Chapter 12: The Conservation of Momentum

Momentum

When discussing interactions between objects, sometimes it is difficult to determine all the forces and variables that would be necessary in order to use Newton's Laws to determine the outcome of a situation. This is especially true in the case of a collision or explosion, where the forces (and thus the accelerations of each piece) are not constant through out the entire event. In such a case, we have a way to discuss the entire situation as one object, allowing us to make some very accurate, yet general, conclusions about the outcome. In order to do this, we need to define a system (the student may remember that this was already done when we discussed energy, but we repeat and expand the definition in this section), which is an object or number of objects that are considered as one thing (notice we say objects and not forces or influences, thus for example we would never count "gravity" as part of a system). An example might be two billiard balls. Although they are two things, it is appropriate to discuss the system of the two balls together as one thing. Another example is a rocket ship. This example shows us that it is important to correctly define our system. If we call the rocket ship a system, we must remember to include all the things in the rocket ship (the fuel, the astronauts, the air, etcetera). Very often we will work with closed systems, which means that none of the objects may leave the system. When we say leave the system, we are not talking in terms of physically moving, we mean that what ever we call our system cannot change over time. In the case of the rocket ship, if our system is the ship, then our system must contain all the fuel in the ship, even after it has been combusted and ejected. The fact that the fuel will be miles away is of no consequence, it must be continually considered if the system is closed.

If we apply Newton's Second Law to a system as a whole, we get the following:

$$\Sigma F=ma$$

Dropping the sigma (but remembering that the F is the sum of all the outside forces applied to the system) and making the substitution $a=\Delta v/\Delta t$, we get:

$$F=m\Delta v/\Delta t$$

or

$$F\Delta t=m\Delta v.$$

We call the term $F\Delta t$, the impulse applied to the system, the force multiplied by the time it is applied (one note: if the force varies the average force may be used) and we call the quantity $m\Delta v$ the change in momentum. Momentum is defined as mv, or mass times velocity and is usually abbreviated with a p.

$$p=mv.$$

Thus,

$$\Delta p=m\Delta v$$

or more correctly,

$$\Delta p=\Delta mv.$$

We move the delta out front in the most general case, since there can arise situations where the mass of the system we are considering is changing. It is interesting to take a side track here and examine that previous equation derived from Newton's Laws.

$$F\Delta t=\Delta p$$

or,

Impulse = Change in Momentum.

The impulse contains the forces involved and the time of the event, while the momentum contains the change in velocity that the system undergoes. Consider punching a pillow compared to punching a wall. In both cases, your hand goes from some velocity to zero. Thus if you throw both punches at the same speed, the change in momentum for your hand is the same in both cases. According to our equation, that makes the impulse given to your hand by the wall or the pillow the same. However, since the pillow is softer, the time in that case is greater. If two impulses are the same and the time is greater in one, it follows that the force in that case must be less (as your hand can attest to). The same concept applies in the case of hitting an air bag in a car, or more interestingly during a boxing match. During a match, you will notice that the boxers "roll with the punches". If, for example, a boxer is hit in the head, he will allow his head to go back with the punch. This extends the time of contact, lowering the force his face must give to stop his opponents hand (and by Newton's Third Law, this is the same force his face receives).

Another interesting side track on this matter involves bouncing objects. Consider a bullet that is fired at a wall and sticks into it. The change in momentum is then:

$$\Delta p=m\Delta v$$

or

$$\Delta p=m(v_f-v_i)$$

but since $v_f=o$,

$$\Delta p=-mv_i$$

(Why negative?). Now consider the same bullet being fired at a solid steel wall and imagine the extreme (impossible) case where the bullet bounces back at the same speed with which it hit the wall. Now the change in momentum is:

$$\Delta p=m\Delta v$$

$$\Delta p=m(v_f-v_i)$$

But since $v_f=-v_i$ (Why negative?)

$$\Delta p=m(-v_i-v_i)$$

$$\Delta p=-2mv_i$$

Thus for bouncing, <u>the change in momentum is double</u> the value of the case where the bullet sticks in the wall (in the more realistic case where the final velocity is less than the initial, the change in momentum is still greater, somewhere between one and two times the original). Since the change in momentum is greater, the impulse required is greater. This means the either the force or the time is greater in the case of bouncing as opposed to sticking together. It is usually the case that the force is greater, instead of the time (although it could be any combination). So bouncing objects require more force than objects that stick together. This can be seen conceptually by considering the fact that if the bullet sticks, the wall only had to stop it. But if the bullet bounces, the wall had to bring it to a stop and then "throw" it back. One interesting application of this is when police occasionally use rubber bullets to disperse a violent mob. The bullets do not penetrate, but they do bounce off the victims. Rubber bullets actually <u>hurt</u> more than regular bullets, but since they do not enter the body, they do not do anywhere near as much damage.

Thus far we have been discussing momentum changes in closed systems. The most useful aspect of momentum, however, is in closed, isolated, systems. An isolated system in one that has no outside forces affecting it. Thus it would be difficult to discuss isolated systems on the earth, since we always have gravity acting on the system. Another way to define isolated systems is one where all of the forces involved in the system are <u>internal</u> and none originate from the outside. Although in practice we rarely see these situations, many real life events come close enough to approximate them with this technique. If there are no outside forces, then

$$F=0$$

and

$$F\Delta t=\Delta p$$

becomes

$$\Delta p=0.$$

This rather simple looking equation is one of great importance and practicality. It is commonly called the Conservation of Momentum. It simply states that the total momentum of a closed, isolated system remains constant at all times, or that the momentum never changes. The student should be aware of the differences between this situation and the ones described earlier. The previous examples often viewed a system consisting of only one object. Thus they were not isolated. Consider the following example to compare the differences.

EX DA2.) Consider three magnets resting on a frictionless table. Considering the three together as a system, discuss the following:

a.) The forces on the system and the internal forces
b.) The motion of the individual magnets and the motion of the system.
C.) The application of the conservation of momentum.

Now repeat the question considering the magnets being placed near a very strong fourth magnet, which is not considered to be part of the system.

A tabular comparison of the two situations (how to view things as either isolated or not isolated) is presented below.

Summary Table of Momentum Concepts

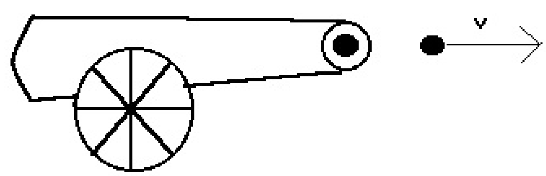
Situation:	two bowling balls collide	two bowling balls collide
System:	both balls	one ball
Outside Forces:	no	yes (force of second ball acting on first)
Momentum:	conserved	not conserved
Equation:	$\Delta p=0$	$F\Delta t=\Delta p$

It is important for the student to be able to differentiate what system is isolated or closed so that they can tell which equation to use and how to solve problems using the conservation of momentum. Below are some examples:

1.) A child kicks a ball: If we wish to look at the ball, the system is not isolated. We need to use $F\Delta t=\Delta p$.
2.) A child on ice skates throws a basketball forward: If we look at both the child and the ball as one system, it is closed and isolated (we can ignore gravity since there is another force, the normal force, to counteract it). We us $\Delta p=0$.
3.) A car hits a barrier: Not isolated.
4.) One car hits another: Not isolated because friction will act on the cars.
5.) A hand grenade explodes: Isolated, provided that friction can be ignored as well as gravity.

We will now begin to look at situations like the one shown in the middle column of the table, where momentum is conserved. Recalling that the Δp in the equation $\Delta p=0$ is the total momentum of the <u>system</u>, we can begin to make us of this equation.

EX DB.) Suppose a 1300 kg cannon fires an 80 kg cannon ball horizontally at 60 m/sec (with respect to the earth). What happens to the cannon? (consider the cannon to be resting on a frictionless surface)

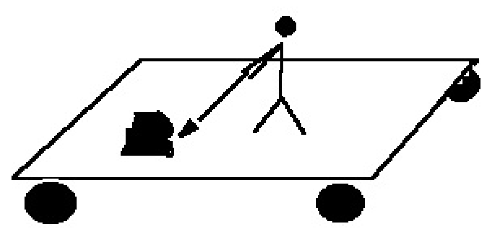


Thus we see that the cannon fires and moves backwards in order to conserve momentum. This result should come as no surprise, since our knowledge of Newton's Laws should have told us this beforehand. What is interesting, however, is that to use Newton's Laws directly, we would have had to know the exact force of the cannon (which would not have been constant) and thus the problem would have required a thorough knowledge of Calculus (unless we knew the average force and the time during which the force was in effect) to solve.

EX DC.) If a 10,000 kg railroad car full of coal rolls down a hill and collides (at v = 4 m/s) with an empty car (3000 kg) at the bottom in such a manner that they stick together, what will be the final velocity of the two cars?

Now consider another example, one with multiple events.

EX DD.) A boy (100 kg) stands on a 2000 kg railroad car filled with coal. What would happen if he began shoveling the coal out of the car? How fast wold he be going if he shoveled out on 100 kg shovel full at 2 m/sec (a 100 kg shovel of coal is obviously unrealistic, but it is used to make a point)? How fast would he be going after the second shovel?



This problem could have been done in two different ways, depending on the frame of reference that was chosen. It is an interesting exercise to try it in both frames and then compare your answers. When we deal with multiple situations, it is important to keep your frame of reference in mind, as in the previous problem. Velocities are always relative to something, and that something can be different for different objects.

As a review concerning momentum, as well as a number of other topics discussed previously, consider the next example.

EX DE.) Two magnets are resting on a frictionless table as shown below, with $m_1>m_2$ (where the m_1 is the mass of magnet 1 and m_2 is the mass of magnet 2. Discuss the following:

a.) Label the forces and discuss their magnitudes at the instant they are released.

At the instant before they collide,

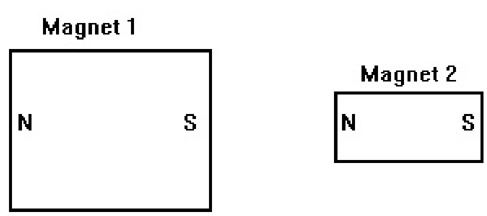
b.) Discuss their momenta.

c.) Discuss the impulse they underwent.

d.) Discuss their changes in momentum.

e.) Discuss their kinetic energies.

f.) Supposing the two collide, what can be said about their resulting motion?

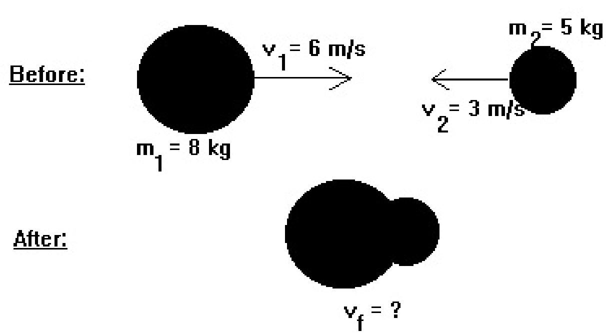


It is easy to become confused in the above problem, because the questions cause you to shift your thinking between three different objects. Some questions require that you think about the two magnets separately, some require that you consider them to be a system. Each magnet individually has outside forces, a change in momentum, and an impulse imparted to it. The system of the two magnets has no outside forces, no impulses and thus momentum is conserved. Be sure you are clear in your mind about these concepts when discussing these types of problems.

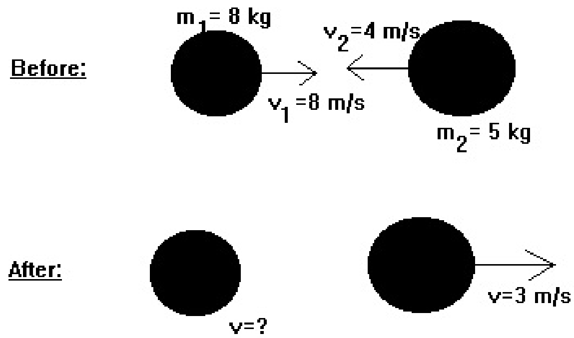
A few other comments should be made regarding the previous problem. First, the question about their change in momentum can be answered two ways, either directly (they began at zero and ended up with opposite momenta) or in a roundabout fashion (they underwent the same force for the same amount of time, thus having the same impulse in opposite directions and therefor had the same change in momentum because change in momentum equals impulse). This points out rather clearly that all of these concepts are interrelated, they are all connected to each other in some fashion. The second note applies to the last question. Since it is a closed, isolated system, the system as a whole can undergo no changes in momentum. It began with zero momentum, it must end with zero momentum.

Perhaps the greatest utilitarian purpose of the Conservation of Momentum involves collisions. Collisions are either elastic or inelastic. An elastic collision is one where kinetic energy is conserved (an impossible situation in real life, but one where very often the deviation from perfection can be easily ignored if it is small) and a perfectly inelastic collision is one where there is no kinetic energy conserved (or alternatively, where the minimum amount of kinetic energy is left over after the collision that still allows for the conservation of momentum). In the real world, most collisions fall somewhere in between the two, but for our purposes we will call a collision where the Kinetic energy is conserved an elastic collision and any other one will get the label inelastic. We will begin collision problems by doing two very simple inelastic type problems and then using the concept of an elastic collision to determine a general result.

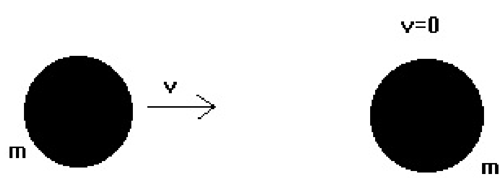
EX DF.) Two balls collide and stick together as shown in the diagram below. Determine the resulting velocity of the composite object.



EX DG.) The two balls below collide as shown. Determine the velocity of the ball labeled m_1 after the collision. <u>Explain the result?????</u>



EX DH.) Consider two balls of mass m. One is initially at rest and the other is moving at v. If they undergo an elastic collision, show that the v_f of ball 2 equals v and the v_f of ball 1 is 0. Also show that there are an infinite number of other possibilities that satisfy the conservation of momentum, but would not satisfy the elastic condition.



By showing that there are an infinite number of solutions that satisfy the conservation of momentum, but only one that satisfies both conservation of momentum and the elastic condition, we see why both are needed to solve a problem of this nature. Consider it this way. The conservation of momentum chooses all the correct answers, and the amount of elasticity in the collision determines exactly which one of the correct answers is the solution in a certain situation.

After looking at this problem, the next logical step would be to try the case of both balls moving towards each other. If we try this directly, we will find that we are quickly bogged down in an algebraic mess. However, if we switch reference frames, we will find that we have already done all the work.

EX DI.) Switch reference frames to find the result of the following elastic collision.

We see what happened here is that the two balls switched velocities. This is indeed a general result, provided that the two balls are the same mass and that the collision is elastic. We should have also learned a number of things about switching reference frames in momentum problems.

Notes on Switching Reference Frames for Momentum Problems

1.) In order to switch into a reference frame where one object is standing still, you need to add the negative velocity of that object to all objects in the problem.
2.) At the end of the problem, switch back out of your temporary frame by adding the original velocity back to every object.
3.) This technique can be used for all momentum problems.
4.) This technique is vector in nature, and can be used even when the collision does not occur along a straight line, provided the velocities are added as vectors.
5.) This technique shows something very important: momentum is a relative quantity, it depends on your frame of reference.

Going back to our original method of derivation, let us now look at a case where the balls are not of the same mass.

EX DJ.) Find an equation for the general case of two unequal masses colliding, with one at rest. Then analyze the result for $m_1>m_2$ and $m_2>m_1$.

Once we have derived the results above, namely:

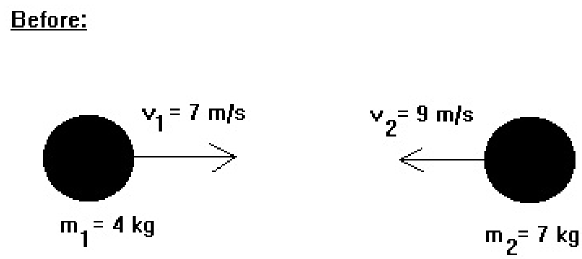
$v_{1f}/v_{1i} = (m_1-m_2)/(m_1+m_2)$

and

$v_{2f}/v_{1i} = 2m_1/(m_1+m_2)$

we can use them for <u>any elastic collision.</u> Although the formula were derived for a case of m_2 at rest, we can always switch into a frame where this is case, solve the problem and switch back out.

EX DK.) Switch reference frames to determine the result of the following elastic collision.



Looking at the above equations, we should take a moment to examine some special (or extreme) cases. I will warn the student ahead of time that the result found here are only valid in <u>extreme</u> situations, but are useful for determining approximate answers.

EX DL.) Using our derived general results, determine the final velocities for both objects in the following collisions:

a.) A marble is fired at a bowling ball ($m_1 << m_2$)
b.) A bowling ball is fired at a marble ($m_1 >> m_2$)
c.) A marble is fired at v_1 towards a bowling ball moving at v_2.

These results can be used for elastic collisions that fit the extreme conditions, but no other time. It is interesting (but perhaps a little confusing) to consider these results in another way. These results give us the boundaries for elastic collisions. For example, part a.) gives us the boundaries for the lowest possible mass of the projectile, part b gives us the bounds for a huge projectile (remembering that what really counts is the comparison of one mass to the other, not the value of the mass). The chart below summarizes our results.

Solutions for Elastic Momentum Problems

Collision Conditions	Final Velocity of Projectile (m_1)	Final Velocity of Target (m_2)
$m_1 << m_2$	$-v_i$	0
$m_1 = m_2$	0	v_i
$m_1 >> m_2$	v_i	$2v_i$

From the chart we see that the projectile will always have a velocity between $-v_i$ and v_i and the target will be between 0 and $2v_i$. If you ever get an answer out of these ranges, you screwed up somewhere. Now let us use the extreme cases to solve an interesting example.

EX DM.) What would happen if you placed a tennis ball on top of a basket ball and dropped the combination? (Consider the collision to be elastic, and use the $m_1 >> m_2$ approximations. Also assume there will be a minute amount of space between the two balls when the basket ball hits the floor.)

Momentum, as mentioned previously, is a vector quantity, and is thus applicable in two or three dimensions if we deal with momentum as a vector. The procedure for dealing with a two dimensional case is to resolve the components of the momenta involved onto x and y axes (which are arbitrarily positioned) and then to use the fact that if the Conservation of momentum is true in two dimensions, it must be also true for each dimension separately (an astute student might remember that this was also the case for projectile motion, and they might begin to wonder how many other quantities can be dealt with separately in each direction and why this is so). Thus our equation for the Conservation of Momentum is actually two separate equations (as all 2-D vector equations are), as such:

$$\Delta p=0$$

also means:

$$\Delta p_x=0$$

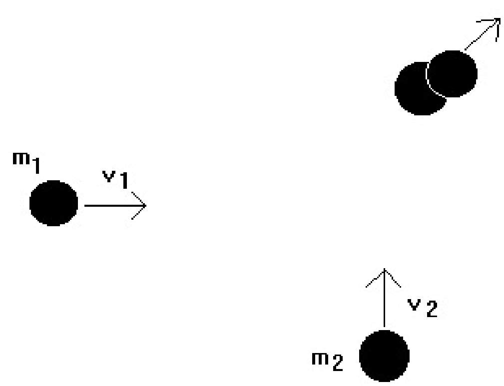
and

$$\Delta p_y=0.$$

An example of how to use this is shown below.

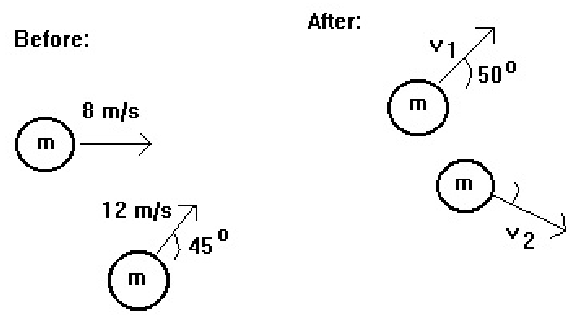
Inelastic collisions are handled in the same way, remembering that you cannot use the conservation of kinetic energy as a guide to constructing equations.

EX DO.) If the two particles below collide and stick together, what will be the velocity of the final combination given m_1 = 2 kg, m_2 = 5 kg, v_1 = 3 m/sec, v_2 = 4 m/sec ?



A second example begins to make things more complicated.

EX DN.) If the collision below is elastic, find the velocity of each ball after the collision given that m = 4 kg and v_2 makes an angle of 40^0 down from the horizontal. (MO7)

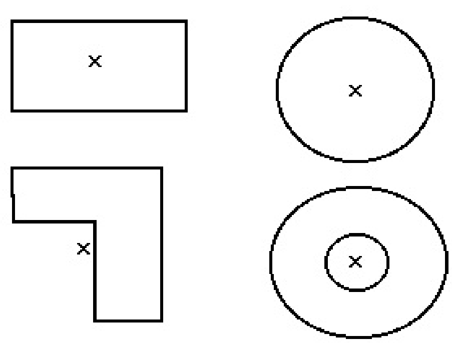


The problems above should show you how to deal with cases in two dimensions. There is an interesting conclusion that is apparent in this problem, although it is not proven. When objects of the same mass collide elastically in two dimensions, they always go off at right angles to each other. This can be easily seen on a billiards table, where the cue always goes off at 90^0 to the ball it strikes. Although the table is not a perfectly elastic situation, and the result can be affected by other factors (such as spin), the outcome is close to the ideal case. There is a way to prove this result, but we will pass it up for now.

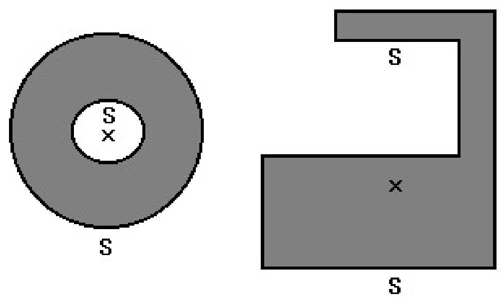
It is important to remember that the above problems are vector in nature (if I haven't reminded you of that recently) and thus can be solved as any vector problem can be solved. It is interesting to redo those past two examples graphically in the spaces below. It is suggested that you do the last example first.

EX DP.) Redo the two previous vector problems graphically in the space below.

Before we leave momentum, let us spend a few minutes looking at some of the interesting aspects of the conservation of momentum. The first involves the concept of the center of mass. The center of mass is an imaginary point (on or off an object) where all the mass appears to be concentrated. Consider for a moment, a brick. If we were to ask where exactly on the brick gravity was acting, the answer would have to be everywhere. Each and every atom in the brick is being pulled by the force of gravity between it and the earth. This obviously makes working with the force of gravity a complicated endeavor. It would be more convenient to average the force of gravity and have it focused on one spot so that we could deal with one force. That one average spot turns out to be the center of mass (actually, it is the center of gravity, but in most instances these will be the same). The center of mass is the one average point where the gravity seems to be pulling on the object. An easy way to understand the center of mass is to say that it is the balancing point of an object. If we try to balance a meter stick horizontally on our finger, we must support it at the 50 cm mark (providing it is uniform). This is the center of mass. Notice that we can also balance the meter stick vertically if the center of mass is directly above a point of support. Below is a diagram of a number of objects with their centers of mass marked with an x. Notice how the center of mass need not be on an object. It is simply an imaginary point.

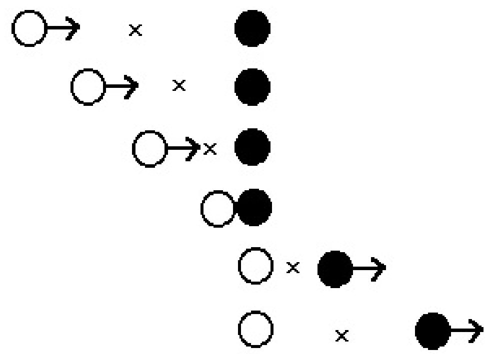


You might ask, "If the center of mass is a balancing point, how can it be off the object?" Reread the line above concerning balancing. It states that the object will balance if the center of mass is above a point of support. An object will balance as long as the support is directly below the center of mass. Not only that, but the object will balance if the center of mass is below the point of support. The diagram below shows two objects that can be balanced by placing your finger at either s (the center of mass is marked with an x).



One other interesting aspect about the center of mass is that if it is supported from above, we generally call that a "stable" situation, meaning that if it is disturbed slightly it will return to normal of its own accord. The actual condition for a stable situation is that the center of mass must rise if disturbed. Examining the two situations above will give you a good general idea of this condition. It is very easy to balance the objects with your fingers at the top S, while balancing with your finger at the bottom S requires some agility.

But what does all this have to do with the Conservation of momentum? Consider the center of mass of two balls of equal mass with one approaching the other at some velocity v. The center of mass of this system (to envision the center of mass of a system, consider all the elements of the system as connected by massless rods and think about balancing it) is midway between the two balls and moving in the same direction as the traveling ball with half the speed (think about this, it does make sense). Momentum tells us that if the collision is elastic, the stationary ball will take off with the speed of the original ball, while the first ball will become stationary. What is the center of mass doing after the collision? It is moving in the same direction as before with the same speed. The diagram below is certainly worth a thousand words in clearing this up. Both balls are the same mass and the center of mass is again located with an x.

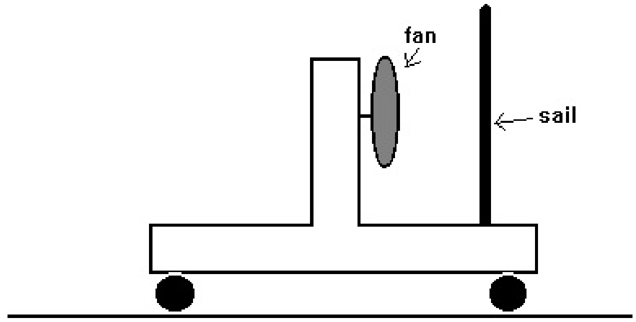


This shows us that during a collision, the center of mass remains totally unchanged. Think about the simplest situation, two balls of the same mass approaching each other at the same speed. The center of mass of the system is in the middle, and at rest. After the collision, the two balls switch velocities and continue, leaving the center of mass at rest. This is true in all situations of closed, isolated systems. Another interesting aspect of this is in the case of an explosion. Consider a hand grenade resting on the ground. When it explodes, its pieces will fly off in such a manner as to keep the center of mass at rest! Furthermore, imagine a hand grenade that is thrown up in the air. When it explodes, it pieces will move in such a manner to keep the center of mass on its intended path (a parabola). Another instance is throwing an unbalanced object, such as a base ball bat. If you throw it straight, it will spin. In actuality, its center of mass moves in a straight line while the object spins around it.

The fact that the motion of the center of mass is unchanged by a collision within the system is not just a fluke. In fact, it reminds us how intertwined all of our formulae are. Consider Newton's Laws as they apply to a system that is closed and isolated. Since no forces act on the system from the outside, the first law tells us that its motion remains unchanged (i.e. no acceleration). We just need to remember that when we say "its" motion, we are referring to the motion of the entire system.

The other note to make is that momentum allows us to solve problems we have done in the past with greater ease. Consider the example below

EX DQ.) Discuss the motion of the fan cart with the sail attached by using the conservation of momentum. Also discuss exactly why the force on the sail is greater than the force on the fan.



The last example to discuss is simply a fun exercise.

EX DR.)   Imagine two astronauts play catch in outer space with a huge medicine ball (the same mass as the astronauts). How long would the game last?

Assignment #12

1.) A 70 g lump of clay falls to the ground from 6 m. If it takes 0.20 sec. to come to a stop, what was its a.) change in momentum? b.) impulse? c.) force experienced?

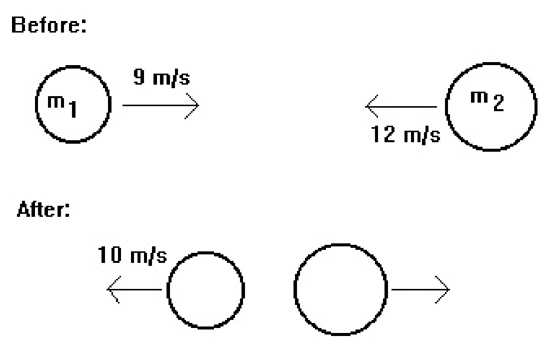
2.) Consider a car accident where a car hits a barrier and comes to a stop. What is the percentage increase in the force involved in a collision where the car began at 55 mph compared to a car traveling at 30 mph. Assume both cars come to a complete stop and that the time for the collision is the same in both instances.

3.) If you applied a force of 30 N to a 15 kg object for 20 sec, what is the maximum impulse you could impart? What is the maximum change in momentum that you could effect? What is the highest velocity it could reach if it began at rest?

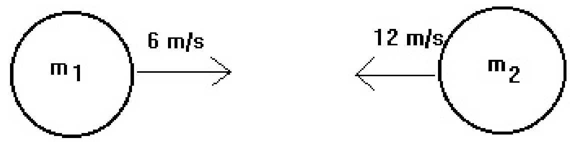
4.) Consider the motion of a pendulum that swings back and forth without friction. Does its motion violate the conservation of momentum? Why or why not?

5.) In a lab experiment, a cart (400 g) traveling at 2 m/s strikes another cart at rest. After the collision the first cart continues at 1.1 m/s. If the collision is elastic, find the mass of the second cart and its new velocity.

6.) Determine the velocity of M_2 after the collision, given that M_1=5 kg and M_2=7 kg. (MO3)



7.) If the two balls collide and stick together, what will be the final velocity given that m_1 = 9 kg and m_2 = 7 kg?(MO8)

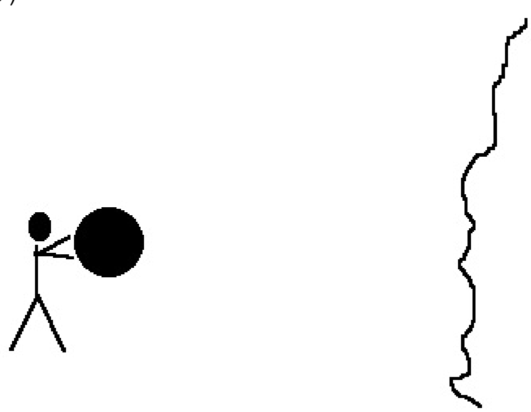


8.) A 15 kg ball traveling to the right at 12 m/sec collides elastically with a 10 kg ball traveling to the left at 20 m/sec. Determine the velocities of the balls after the collision.

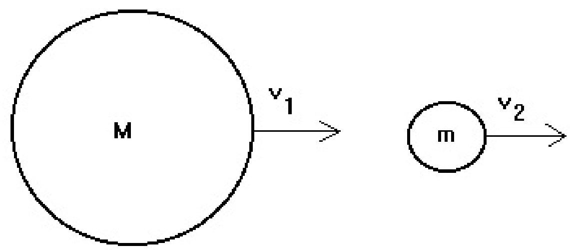
9.) Consider a wagon rolling across a frictionless surface in the rain. As the wagon fills up, the velocity decreases. Is this a closed, isolated system? Is momentum conserved? Do not just answer each of the above questions, explain and justify your answers.

10.) A 6 kg ball travels at 12 m/sec to the right catches up and collides elastically with a 8 kg ball traveling the same direction at 7 m/sec. What are the final velocities of the balls after the collision?

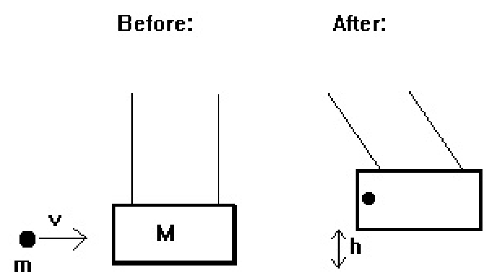
11.) Explain in detail what would happen if an astronaut tried to play catch by bouncing (elastically) a very heavy ball (the same mass as the astronaut) against an asteroid (much more massive than the astronaut). (MO6)



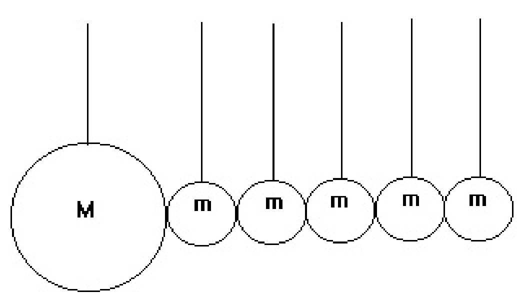
12.) By switching reference frames and using the fact that M>>m, determine the velocity of m after the collision (assume elastic). (MO5)



13.) Imagine that a small bullet (m) is fired into a heavy block of wood (M) at velocity v. The bullet sticks in and the block, suspended by two strings, swings like a pendulum up to a height h. Use the conservation of energy and momentum to determine an equation for the speed of the bullet before impact (v) in terms of m, M, h, and g.

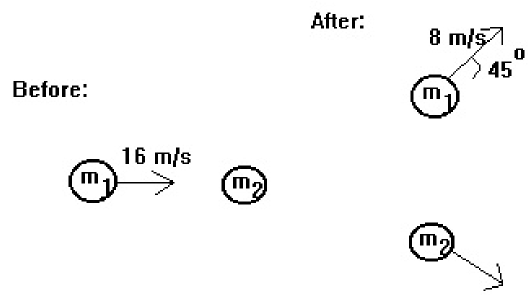


14.) One of the most common examples of an elastic collision is the five suspended metal balls. Suppose that we had a variation of these as shown below. Instead of five equal masses we now have one large mass (M) and a number of smaller masses (m). Prove that if the large ball is set swinging, a number of smaller balls must leave the other side such that X = M/m where X is the number of small balls. Conditions: X must be an integer, the collision is elastic and assume that M comes to rest. (E3*)

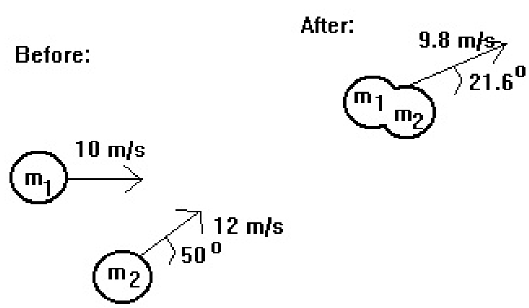


15.) A proton with a speed of 300 m/sec collides elastically with another proton at rest. The two protons bounce off each other and the incoming proton goes off at a 60 degree angle to the horizontal. (a.) What is the direction of the velocity of the stationary proton after the collision ? (b.) What are the speeds of the two protons after the collision ? (HR86)

16.) In the collision below, momentum is conserved but not kinetic energy. What is (a.) the velocity of ball B after the collision and (b.) the percentage of kinetic energy left after the collision? (M_1= 6 kg, M_2=5 kg) (MO4)

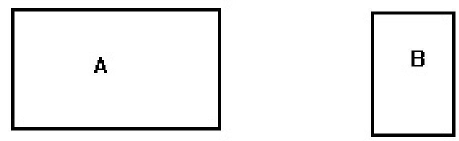


17.) Determine the mass of particle 1 if m_2 = 2 kg in the collision below.



18.) Consider two astronauts in space. What will happen if they play catch with a baseball? Eventually, the game will end with both of them and the ball flying off into space. Imagine that they used a ball that was 1/3 of their mass and that the two astronauts had the same mass. Determine how many throws they could throw and catch before the game would be over.

19.) Consider the two boxes below on a frictionless surface. Box A has twice the mass of box B and both boxes start at rest.



a.) Describe what would occur, in terms of momentum and velocity, if you applied an equal impulse to both boxes.

b.) Describe what would occur, in terms of impulse, momentum and velocity, if you applied an equal force to both boxes but applied the force to box A for twice as long.

c.) Describe what would occur, in terms of force, impulse and momentum, if you brought both boxes to the same speed in the same amount of time.

20.) Decipher: "Arien precipitation is conducive to spurring the production of dulcet Taurian flora." (DNCTHWG)

Activity #12
Momentum with Soda Tops

The purpose of this activity is to investigate the conclusions made about the conservation of energy by staging collisions between two white plastic soda bottle tops.

Procedure:

1.) Using two identical bottle caps on a smooth table top, put one at rest and send the other sliding across the table at it so that it hits it straight on. Observe what happens after the collision.

2.) Slide the two caps at each other at different speeds and observe what happens.

3.) Put a small piece of tape on the table and place one cap on the tape. Send the other cap sliding at it so that it glances it and the two caps go off at angles. Put small pieces of tape under the landing spot of each cap. Using a ruler and chalk, mark a line from the starting point to each landing spot and measure the angle between the two paths.

4.) Add a small amount of sand (or a small weight) to one cap and use that cap as a target. Slide the other cap at it (so that it hits head on) at different speeds and observe the results.

5.) Repeat the previous step using the heavier cap as the projectile.

Conclusions: How close were your results to the expected in each case? How elastic were the collisions? Were the angles in step 3 90° as expected? Was it possible in step 4 for the projectile to ever continue forward? Was it possible in step 5 for the projectile to ever bounce backwards? Was this system closed? Was this system isolated? Was momentum conserved? Was kinetic energy conserved?

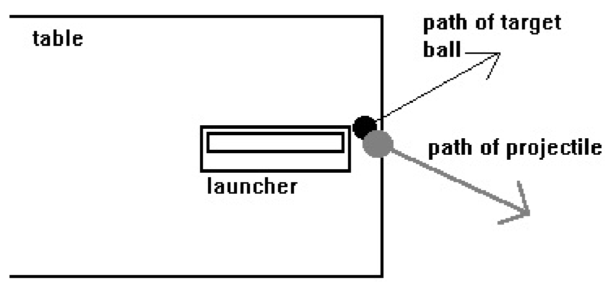
Lab #9 - Two Dimensional Momentum Conservation

In this lab, you will determine the percent of kinetic energy conserved in a number of collisions to determine the extent of elasticity in each case. You will also use the conservation of momentum to determine the initial velocity of an incoming ball.

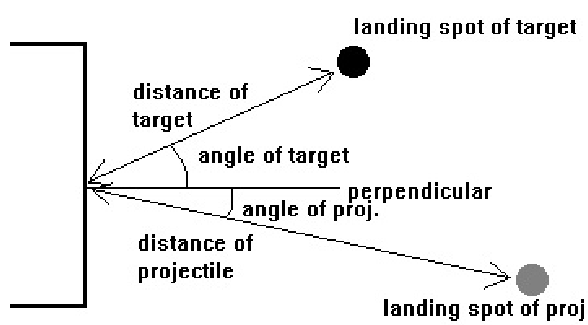
Procedure:

1.) Set up the projectile launcher horizontally on a table.

2.) Place a target ball (aluminum) on the edge of the table so that when the projectile is launched, it will strike the ball a glancing blow and the two balls will go off at angles. (see diagram)



3.) Fire the projectile and note the location where both balls land. From this information measure the distance traveled by both balls (along the floor) and the angle they made from the perpendicular to the table. (see diagram 2)



4.) Measure the height of the table and the mass of each ball.

5.) From the information measured, determine the velocity of each ball after the collision and use the conservation of momentum to determine the velocity of the projectile.

6.) Repeat this procedure four times with target balls of different materials.

7.) Fire the launcher five times without a target ball to determine an accepted value of the projectiles velocity. Average those found in the above step and determine a percent error.

8.) Determine the percent of kinetic energy conserved in each collision.

9.) Use the conservation of momentum in the y direction (parallel to the edge of the table) to see if the y components of the two balls match (why should they?) Using the projectile's y velocity as the accepted, determine a percent error for the targets y velocity.

Height of table	
Mass of projectile	

Ball	Mass of Ball	Target dist.	Target angle	Target veloc.	Proj. dist.	Proj. angle	Proj. veloc.
Al							
Brass							
Wood							
Steel							
Cork							

Ball	x comp Target Momen.	y comp Target Momen.	x comp Proj. Momen.	y comp Proj. Momen.	Initial veloc. of Proj	% of Conser. Kinetic Energy
Al						
Brass						
Wood						
Steel						
Cork						

Table for Determining Accepted Projectile Velocity

Trial	Distance Proj. Travels	Proj. Velocity
1		
2		
3		
4		
5		
Average Velocity		

Hints on Conclusions: Be sure to mention all aspects of the lab in your conclusions. There are three areas being investigated: 1.) the elasticity of collisions between balls of different materials, 2.) the accuracy of determining initial velocity using the conservation of momentum and 3.) the accuracy of using the conservation of momentum in this particular set of collisions. Each area raises its own questions. A few examples of the questions raised are:

1.) Does material play a role in elasticity?
2.) Did the conservation of momentum yield the correct result for the velocity of the projectile before the collision? How would this affect your results in part 1? Was the device very precise in giving the same velocity each time?
3.) Was the conservation of momentum satisfied? If it was not perfect, why not? Was it different for each of the different balls? If so, why? What does this tell you about our system? What does this say about outside forces?

Impulses and Jumping and Landing

In this short activity, you will measure the impulses and changes in momentum for a person jumping and a person landing using a force platform.

Note: You will have to use your weight in this activity. If you don't want to reveal your weight, have someone else in the group do the activity for you.

Procedure:

1.) Find your weight by using the bathroom scale and covert your weight to mass using 1 pound = 0.454 kg. Record this information.

2.) Stand on the force platform and have someone zero the platform out on the computer (so that your weight is taken out of the calculations). You do this by going to "calibrate sensor>one point (off set)>stand on platform>click on "read from sensor". Set the sensor to 100 Hz, low sensitivity.

3.) Start the computer and jump as high as you can in the air and land back on the platform. Stop the recording. Have the computer graph force versus time.

4.) Check and make sure that the graph is acceptable, then find the area under each spike (ask your teacher how to do this, the computer can do it for you by highlighting the points and having it integrate). Make sure both calculations show up on the graph.

5.) Print out the graph and answer the following questions on a separate sheet of paper. Turn in the answers attached to the original graph.

Questions

1.) On your graph, clearly label each of the following points: a.) You begin your jump, b.) you leave the platform, c.) you hit the platform on the way back down, d.) you come to a complete stop.
2.) From your graph, determine the time you were in the air.
3.) Using half of this time and the equations from a previous chapter, find the velocity with which you left the platform.
4.) The integrated areas on the graph correspond to the impulses of you jumping and landing. What is the value of each impulse?
5.) What is your change in momentum for jumping?
6.) What is your change in momentum for landing?
7.) What was your speed when you left the platform? (Use answer from problem 5 for this.)

8.) What was your speed when you landed? (Use answer from problem 6 for this.)

9.) Find a percent error for your speeds from 7 & 8 by comparing them to the answer to question 3 (use the answer to question 3 as the accepted value).