the acceleration of the block on the frictionless incline below and derive an equation to find the time it would take the block to cover a certain distance (starting from rest). In the process, also determine an equation for the normal force on the block.



A few comments should be made regarding the last problem. First, it shows us why it is sometimes easier to work with rotated coordinate systems, where the motion is along the x axis. Secondly, it shows us that we should work in variables first, then plug the numbers in. The reason we do this is obvious if one takes the time to look at our results. Our final formula does not involve mass at all, thus the mass of the block should not matter. We would miss this result if we had simply plugged in numbers from the beginning. Also, by working with variables, we had made a general equation which is good for any similar physical situation. In other words, the formula we have written is now applicable for any block on any frictionless incline plane (and we know how many of them we encounter in our everyday lives!), regardless of angle or mass of block. This result also allows us to notice patterns in the situation. For example, consider raising the plane and consider what would happen as θ approaches 90^0. By knowing the result, we can see if our equations for the normal force and the acceleration behave as we think they should.

EX EA.) Consider the pulley system below.
a.) find the acceleration of block A.
b.) find the tension in the rope.
Imagine the pulley to be massless and frictionless.



This problems is illustrative of a number of things. First, it shows us how to deal with problems involving pulleys, or any other problem with multiple masses or components. For each mass, we need to apply Newton's Laws separately and then look for common variables to solve the set of equations. Secondly, this problems reminds us of the "mechanistic nature" of the equations we are using. Force problems follow a very predictable format. We simply write $\Sigma F=ma$ for each direction and mass and then look at how many variables we need to solve for. For each variable, we need one equation. In the above instance, the two variables were tension and acceleration and $\Sigma F=ma$ applied to each block gave us two equations to solve for those variables. Thirdly, it shows us that when working with pulleys, we must pick a direction <u>along the string</u> and call it positive. This is a common error when attempting these types of problems. Notice that on one side the weight is positive and on the other it is negative, but what is important is that the acceleration is the same sign on both sides.

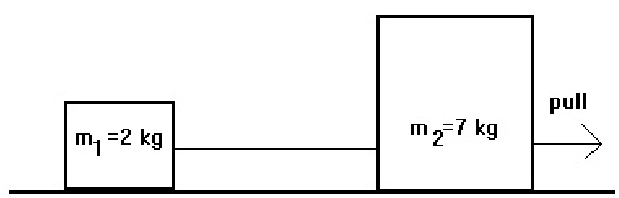
Once again, by working with variables, we have produced a general result that can be analyzed further.

EX EB.) Use the result from the previous problem, and analyze it for the special cases of $m_a=m_b$, $m_a>>m_b$, and $m_a<<m_b$.

This form of analysis of special situations is very important in physics, since it shows things about the solution. It is also a way to check the solution to see if it make sense.

Let us finish this section with one final problem, which can be solved a few different ways.

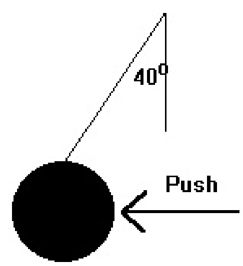
EX ED.) Two blocks are connected as shown in the drawing below. If they are pulled with a force of 30 N, what is the tension in the cord between them? Consider the rope to be massless and the surface frictionless.



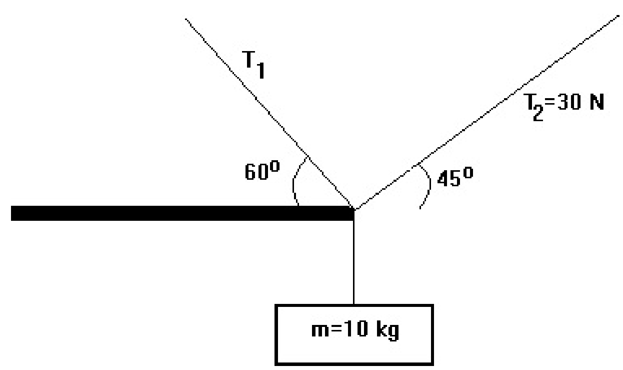
Assignment #13

1.) A car weighing 1500 kg is on a hill that rises 3 m for every 100 m of horizontal distance. If a person is attempting to hold the car from rolling down the hill, how much must the push, parallel to the ground (no friction)? (O1)

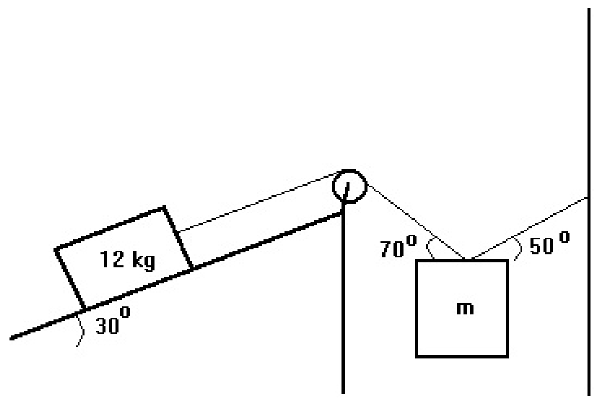
2.) A 30 kg ball is suspended from the ceiling by a string (considered massless). A person pushes the ball, as shown below and holds it in place. Find the magnitude of the push.



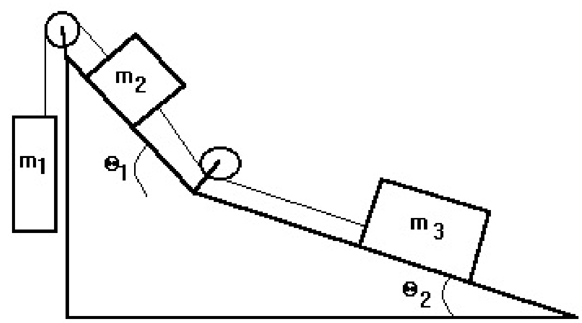
3.) In the diagram below, a rod holds the three strings in place. Calculate the tension in the second string (labeled T_1) and the force that the rod exerts. (O9)



4.) Determine the mass of m that will keep the system below in equilibrium (all surfaces are frictionless). (O10)



5.) Find the value of M_3 in the diagram below, if M_1 = 30 kg, M_2 = 10 kg, the bottom plane makes an angle of 20° with the horizontal and the upper plane makes an angle of 60° with the horizontal and the system is in equilibrium. Assume all surfaces are frictionless. (O22)



6.) What is the average pressure exerted on the floor by a lady standing in high heel shoes? If an elephants feet are approximately 0.6 m^2 each, how much force does an elephant exert on the same floor.

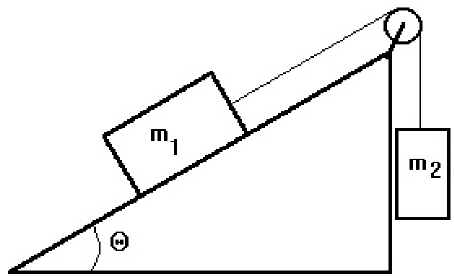
7.) If an average car has a mass of 1400 kg and a tire pressure of 35 lbs/in^2, how much surface are of the tire is in contact with the road?

8.) Imagine three forces acting on a 15 kg object. Force 1 is pushing with 30 N in an easterly direction, force 2 is pushing with 30 N at 35° north of west and force 3 pushes with 15 N in a westerly direction. Represent the objects displacement after 30 sec as a vector (the object is at rest before the forces act). (O5)

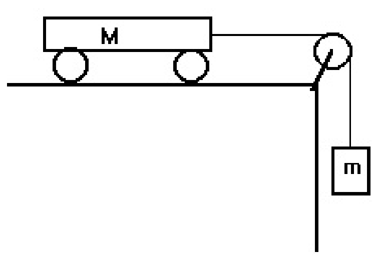
9.) A block is projected up a frictionless incline plane with a velocity of v. The angle of the incline is A. Give your answers to parts a, b and c in the form of algebraic expressions.

a.) How far up the plane does the block slide before stopping?
b.) How much time elapses before it stops?
c.) What is its velocity when it is halfway up the plane?
d.) Give numerical answers for v = 4.0 m/sec and A = 38 degrees.
(O4)

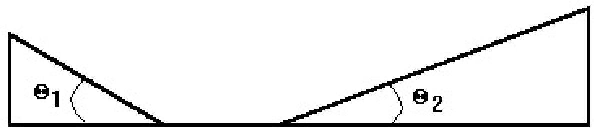
10.) In the diagram below, m_1 = 6 kg and m_2 = 3.5 kg. If the incline is frictionless and has an angle of 30°, determine the acceleration of each block. (O26)



11.) Consider the set up below, consisting of a cart of mass M connected to a smaller mass m by a string that passes over a frictionless pulley. Imagine that the <u>weight</u> of mass m was an unknown and derive an equation that allows you to solve for W given M and the resulting acceleration (in other words, W=f(M,a) where f is a function involving only a, M, g). You will find that it is impossible not to have m in this equation, so introduce a new variable, M_2=M+m and use it in your equation. How does this situation explain a strange part of the procedure in lab #3? Assume no friction. (O24)



12.) One of Galileo's most famous experiments showed that a when a ball rolls down a plane and then up another plane of a different angle, it will rise to the same vertical height, regardless of the distance up the plane the ball must travel (without friction, of course). **PROVE,** using only the equations of one dimensional motion and the sum of the forces, that his hypothesis was true. (O13)



11.) Decipher: "Eleemosynary deeds have their incipience intramurally." (DNCTHWG)

Activity #13 - Pulley Systems

In this activity, you will construct different pulley systems and learn how they are used to lift heavy objects with ease. You will also learn the term mechanical advantage and why pulley systems work.

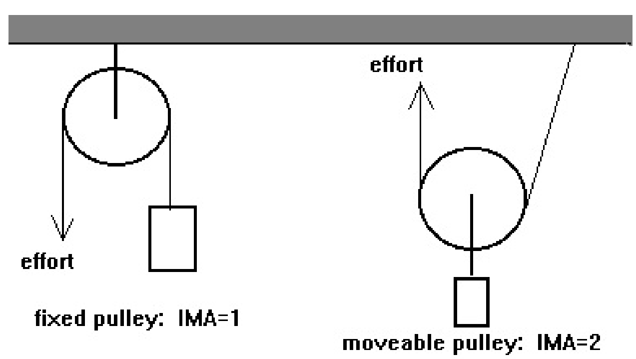
Introduction: A pulley system is a combination of pulleys with a common string wrapped around all of them. Usually a weight is placed on one of the pulleys and the string is pulled to lift the weight. The mechanical advantage of a pulley system is the number of times that the system multiplies your effort force. For example, it is possible to construct a system that requires you only to pull with 20 N to lift a 100 N object. In this case the MA of the system is 5. There are two types of mechanical advantages, ideal mechanical advantage, or IMA (the number of times the machine should multiply your force) and actual mechanical advantage, or AMA (the number of times it actually does multiply your force. The AMA is found by measuring the weight of the object lifted and the force needed to lift it. The IMA is found by measuring the distance the object is lifted when the string is pulled a certain distance.

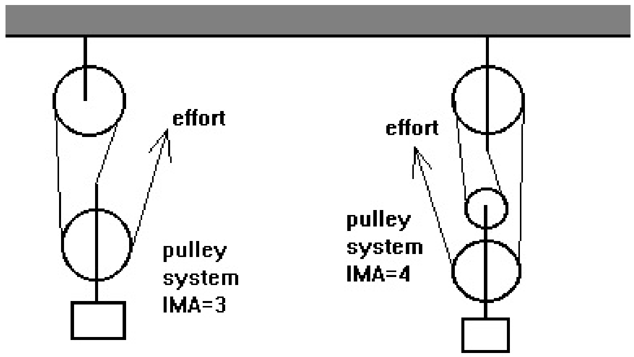
AMA = weight of object/force needed to lift

IMA = distance string pulled/height object lifted

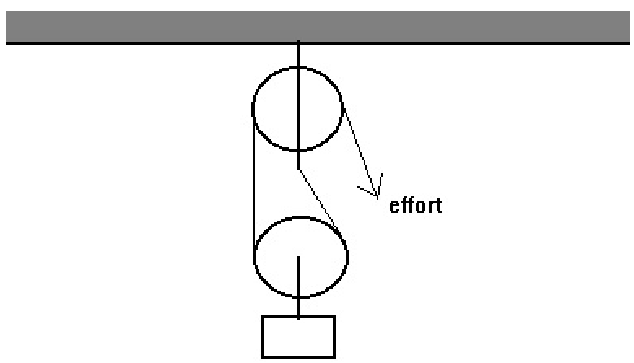
In this activity, you will measure both the IMA and AMA of five pulley systems that you construct and see how close they are to each other (a frictionless systems would have IMA=AMA).

The systems to build are shown below. Notice that when multiple pulleys are needed they are drawn one on top of the other for clarity, but they could just as well be placed side by side.



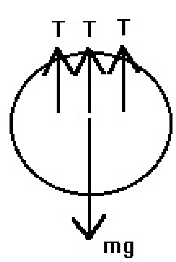


You will notice from these diagrams that the IMA of the system can be calculated easily by simply counting the number of strings directly supporting the weight. Take another look at those diagrams to see the pattern. One word of warning, however, you must only count the strings directly supporting the weight. For example, what is the IMA of the systems below?



In this case, it is only 2, not 3, as many students might guess. The final portion of the string, labeled effort, is not directly supporting the weight.

The reason for this correlation between string number and IMA is simple. Consider a free body diagram for the bottom pulley in the system with an IMA = 3. It would look like this:



Thus in equilibrium we have:

$$T + T + T = mg$$

and since all the Ts are equal (they are all the same string), we have:

$$3T = mg$$

or

$$T = mg/3.$$

Using this logic, the student can see why in the last example the portion of the string labeled effort did not contribute to the IMA.

Procedure:

1.) Construct a pulley system with an IMA = 1.

2.) Measure the effort required to hold a mass in equilibrium and measure the weight of the mass.

3.) Pull the string a certain distance and measure the how high the mass rises (it might be better to measure how far the string needs to by pulled to raise the mass a certain distance).

4.) Repeat the procedure for IMAs of 2, 3, 4, and 5. You will need to design the system with IMA of 5 yourself.

5.) Calculate the IMA and AMA and compare them to see how efficient each system is. Draw any general conclusions you can.

6.) Devise a method of measuring the IMA of the pulley system given by your instructor.

Pulley System	Effort Force	Object Weight	Effort Dist.	Object Dist.	IMA	AMA
1						
2						
3						

4						
5						

Mechanical Advantage of Instructors System:

Activity #14 - Forces in an Elevator

In this activity, you will use a remote computer interface to measure the forces on an object in an elevator. The apperatus will consist of a remote interface, a force sensor and a mass tied to a string and attached to the force sensor. You will measure the forces on the string while the elevator goes up to the second floor and then measure them again on the ride down. You will use this information, along with a force diagram and analysis to find the maximum acceleration of the mass both upwards and downwards.

Procedure:

1.) Familiarize yourself with the operation of the remote interface and program it to measure force versus time.

2.) Bring the equiptment in the elevator and begin recording the instant that you press the button for the second floor. Be sure to either hold the device very still, or mount it on a ring stand.

3.) Stop recording when you are fully at rest.

4.) Repeat the procedure while the elevator goes down.

5.) Return to the computer and download the data as quickly as possible, since another group might be waiting on the device.

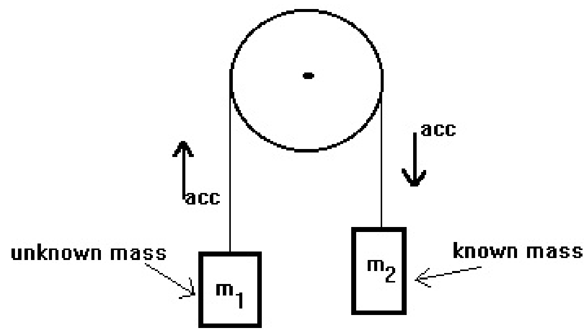
6.) From this data, graph force versus time for each run.

7.) On these graphs, label all the signifigant sections of the elevator ride (i.e. acceleration begins, second floor reached, etc.)

8.) Using the graphs and a force analysis, determine the maximum acceleration achieved by the block.

Lab #10 - Resolution of Forces

In this lab we will be using a combination of our laws of motion along with a resolution of forces from Newton's Second Law to determine the mass of an unknown object. Consider the system shown below. Suppose we attach an unknown mass to one side of the pulley system (m_1) and attach a known mass to the other side (m_2) so that the system accelerates in the direction shown.



If we accurately measure how long it takes the known mass to cover a certain distance, we can determine its acceleration using our equations of motion. Once an acceleration is determined, we can use our resolution of forces for pulleys to solve for the unknown mass (given the acceleration and the known mass).

Procedure:

1.) One person should stand on the balcony above the commons and the other should stand below. Place a few books on the area where the masses will be striking the floor so that you don't damage any bricks when the masses land.

2.) Attach the known mass to one end of the pulley system and attach an unknown mass to the other. Experiment with the known masses so that the system will not accelerate too fast for your partner to measure or too slow so that friction becomes a factor.

3.) When you are ready to time the mass falling, have the partner on the floor hold the unknown mass so that it is just touching the floor. The person on the balcony should then secure the string and the person on the floor can step back to time the event and to make sure no innocent bystander is approaching. The person on the top should also catch the unknown mass as it comes up so that the string does not break.

4.) When the person on the top lets go of the string the other person should use a stop watch to time how long the mass takes to reach the floor.

5.) The entire procedure is repeated three time with the same mass and an average is taken.

6.) The procedure is repeated three times with different known masses.

7.) When you are done, you must measure the distance that the mass fell. This needs to be done as accurately as possible. The best method to use is to note exactly where the top of the mass is hanging before it is released, then release the mass and mark the string when the mass is on the floor. Cut off the string and measure the string with a meter stick.

8.) Using the average time for each known mass, the mass of the unknown object is calculated.

9.) The unknown mass is then taken to a balance and its mass is determined.

10.) The percent error for each of the three known masses is calculated.

Hints for conclusions: The theory section should include a derivation of the pulley equations and should show the equation that you use to determine the unknown mass (solved for m_1). Then solve your equation of motion for the acceleration and combine these two so that your have one equation that allows you to calculate the unknown mass using the known mass, the distance traveled and the time it took. Calculate percent error and comment on any discrepancies.

Lab Extension #2 - Expected Error Ranges

In our previous extension, we discussed how to evaluate the precision of an experiment to determine how well it was performed (it also tells you quite a bit about the precision of the equipment used). In this extension we will focus on accuracy.

There are two types of accuracy in an experiment: accuracy related to the measurability of quantities and accuracy due to human error. For our purposes we will call these two things expected accuracy and experimental accuracy, respectively. Experimental accuracy is measured by percent error, which tells you how far off the accepted value your result was. Expected accuracy deals with the built in limitations of the materials used and the measurements made.

Determining Expected Accuracy.

In this section, we will outline a very crude, yet effective, method of determining expected accuracy. This is a theoretical measurement, meaning it is derived mathematically (preferably) before the experiment is carried out. Expected accuracy tells you ahead of time within what range you can expect your answer to fall. No experiment is perfect, since they all rely on human performance and ability as well as relying on the precision of your measuring devices. This is best illustrated by example.

Consider the experiment just carried out. In this case, three measurements were taken to arrive at an answer: time, distance and known mass. Imagine that a small error was made in the timing (a very possible situation, since humans all have some measurable reaction time between when the mass began to move and when the timer was started). If a mistake was made in the timing, you would expect that mistake to cause an error in your final answer. Suppose you knew that your reaction time was 0.1 sec. If you assumed that, then you would expect a certain amount of error in your experimental mass. That expected error is the expected accuracy of the lab. Philosophically, what we are saying is this: "I know I will have some errors in my measurements, and therefor some errors in my answers. If my answers are within the error I expect my measuring mistakes to cause, I can say the lab was a success".

The expected error concept gives you an idea of the overall success of a lab. Imagine that you did a lab and came up with a 50% error. Was that good or bad? Most students would say bad, but it is possible that a lab might have an expected error of 80%. This could arise if the lab was very complicated (mathematically) and relied heavily on one measurement. One small error in that measurement might be magnified by using that measurement in the calculations. On the other hand, a 1% error might be terrible in a lab that had an expected error of 0.02% (such a lab might have extremely precise measuring devices).

To calculate the expected percent accuracy is a very difficult task (mathematically), so we will engage in a simplified version for this lab. What we do is to take the average measurement for one of our quantities (in this case time) and calculate the unknown using that quantity (the unknown mass). We will call this m_1. We then determine how inaccurate we might have been in measuring the time (say 0.1 sec) and add it to our average time. We use this time to find the unknown and call it m_2. Next we do the same thing after we subtract the inaccuracy from the average and call the unknown m_3. What we are saying is that our expected inaccuracy if our time was too high it would yield a result of m_2 and if our time was too low, it would yield m_3. We then find a percent error for m_2 using m_1 as the accepted and another percent error for m_3 with m_1 as the accepted. These percents (one positive and one negative) tell us that we can expect this lab to yield results between these two values. They can also tell you if your error was too high or too low. It is possible that a time that is too high might mean a mass that is too low, your expected accuracy will tell you which direction your timing error will send your experimental value off the mark. Before we calculate this for this particular lab, we should mention that to do this properly, you would need to add in the expected errors for each measurement (and determine which way it would cause the error to go - in some cases you would need to add errors in one measurement and subtract them in another to calculate the answer properly).

Lab Calculations:

Calculate the expected accuracy for this lab after determining the expected error in timing. Do your experimental errors fall within this range? Was your timing too quick or too long? How do your errors compare with the accepted ones? Comment on any discrepancies.

Chart to Assist in Calculating Expected Accuracy Bounds

Average time value	
"Accepted" mass value	
Predicted error in time	
High time value	
Resulting mass value	
Low time value	
Resulting mass value	
"High time" percent error	
"Low time" percent error	